Black Holes in an Expanding Universe

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An exact solution representing black holes in an expanding universe is found. The black holes are maximally charged and the universe is expanding with arbitrary equation of state ($P = w\rho$ with $-1 \leq \forall\ w \leq 1$). It is an exact solution of the Einstein-scalar-Maxwell system, in which we have two Maxwell-type U(1) fields coupled to the scalar field. The potential of the scalar field is an exponential. We find a regular horizon, which depends on one parameter [the ratio of the energy density of U(1) fields to that of the scalar field]. The horizon is static because of the balance on the horizon between gravitational attractive force and U(1) repulsive force acting on the scalar field. We also calculate the black hole temperature.

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Black holes play a central role in astrophysics as well as string theory, and may possibly be created at the LHC [1]. Not surprisingly, they have been studied intensively over the past 35 years. Important progress has been made in understanding the Hawking process [2] and the states responsible for black hole entropy at the microscopic level [3]. However many problems remain unresolved: does cosmic censorship [4] hold, what happens when black holes collide, how does accretion of matter affect the thermodynamics of black holes, and how does it affect the growth of black holes? To answer these dynamical questions one needs time-dependent solutions of the Einstein equations containing black holes. In this Letter we shall focus on solutions representing black holes in a background Friedmann-Lemaître-Robertson-Walker (FLRW) universe.

There have been many previous attempts to obtain black holes embedded in the FLRW universe. The Einstein-Straus model is perhaps the simplest one [5]. It is a patchwork of Schwarzschild black holes with an FLRW universe. However, these black holes are time symmetric, and so they do not describe a dynamical black hole in a universe.

One well known black hole candidate in the FLRW universe is the McVittie solution [6], which is a spherically symmetric, time-dependent solution of the Einstein equations. The solution approaches an FLRW universe at "infinity", and looks like a black hole near the "horizon." However, as shown in [7], the McVittie solution is disqualified as a black hole (or a point mass singularity) in the FLRW universe. Recently, Sultana and Dyer constructed a more sophisticated black hole solution in a dynamical background by a conformal technique [8]. The matter content is null dust and ordinary dust. The solution tends to an Einstein–de Sitter spacetime asymptotically. This model, however, violates the dominant energy conditions.

Assuming self-similarity, we can show that a regular black hole may exist only in an accelerating universe, but this requires numerical study [9]. The analytic solution found by Carr and Hawking describes a self-similar spacetime with a regular black hole but it approaches asymptotically a "quasi" FLRW spacetime which has a deficit angle, but not an exact flat FLRW spacetime [10]. There are also discussions of "dark energy" accretion onto a black hole in a universe [11].

If a cosmological constant is present, we have the Schwarzschild–de Sitter (SdS) and Reissner–Nordström–de Sitter (RNdS) solutions [12,13]. Although these spacetimes are static, they may be converted by a coordinate transformation into the form of a black hole in an exponentially expanding universe [14]. Multi-black-hole solutions in a de Sitter universe were found by use of extremely charged black holes and their collision discussed [15,16]. This Kastor-Traschen (KT) solution is a time-dependent generalization of the Majumdar-Papapetrou solution, which describes extremely charged RN black holes [17]. Similar solutions were given in [18,19].

Another time-dependent cosmological black hole system was found from the compactification of intersecting brane solutions in higher-dimensional unified theory [20]. As clarified in [21] the global picture of dynamical solution describes a multi-black-hole system in the expanding universe filled by "stiff matter." We shall call it the MOU solution.

Here we generalize these two solutions and present an exact solution describing a cosmological multi-black-hole system with an arbitrary power law expansion. This is a solution of general relativity with a scalar field with an exponential potential and two Maxwell-type U(1) fields coupled to the scalar field. The solution has regular event horizons, approaches asymptotically an exact flat FLRW spacetime without a deficit angle, and no singularity exists outside the horizons.

The above known solutions take the following form:

$$ds^2 = -\bar{U}^{-2}d\bar{t}^2 + a^2(\bar{t})\bar{U}^2 d\boldsymbol{r}^2. \tag{1}$$

The KT solution with N black holes located at the coordinate position $\boldsymbol{r}_{(i)}$ ($i = 1, \cdots, N$) is given by

$$\bar{U} = 1 + \sum_{i=1}^{N} \frac{Q_{(i)}}{a|\boldsymbol{r} - \boldsymbol{r}_{(i)}|}, \tag{2}$$

where $Q_{(i)}$ is the charge of the ith black hole, and the scale factor of the background universe is given by $a(\bar{t}) \propto \exp(H_0 \bar{t})$ (H_0: constant). It is asymptotically de Sitter spacetime. The MOU solution discussed in [20,21] is given by

$$\bar{U} = \left[1 + \sum_{i=1}^{N} \frac{Q_{T(i)}}{a^4|\boldsymbol{r} - \boldsymbol{r}_{(i)}|}\right]^{1/4} \left[1 + \sum_{i=1}^{N} \frac{Q_{S(i)}}{|\boldsymbol{r} - \boldsymbol{r}_{(i)}|}\right]^{3/4},$$

where $Q_{T(i)}$ and $Q_{S(i)}$ are the conserved charges of time-dependent and static branes, respectively, and the scale factor is given by $a = (\bar{t}/\bar{t}_0)^{1/3}$, which also holds for an expanding universe with stiff matter.

By changing the time coordinate, these solutions can be rewritten in the form of a brane system, discussed in detail in [20], as

$$ds^2 = -U^{-2}dt^2 + U^2 d\boldsymbol{r}^2, \tag{3}$$

where

$$U = H_T^{n_T/4} H_S^{n_S/4}, \tag{4}$$

$$H_T = \frac{t}{t_0} + \sum_{i=1}^{N} \frac{Q_{T(i)}}{|\boldsymbol{r} - \boldsymbol{r}_{(i)}|}, \tag{5}$$

$$H_S = 1 + \sum_{i=1}^{N} \frac{Q_{S(i)}}{|\boldsymbol{r} - \boldsymbol{r}_{(i)}|}. \tag{6}$$

Here n_T and n_S are appropriate non-negative real numbers with the constraint $n_T + n_S = 4$, and t_0 is a constant. The transformation of the time coordinate is given by $t/t_0 = a^{4/n_T}(\bar{t})$, where t_0 is fixed as $t_0 = H_0^{-1}$ for the KT solution and $3\bar{t}_0/4$ for the MOU solution, respectively. Setting $n_T = 4$, we find the KT solution, while the case with $n_T = 1$ corresponds to the MOU solution.

Assuming the metric form (3)–(6), if we take an arbitrary real value for n_T (or n_S), we find that the scale factor a in the form of (1) is given by any power function, i.e., $a \propto \bar{t}^p$, where $p = n_T/n_S$. (We regard $p = \infty$ as an exponential expansion.)

When the universe expands with an arbitrary power by a scalar field, one needs an exponential-type potential [22]. In fact, if we have the universe filled by a scalar field with the potential

$$V = V_0 \exp(-\alpha\kappa\Phi), \tag{7}$$

the scale factor increases as $a \propto \bar{t}^p$ with $p = 2/\alpha^2$, where $\kappa^2 = 8\pi G_N$ is the gravitational constant. Thus we should choose $\alpha = \sqrt{2/p} = \sqrt{2n_S/n_T}$.

We shall therefore adopt the following action:

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{2\kappa^2}R - \frac{1}{2}g^{\mu\nu}(\partial_\mu\Phi)(\partial_\nu\Phi) - V(\Phi) - \frac{1}{16\pi}\sum_{I=S,T} n_I e^{\lambda_I\kappa\Phi}(F_{\mu\nu}^{(I)})^2\right], \tag{8}$$

where $g_{\mu\nu}$ is a spacetime metric, Φ is a scalar field with the potential $V(\Phi)$ given by (7), and $F_{\mu\nu}^{(I)} (I = S, T)$ are two Maxwell-type U(1) fields, which couple to the scalar field with the coupling constants λ_I. The vector potentials are given by $A_\mu^{(I)}$, and n_I are their degeneracy factors.

The metric (3) with (4) plus

$$\kappa\Phi = \frac{1}{2}\sqrt{n_T n_S/2}\ln(H_T/H_S), \tag{9}$$

$$\kappa A_t^{(T)} = \sqrt{2\pi}H_T^{-1}, \qquad \kappa A_t^{(S)} = \sqrt{2\pi}H_S^{-1}, \tag{10}$$

with (5) and (6) and $\kappa^2 V_0 t_0^2 = n_T(n_T - 1)/4$ is really an exact solution of the system (8), if we assume

$$\alpha = \lambda_T = \sqrt{2n_S/n_T}, \qquad \lambda_S = -\sqrt{2n_T/n_S}, \tag{11}$$

and $n_T + n_S = 4$ [23]. For $n_T = 4$ and $n_T = 1$, we recover the KT and MOU solutions, respectively.

The above solution with arbitrary n_T gives a multi-black-hole system in an expanding universe for which the scale factor and effective equation of state are given by $a \propto \bar{t}^p$ with $p = n_T/n_S$, and $P = w\rho$ with $w = \frac{2n_S}{3n_T} - 1$, respectively. Note that w takes an arbitrary value in the range of $-1 \le w \le 1$, corresponding to the value of $1 \le n_T \le 4$.

We summarize some typical solutions in Table I.

In order to discuss the spacetime found here in detail, in what follow, we consider a single black hole system. For simplicity, we assume that two charges are equal, i.e., $Q_T = Q_S =: Q$. We shall rewrite the metric (3) as

$$d\tilde{s}^2 = -\tau^2 U^{-2} d\tilde{t}^2 + U^2(d\tilde{r}^2 + \tilde{r}^2 d\Omega_2^2) \tag{12}$$

with (4) and $H_T = \tilde{t} + \tilde{r}^{-1}$, $H_S = 1 + \tilde{r}^{-1}$, where $\tau = t_0/Q$, $d\tilde{s}^2 = ds^2/Q^2$, $\tilde{r} = r/Q$, and $\tilde{t} = t/t_0$ are dimensionless variables. The metric (12) depends on only one parameter τ, whose physical meaning is given as follows: The energy density of the scalar field is uniform at $t = t_0$, which is given by $\rho_\Phi(t_0) = 3n_T^2/16t_0^2$. While the total density of the U(1) fields is evaluated on the horizon for the static extreme RN black hole with the charge Q as $\rho_{U(1)}|_H = 1/Q^2$. For the time-dependent black hole, $\rho_\Phi|_H$ and $\rho_{U(1)}|_H$ are different from the above values, but their orders of magnitude are still the same. Thus we can claim that τ is related to the ratio of two energy densities at the horizon as $\tau^2 \sim \rho_{U(1)}/\rho_\Phi|_H$ (see [21,24]). The limit of $\tau \to \infty$ gives the static extreme RN black hole.

The circumference radius $R = Q\tilde{R}$, which is a geometrically invariant variable, is given by

TABLE I. Some values of the typical parameters of a black hole system in a universe, for which expansion law and the equation of state are given by $a \propto \bar{t}^p$ and $P = w\rho$, respectively.

Type	n_T	n_S	α	λ_T	λ_S	p	(expansion law)	w	$\kappa^2 V_0 t_0^2$
I	0	4	∞	∞	0	0	(static)	0	0
	1	3	$\sqrt{6}$	$\sqrt{6}$	$-\sqrt{6}/3$	$1/3$	(stiff matter)	1	0
	$4/3$	$8/3$	2	2	-1	$1/2$	(radiation)	$1/3$	$1/9$
	$8/5$	$12/5$	$\sqrt{3}$	$\sqrt{3}$	$-2/\sqrt{3}$	$2/3$	(dust)	0	$6/25$
II	2	2	$\sqrt{2}$	$\sqrt{2}$	$-\sqrt{2}$	1	(Milne)	$-1/3$	$1/2$
III	3	1	$\sqrt{6}/3$	$\sqrt{6}/3$	$-\sqrt{6}$	3	(quintessence)	$-7/9$	$3/2$
	4	0	0	0	$-\infty$	∞	(de Sitter)	-1	3

$$\tilde{R} = \tilde{r}U = (1 + \tilde{t}\,\tilde{r})^{(n_T/4)}(1 + \tilde{r})^{(n_S/4)}. \qquad (13)$$

The curvature singularity appears at $\tilde{r} = -1$ and $\tilde{r} = -1/\tilde{t}$, where $\tilde{R}$ vanishes. Analyzing the behavior of trapping horizons in the limit of $\tilde{r} \to 0$ and near horizon geometry as in [21], we find the horizon radius ($\tilde{R}_+$ or $\tilde{R}_-$) which satisfies the following equation:

$$\tau(\tilde{R}_+^{(4/n_T)} - 1) = \tilde{R}_+^2, \qquad \tau(\tilde{R}_-^{(4/n_T)} - 1) = -\tilde{R}_-^2. \quad (14)$$

The spacetimes are classified by their causal structure into three types: Type I ($n_T < 2$), Type II ($n_T = 2$), and Type III ($n_T > 2$).

In Type I, there are two horizons, $\tilde{R}_+$ and $\tilde{R}_-$, which are the roots of Eqs. (14). Since $\tau > 0$, we find $\tilde{R}_+ > 1 > \tilde{R}_- > 0$. We show the horizon radii in terms of τ in Fig. 1(a). For Type II, if $\tau > 1$ there are two horizons, $\tilde{R}_+ = \sqrt{\tau/(\tau - 1)}$ and $\tilde{R}_- = \sqrt{\tau/(\tau + 1)}$, but if $\tau \le 1$, we find only one horizon, $\tilde{R}_-$. In Type III, if $\tau > \tau_{\rm cr}$, we find two roots $\tilde{R}_{+,1}$ and $\tilde{R}_{+,2}$ ($\tilde{R}_{+,1} < \tilde{R}_{+,2}$) for the equation for $\tilde{R}_+$ (14), where $\tau_{\rm cr} = n_T^{n_T/2}(n_T - 2)^{-(n_T-2)/2}/2$. From the detail analysis of spacetime structure, we find there exist two horizons; $\tilde{R}_-$ and $\tilde{R}_H = \tilde{R}_{+,1}$ ($\tilde{R}_- < 1 < \tilde{R}_H$), but $\tilde{R}_C = \tilde{R}_{+,2}$ is not a cosmological horizon except for the KT solution [24]. The cosmological horizon turns out to be time-dependent just as that in an accelerating universe [25]. On the other hand, if $\tau < \tau_{\rm cr}$, we find one horizon, $\tilde{R}_-$. If $\tau = \tau_{\rm cr}$, there are two horizons, $\tilde{R}_-$ and $\tilde{R}_{+,1} = \tilde{R}_{+,2}$ (degenerate) [see Fig. 1(b)]. As we see from Table I, Type I and III correspond to a decelerating and accelerating universes, respectively.

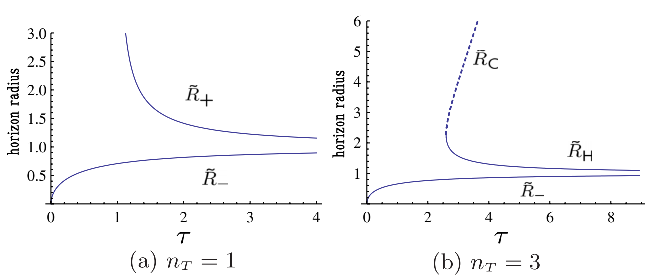

FIG. 1 (color online). Two horizon radii ($\tilde{R}_-$ and $\tilde{R}_+$ or $\tilde{R}_H$) for $n_T = 1$ and 3. $\tilde{R}_H$ and $\tilde{R}_C$, which is not a cosmological horizon, degenerate at $\tau_{\rm cr} = 3\sqrt{3}/2$.

Since the present spacetime is spherically symmetric and the near horizon is "static", we can calculate the surface gravity (see [21] for details). Hence we find the black hole temperatures on the horizons ($T_{\rm BH}$) by the surface gravity $\kappa_\pm$ as

$$T_{\rm BH}^{(\pm)} = \frac{\kappa_\pm}{2\pi} = \frac{n_T \tilde{R}_\pm^{-(2n_S/n_T)+1}}{16\pi\tau^2 Q}\left|2\tau\tilde{R}_\pm^{(n_S/n_T)-1} \mp n_T\right|. \quad (15)$$

We depict the behavior of the temperatures in Fig. 2. They are finite and vanish in the limit of $\tau \to \infty$, i.e., the extreme RN spacetime.

It may be interesting to discuss the thermodynamics because we can define the entropy and temperature in these time-dependent spacetimes. We can easily extend the present solution to arbitrary dimensions [24]. The details including the analysis of global structure and study of thermodynamical properties will be given elsewhere [24]. Some questions for future work are the following: (i) Can we find more realistic black hole solutions? It may be straightforward to include rotation (cf. [19]). This is under investigation. As for neutral black holes in a universe, it may be difficult to obtain the analytic solution because such a system is non-BPS state even in the static case, and the radius of a black hole increases in time due to accretion of matter. (ii) Can we extend black hole thermodynamics to time-dependent spacetimes? The present time-dependent solution may provide a good tool for analyzing this question. (iii) Can we discuss some dynamical process with the

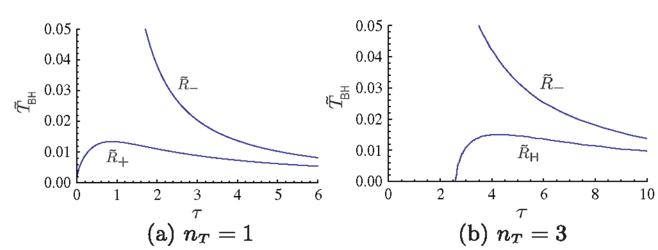

FIG. 2 (color online). Black hole temperatures on two horizons ($\tilde{R}_-$ and $\tilde{R}_+$ or $\tilde{R}_H$) for $n_T = 1$ and 3. $\tilde{T}_{\rm BH} = QT_{\rm BH}$.

present or extended solutions? Black hole collision can be discuss in the contracting universe ($t_0 < 0$) just as the KT spacetime. We can also discuss the brane collisions with multi time-dependent branes, which is a generalization of [26]. (iv) Some solutions have a link to intersecting brane systems in higher-dimensional supergravity model. If n_T is a non-negative integer, we may regard n_T and n_S as numbers of branes. It is true for $n_T = 1$, in which case we can derive the four-dimensional effective action (8) from compactification of the time-dependent M2-M2-M5-M5 brane system in 11-dimensional supergravity theory (see Appendix in [21]). Hence it may be interesting to see whether there is any fundamental or deep reason for this link.

Work along these lines is in progress.

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