Measuring Vacuum Polarization with Josephson Junctions

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We argue that the vacuum polarization by the virtual electron-positron pairs can be measured by studying a Josephson junction in a strong magnetic field. The vacuum polarization results in a weak dependence of the Josephson constant on the magnetic field strength which is within the reach of the existing experimental techniques.

The interaction of the electromagnetic field with the quantum fluctuations of a vacuum is responsible for many remarkable nonlinear phenomena described by the theory of quantum electrodynamics (QED) [1,2]. In particular, the polarization of vacuum by virtual electron-positron pairs results in the dependence of a charged particle interaction with the electromagnetic potential on the field strength and on the characteristic momentum transfer. This intrinsically relativistic effect is strongly suppressed and becomes observable only in very strong fields or in high energy processes. For example, in electron-positron scattering at energy close to the mass of the Z boson, which is 5 orders of magnitude heavier than an electron, the electromagnetic interaction is described by the "running" coupling constant $\alpha(M_Z) \approx 1/128$ rather than by the usual fine structure constant $\alpha \approx 1/137$. In condensed matter the effect of vacuum polarization is so tiny that in most cases it is absolutely indistinguishable against the background of the complex quantum mechanical interactions. The only chance to observe it is to find a relation which is exact in quantum mechanics but can be modified in full QED. A renowned example of an exact result in quantum mechanics is the Josephson frequency-voltage relation. In the seminal papers [3] Josephson studied a system of two superconductors separated by a thin insulating barrier, the Josephson junction. He found, in particular, that a *constant* voltage V across the junction results in an *alternating* current through the junction at the frequency ν proportional to the voltage, $\nu = K_J V$, where K_J is the Josephson constant. A simple quantum mechanical calculation relates it to the electron charge and the Planck constant

$$K_J = 2e/h. \qquad (1)$$

This prediction has been verified experimentally [4] and is known as ac Josephson effect. A remarkable property of the ac Josephson effect is that Eq. (1) is stable against all kinds of perturbations in quantum mechanics because of the gauge invariance [5]. Another famous example of the exact relations in quantum mechanics is the quantization of the Hall conductivity of the two-dimensional electron sys-

tem in a strong transverse magnetic field in the units of $1/R_K$, where

$$R_K = h/e^2 \qquad (2)$$

is the von Klitzing constant [6]. As in the case of the ac Josephson effect, this result is protected against corrections by the gauge invariance [7]. The absence of quantum mechanical corrections makes Eqs. (1) and (2) crucial for metrology. For example, the most accurate value of the Planck constant is currently obtained through the relation $1/h = K_J^2 R_K/4$ [8]. Given the important role that K_J and R_K play for determination of the fundamental constants, much effort is being made to verify Eqs. (1) and (2) experimentally [9].

On the theory side, for a long time these relations were thought to be exact. Recently, however, a deviation from the quantum mechanical result (2) has been discovered [10]. The physics behind this phenomenon is in a modification of the *local* coupling of charged particles to the electromagnetic potential due to vacuum polarization by highly virtual electron-positron pairs in a strong magnetic field. For a typical magnetic field strength of about 10 T it amounts to a tiny 10^{-20} correction. This is well beyond the precision of the current quantum Hall experiments [11], which is about one part in 10^{12} and is limited by the thermal Johnson-Nyquist noise. For Josephson junctions there is no such limit and universality of the frequency-voltage relation has been established to the amazing accuracy of 10^{-19} already two decades ago [12]. Thus one may expect that a similar effect, if it exists, can be experimentally observed in a Josephson junction subject to a strong magnetic field. The purpose of this letter is to show that this is indeed the case.

Our analysis of the ac Josephson effect is based on the following fundamental properties: (i) existence in a superconductor of the macroscopic phase-coherent state of weakly bound electron pairs (Cooper pairs) described by the wave function $\Psi = |\Psi|e^{i\theta}$, (ii) 2π-periodic dependence of the current through the junction on the difference of the phase θ across the junction, and (iii) gauge invariance of the electromagnetic interactions. We also rely on

the fact that a sufficiently strong magnetic field is not screened by the Josephson current and penetrates the junction [13]. To get the correction to Eq. (1) in a closed analytical form we consider a simplified model of the Josephson junction described below. This, however, does not affect the general character of the result.

Interaction of Cooper pairs with the external electromagnetic potential $A^\mu = (A_0, \mathbf{A})$ is dictated by gauge invariance and in quantum mechanics is described by the Hamiltonian of the following general form [14]

$$\mathcal{H} = 2eA_0 + H(\mathbf{B}, \mathbf{E}, \mathbf{D}), \quad (3)$$

where the second term is a function of the spatial covariant derivative $\mathbf{D} = \boldsymbol{\partial} - i2e\mathbf{A}$, the electric field $\mathbf{E}$ and the magnetic field $\mathbf{B}$ corresponding to the potential A^μ. Our analysis does not depend on the specific form of the function H. The gauge invariance ensures that Eq. (3) is linear in A_0 and explicitly depends on $\mathbf{A}$ only through the covariant derivative. The form of the interaction (3) guarantees that the Josephson frequency-voltage relation is exact [5]. This can be proven as follows. The physical observables depend on the gauge invariant combination

$$\phi(\mathbf{r}) = \theta(\mathbf{r}) - 2e \int^{\mathbf{r}} \mathbf{A}(\mathbf{r}') \cdot d\mathbf{r}' \quad (4)$$

rather than on the bare phase $\theta(\mathbf{r})$. At the same time the static scalar potential $A_0(\mathbf{r})$ can be the removed by the gauge transformation

$$A^{\mu'}(r) = A^\mu(r) - \partial^\mu \xi(r), \qquad \xi(r) = tA_0(r), \quad (5)$$

so that

$$A_0'(\mathbf{r}) = 0, \qquad \mathbf{A}'(\mathbf{r}) = \mathbf{A}(\mathbf{r}) + t\boldsymbol{\nabla}A_0(\mathbf{r}). \quad (6)$$

Thus the phase difference between two arbitrary points $\mathbf{r}_1$ and $\mathbf{r}_2$ can be written as

$$\begin{aligned} \Delta\phi &= \Delta\phi_0 - 2et \int_{\mathbf{r}_2}^{\mathbf{r}_1} \boldsymbol{\nabla}A_0(\mathbf{r}') \cdot d\mathbf{r}' \\ &= \Delta\phi_0 - 2et[A_0(\mathbf{r}_1) - A_0(\mathbf{r}_2)], \end{aligned} \quad (7)$$

where $\Delta\phi_0$ is the time-independent phase difference in the absence of the electric field. The Josephson current is 2π-periodic function of ϕ and, therefore, Eq. (7) implies that the potential difference $V = A_0(\mathbf{r}_1) - A_0(\mathbf{r}_2)$ between two superconductors results in the Josephson current oscillations with the angular frequency

$$\omega = 2eV, \quad (8)$$

which gives Eq. (1) in the physical units. Thus gauge invariance leaves no room for corrections to this equation in quantum mechanics.

In full QED the situation becomes more involved since the coupling of charged states to the electromagnetic potential is modified by the radiative corrections though it remains gauge invariant. We are interested in the correc-

tions due to vacuum polarization through creation and annihilation of virtual electron-positron pairs in external magnetic field graphically shown in Fig. 1. Quantitatively the effect is determined by the behavior of the vacuum polarization tensor $\Pi_{\mu\nu}(q)$ at small four-momentum transfer q. It can be expanded in powers of the external field, see Fig. 1. For a homogeneous or slowly varying field this gives a series in the parameter

$$\beta^2 = \left(\frac{eB}{m^2}\right)^2 \ll 1, \quad (9)$$

where $B = |\mathbf{B}|$ and m is the electron mass. The leading $\mathcal{O}(1)$ term of the expansion is ultraviolet divergent and is absorbed by the on-shell renormalization of the physical electron charge e. The $\mathcal{O}(\beta^2)$ correction to the polarization tensor in the limit $q \to 0$ reads [10]

$$\begin{aligned} \delta\Pi_{\mu\nu}(q) = -\frac{\alpha}{\pi}\beta^2 \frac{1}{45}[&2(g_{\mu\nu}q^2 - q_\mu q_\nu) \\ &- 7(g_{\mu\nu}q^2 - q_\mu q_\nu)_\parallel + 4(g_{\mu\nu}q^2 - q_\mu q_\nu)_\perp]. \end{aligned} \quad (10)$$

The correction to the polarization tensor is transverse because of gauge invariance. At the same time the Lorentz and rotational invariance is broken and Eq. (10) includes the transverse projectors in the "parallel" ($q_0, \mathbf{q}_\parallel$) and "orthogonal" ($\mathbf{q}_\perp$) two-dimensional subspaces of the whole four-dimensional Minkowskian momentum space ($q_0, \mathbf{q}$). Here $\mathbf{q}_\parallel$ and $\mathbf{q}_\perp$ components correspond to the spatial momentum parallel and orthogonal to the magnetic field, respectively. The correction (10) cannot be "renormalized out" and results in corrections to the local coupling of charged states to the electromagnetic potential and the correction to the photon propagator, which is nontrivial since the external magnetic field changes the photon dispersion law [15]. The first term of Eq. (10) is Lorentz covariant and has the same structure as the $\mathcal{O}(1)$ vacuum polarization. The last two terms of Eq. (10) violate Lorentz and rotational invariance leaving unbroken axial symmetry in respect to the magnetic field direction. These noncovar-
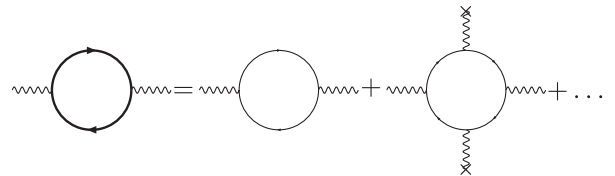

FIG. 1. Feynman diagrams representing the vacuum polarization by virtual electron-positron pairs in the external magnetic field. The arrow and wavy lines correspond to the free electron and photon propagators, respectively. The bold arrow lines correspond to the electron propagating in the external magnetic field. The crossed wavy lines represent the external magnetic field.

iant terms result in the double pole contribution in the photon propagator and contribute to the photon dispersion.

We are interested only in the corrections to the local coupling with the scalar potential A_0, which depends on the magnetic field configuration, i.e., on the particular structure of the Josephson junction. To get this correction in a closed form we consider a simplified model of the Josephson junction shown in Fig. 2. It corresponds to the homogeneous orthogonal electric and magnetic fields inside the infinite and plane insulator layer. For such a field configuration the analysis is simplified because the noncovariant terms vanish and the correction to the dispersion law does not emerge. Thus the correction to the Hamiltonian takes the standard QED form

$$\delta_{v.p.}\mathcal{H} = -e \int \frac{\delta\Pi_{0\nu}(q)}{q^2} \tilde{A}^\nu(q) e^{iqr} \frac{d^4q}{(2\pi)^4} + \dots, \quad (11)$$

where $\tilde{A}^\nu(q)$ is the Fourier transform of the electromagnetic potential and we keep only the contribution with the structure of the first term in Eq. (3). The integral in Eq. (11) gets nonvanishing contribution only from the first term of Eq. (10) and can be evaluated with the result

$$\delta_{v.p.}\mathcal{H} = 2\delta e A_0(\mathbf{r}), \quad (12)$$

where $A_0(\mathbf{r}) = \mathbf{E} \cdot \mathbf{r}$ and

$$\delta e = \frac{1}{45}\frac{\alpha}{\pi}\beta^2 e. \quad (13)$$

Equation (12) has exactly the same form as the first term of Eq. (3) but it is gauge invariant because $\delta\Pi_{\mu\nu}$ in Eq. (11) is transverse. It is easy to check by explicit calculation that Eq. (12) does not change under the gauge transformation (6). This can be understood as a manifestation of the Ward indentity [16], which means that the interaction of the charged states to the pure gauge field configurations is not renormalized. However, this remaining "potential" term can be removed from the Hamiltonian by an additional gauge transformation with the parameter
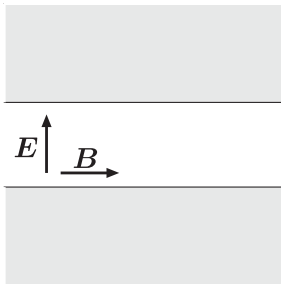

FIG. 2. Model geometry of the Josephson junction. Gray areas correspond to the superconductors separated by the insulator layer. The electric and magnetic fields are homogeneous in the insulator and vanish in the superconductors.

$$\delta\xi(r) = \frac{\delta e}{e}\xi(r). \quad (14)$$

Then, by using the same arguments as before, we derive the new result for the Josephson current frequency

$$\omega = 2e^*V, \quad (15)$$

which differs from Eq. (8) by the "effective charge" $e^* = e + \delta e$. This is not a surprising result because $2e^*V$ is nothing but the difference $\Delta\mu$ of the electrochemical potential between two identical superconductors and the general form of the Josephson relation

$$\omega = \Delta\mu \quad (16)$$

is not changed.

The vacuum polarization effect can be accounted for by introducing an effective field-dependent Josephson constant

$$K_J(B) = K_J\left[1 + \frac{1}{45}\frac{\alpha}{\pi}\beta^2\right], \quad (17)$$

or in physical units

$$K_J(B) = K_J\left[1 + \frac{1}{45}\frac{\alpha}{\pi}\left(\frac{\hbar e B}{c^2 m^2}\right)^2\right]. \quad (18)$$

This result is not exact since it was obtained for a simplified model of the junction. In reality the magnetic field in the junction is partially screened by the Josephson current and oscillates about some average value [13], which should be used in Eq. (18). At the same time the result is not affected by the sharp variation of the magnetic field near the superconductor surface because A_0 is continuous and the thin boundary region does not contribute to the potential difference. In general, the frequency-voltage relation is not sensitive to the interaction inside the superconductors where A_0 is constant.

A more subtle problem is that in real experiments the magnetic field does not vanish only in a finite volume. One may argue that the electric charge of a Cooper pair measured by means of the Gauss law at spatial infinity in this case does not differ from $2e$, that is in apparent contradiction with Eq. (15). Similar argument has been used in Refs. [17,18] to prove the absence of the corrections to K_J through the electron coupling modification due to the interaction of the electrons inside the superconductor. In our case, however, this argument does not work since the coupling is modified outside the superconductor, where the scalar potential varies. Indeed, the chemical potential difference between two superconductors is given by $\Delta\mu = 2e^*V$ if the magnetic field is homogeneous in the region of the nonvanishing electric field giving rise to V, i.e., in the vicinity of the junction, regardless to its behavior at the infinity. Here the following analogy may be useful: the vacuum polarization in the interatomic electric and magnetic fields does not change the total charge of the atom but does change the electron binding energy.

Let us now examine prospects to detect the magnetic field dependence of K_J experimentally. A relevant technique has been elaborated long time ago and consists in comparison of the voltage difference between two junctions which are phase locked to a source of microwave radiation [19]. The junctions are connected by the superconducting links to form a loop. A nonvanishing voltage difference results in a loop current increasing linearly in time, which can be monitored by a sensitive SQUID detector. The time-independent magnetic field does not change the frequency of the plane waves, though it changes their dispersion [15]. Thus if one of the junctions is embedded into magnetic field, the correction to the frequency-voltage relation results in a net electromotive force around the loop. Note that the junctions themselves do not contribute to the net electromotive force because of Eq. (16). To estimate the size of the effect we rewrite the correction term of Eq. (18) as follows

$$\frac{1}{45}\frac{\alpha}{\pi}\left(\frac{B}{B_0}\right)^2, \tag{19}$$

where $B_0 = c^2 m^2/(\hbar e) \approx 4.41 \times 10^9$ T. The critical magnetic field which destroys the quantum coherence in the Josephson junction is close to the one of the superconductors and could be as large as a few units times 10 T. This gives approximately 10^{-20} variation of the Josephson constant. On the other hand a pair of junctions had been compared with relative accuracy of 10^{-19} to test the equivalence principle for charged particles [12] and the accuracy can probably further increased. Thus the effect is likely to be within the reach of the existing experimental techniques.

In summary, the vacuum polarization alters the Josephson frequency-voltage relation in the presence of a strong magnetic field and results in a weak dependence of the Josephson constant on the magnetic field strength. This remarkable manifestation of a fine nonlinear quantum field effect in a collective phenomenon in condensed matter could be observed in a dedicated experiment, that would literally be a measurement of the vacuum polarization with a voltmeter.

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