Maxwell Superalgebra and Superparticles in Constant Gauge Backgrounds

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We present the Maxwell superalgebra, an $N = 1$, $D = 4$ algebra with two Majorana supercharges, obtained as the minimal enlargement of a Poincaré superalgebra containing the Maxwell algebra as a subalgebra. The new superalgebra describes the supersymmetries of generalized $N = 1$, $D = 4$ superspace in the presence of a constant Abelian supersymmetric field strength background. Applying the techniques of nonlinear coset realization to the Maxwell supergroup we propose a new κ-invariant massless superparticle model providing a dynamical realization of the Maxwell superalgebra.

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Introduction.—Recently, after the discovery of the cosmic microwave background (CMB) and the mystery of dark energy [1], it is interesting to consider some field densities uniformly filling space-time. One such modification of empty Minkowski space is obtained by adding a constant electromagnetic (EM) field background, parametrized by the additional field degree of freedom $f_{\mu\nu}$. The presence of a constant EM field modifies the Poincaré symmetries into the so-called Maxwell symmetries [2–9]. The difference from the Poincaré algebra consists in the de Sitter-like substitution (recall that dark energy is sometimes described by the addition of a cosmological term, or replacement of "empty" Minkowski space by de Sitter space)

$$[P_\mu, P_\nu] = iZ_{\mu\nu}. \tag{1}$$

The additional tensorial generators $Z_{\mu\nu}$ are, however, Abelian and satisfy the relations

$$[M_{\mu\nu}, Z_{\rho\tau}] = -i(\eta_{\nu\rho}Z_{\mu\tau} - \eta_{\nu\tau}Z_{\mu\rho} + \eta_{\mu\tau}Z_{\nu\rho} - \eta_{\mu\rho}Z_{\nu\tau}),$$
$$[P_\mu, Z_{\nu\rho}] = 0, \quad [Z_{\mu\nu}, Z_{\rho\tau}] = 0. \tag{2}$$

The Bacry-Combe-Richard (BCR) algebra [2] is a subalgebra of the Maxwell algebra in which $Z_{\mu\nu}$ takes fixed numerical values. In the same way as the Poincaré algebra is the $R \to \infty$ limit ($R =$ dS radius) of de Sitter algebra, the Maxwell algebra $\mathcal{M}_4 = (M_{\mu\nu}, P_\mu, Z_{\mu\nu})$ given in (1) and (2) can be obtained by a suitable contraction of the de Sitter algebra $(\tilde{M}_{\mu\nu}, P_\mu)$ enlarged in a semisimple way by the Lorentz generators $M_{\mu\nu}$ (see also [8]). Performing the rescaling $P_\mu \to \alpha^{-1}P_\mu$, $\tilde{M}_{\mu\nu} \to \alpha^{-2}Z_{\mu\nu}$, $M_{\mu\nu} \to M_{\mu\nu}$ one obtains in the limit $\alpha \to 0$ the Maxwell algebra $\mathcal{M}_4$.

In order to interpret the Maxwell algebra and the corresponding Maxwell group, a Maxwell group-invariant particle model on the extended space-time $(x^\mu, \phi^{\mu\nu})$ with the translations of $\phi^{\mu\nu}$, generated by $Z_{\mu\nu}$ has been studied [6–9]. The interaction term described by a Maxwell-invariant one form introduces new tensor degrees of freedom $f_{\mu\nu}$—momenta conjugate to $\phi^{\mu\nu}$. In the equations of motion they play the role of a background EM field which is constant on-shell and leads to a closed, Maxwell-invariant two form.

The aim of this Letter is to obtain the supersymmetric extension of the Maxwell symmetries with new $N = 1$ superMaxwell algebra and to investigate the corresponding superMaxwell-invariant massless superparticle model. (For massive superparticles one has to consider the $N = 2$ supersymmetries in $D = 4$ [10].) Analogously to the Maxwell case, one can introduce the generalized phase space with coordinates $(x^\mu, \theta^\alpha, \phi^{\mu\nu}, \phi^\alpha, \phi)$ and conjugate momenta $(p_\mu, \zeta_\alpha, f_{\mu\nu}, \tilde{\lambda}_\alpha, D)$. Since $(\phi^{\mu\nu}, \phi^\alpha, \phi)$ are cyclic coordinates the conjugate momenta $(f_{\mu\nu}, \tilde{\lambda}_\alpha, D)$ are constant on shell describing the constant Abelian SUSY $N = 1$ gauge field background. In this way one gets the massless superparticle interacting with x independent field strength superfield $W_\alpha(\theta)$

$$W_\alpha(\theta) = i\tilde{\lambda}_\alpha - \frac{i}{2}f_{\mu\nu}(\bar{\theta}\gamma^{\mu\nu})_\alpha - iD(\bar{\theta}\gamma_5)_\alpha. \tag{3}$$

We see, therefore, that the superMaxwell symmetries describe the geometry of $N = 1$ superspace (x^μ, θ^α) in the presence of constant SUSY gauge field background $(f_{\mu\nu}, \tilde{\lambda}_\alpha, D)$. It is also noted that the superparticle model is invariant under κ transformations, which eliminate half of the Grassmann superspace coordinates θ^α.

Particle model with Maxwell symmetry.—To formulate a relativistic particle model, invariant under the Maxwell group, it is convenient to use the nonlinear coset realizations method [11]. The coset $G/H =$ Maxwell/Lorentz which we employ is parametrized as in [6–9], $g = e^{iP_\mu x^\mu} e^{(i/2)Z_{\mu\nu}\phi^{\mu\nu}}$. The basic Maurer-Cartan (MC) form is

$$\Omega = -ig^{-1}dg = P_\mu L^\mu + \tfrac{1}{2}Z_{\mu\nu}L_Z^{\mu\nu} + \tfrac{1}{2}M_{\mu\nu}L_M^{\mu\nu}, \quad (4)$$

where

$$L^\mu = dx^\mu, \qquad L_Z^{\mu\nu} = d\phi^{\mu\nu} + \tfrac{1}{2}(x^\mu dx^\nu - x^\nu dx^\mu), \quad L_M^{\mu\nu} = 0. \quad (5)$$

The particle action invariant under the Maxwell algebra (1) and (2) is described by the following Lagrangian:

$$\mathcal{L} = \frac{\dot{x}_\mu \dot{x}^\mu}{2e} - \frac{m^2}{2}e + \frac{1}{2}f_{\mu\nu}L_Z^{\mu\nu *}, \quad (6)$$

where e is the einbein implementing the diffeomorphism invariance, $f_{\mu\nu}$ is a tensorial variable canonically conjugate to the new coordinates $\phi^{\mu\nu}$, and $L_Z^{\mu\nu *}$ is the pullback of $L_Z^{\mu\nu}$. In the proper time gauge, one obtains from (6) the equations of motion

$$m\ddot{x}_\mu = f_{\mu\nu}\dot{x}^\nu, \qquad \dot{f}_{\mu\nu} = 0, \qquad \dot{\phi}^{\mu\nu} = -\tfrac{1}{2}(x^\mu \dot{x}^\nu - x^\nu \dot{x}^\mu). \quad (7)$$

They describe the motion of a particle in an EM field $f_{\mu\nu}$, which is constant on shell. The EM potential is described by the one form $\mathcal{A} = \frac{1}{2}f_{\mu\nu}L_Z^{\mu\nu}$. In the closed two form field strength

$$\mathcal{F} = d\mathcal{A} = \tfrac{1}{2}f_{\mu\nu}L^\mu \wedge L^\nu + \tfrac{1}{2}df_{\mu\nu} \wedge L_Z^{\mu\nu} \quad (8)$$

the second term vanishes on shell due to (7) and the field strength components are constants $f_{\mu\nu}$.

From Maxwell algebra to superMaxwell algebra.—We start with the following extension of the superPoincaré algebra in $D = 4$ with Majorana supercharges Q_α ($\alpha, \beta = 1, 2, 3, 4$)

$$\{Q_\alpha, Q_\beta\} = 2(C\gamma^\mu)_{\alpha\beta}P_\mu, \qquad [P_\mu, P_\nu] = iZ_{\mu\nu}. \quad (9)$$

In order to verify the (P, Q, Q) Jacobi identity, P_μ cannot commute with Q_α but requires a new Majorana charge Σ_α defined as

$$[P_\mu, Q_\alpha] = -i\Sigma_\beta(\gamma_\mu)^\beta{}_\alpha. \quad (10)$$

One can show from Jacobi identities that

$$\{Q_\alpha, \Sigma_\beta\} = \tfrac{1}{2}(C\gamma^{\mu\nu})_{\alpha\beta}Z_{\mu\nu}. \quad (11)$$

Σ_α, as well as Q_α, transforms as a spinor under Lorentz transformations,

$$[M_{\rho\sigma}, Q_\alpha] = -\frac{i}{2}(Q\gamma_{\rho\sigma})_\alpha, \qquad [M_{\rho\sigma}, \Sigma_\alpha] = -\frac{i}{2}(\Sigma\gamma_{\rho\sigma})_\alpha. \quad (12)$$

Together with relations (1) and (2) the superalgebra $\mathcal{G} = (M_{\mu\nu}, P_\mu, Z_{\mu\nu}, Q_\alpha, \Sigma_\alpha)$ is shown to close due to the gamma matrix identity $(C\gamma^\mu)_{(\alpha\beta}(C\gamma_\mu)_{\gamma\delta)} = 0$ ($\alpha\beta\gamma\delta$ symmetric sum) valid in $D = 4$. $\mathcal{G}$ defines the minimal Maxwell superalgebra containing the Maxwell algebra $\mathcal{M}_4$ as a subalgebra.

Consistently with the Jacobi relations one can also add a scalar central charge B in (11) as

$$\{Q_\alpha, \Sigma_\beta\} = \tfrac{1}{2}(C\gamma^{\mu\nu})_{\alpha\beta}Z_{\mu\nu} + (C\gamma_5)_{\alpha\beta}B \quad (13)$$

and obtain the centrally extended algebra $\tilde{\mathcal{G}} = (M_{\mu\nu}, P_\mu, Z_{\mu\nu}, Q_\alpha, \Sigma_\alpha, B)$. It can be shown that the central charge B corresponds to the constant mode of an auxiliary scalar in the "off shell" supersymmetric $U(1)$ gauge field theory.

Two Casimir operators of the Maxwell algebra obtained in [2,3],

$$\mathcal{C}_2 = Z_{\mu\nu}Z^{\mu\nu}, \qquad \mathcal{C}_3 = Z_{\mu\nu}\tilde{Z}^{\mu\nu}, \qquad (\tilde{Z}^{\mu\nu} \equiv \tfrac{1}{2}\epsilon^{\mu\nu\rho\sigma}Z_{\rho\sigma}) \quad (14)$$

are also Casimir operators of the Maxwell superalgebra $\mathcal{G}$, but the third mass Casimir operator requires a fermionic term

$$\mathcal{C} = P^2 + M_{\mu\nu}Z^{\mu\nu} + i\Sigma C^{-1}Q. \quad (15)$$

For the centrally extended algebra $\tilde{\mathcal{G}}$ the Casimir operator $\mathcal{C}$ ceases to commute with Q and Σ. However, in the presence of an additional chiral symmetry charge B_5 satisfying

$$[B_5, Q_\alpha] = -i(Q\gamma_5)_\alpha, \qquad [B_5, \Sigma_\alpha] = i(\Sigma\gamma_5)_\alpha, \quad (16)$$

we can construct the extension of Casimir $\mathcal{C}$

$$\tilde{\mathcal{C}} = P^2 + M_{\mu\nu}Z^{\mu\nu} + i\Sigma C^{-1}Q - B_5 B, \quad (17)$$

which becomes a Casimir operator of the algebra $\mathcal{G}_5 = (M_{\mu\nu}, P_\mu, Z_{\mu\nu}, Q_\alpha, \Sigma_\alpha, B, B_5)$. The super algebra $\mathcal{G}_5$ will be realized in a massless particle model in the next section.

Massless superparticle model with Maxwell supersymmetry.—We construct a massless superparticle model using a nonlinear realization of the superMaxwell algebra $\mathcal{G}_5$. The supergroup element $\tilde{g}$ is parametrized as

$$\tilde{g} = e^{(i/2)Z_{\mu\nu}\phi^{\mu\nu}}e^{iP_\mu x^\mu}e^{i\Sigma_\alpha\phi^\alpha}e^{iQ_\alpha\theta^\alpha}e^{iB\phi} \quad (18)$$

using the supercoset $G/H = \mathcal{G}_5/(M \times B_5)$ [12]. Here the chiral generator B_5 is in the unbroken subgroup because we construct a massless particle. The components of the MC form $\tilde{\Omega} = -i\tilde{g}^{-1}d\tilde{g}$ are

$$\begin{aligned}
&\tilde{L}^\mu = dx^\mu + i(\bar{\theta}\gamma^\mu d\theta), \quad \tilde{L}^\alpha = d\theta^\alpha, \quad \tilde{L}_M^{\mu\nu} = 0,\\
&\tilde{L}_Z^{\mu\nu} = d\phi^{\mu\nu} + i(\bar{\theta}\gamma^{\mu\nu})_\alpha d\phi^\alpha + \frac{1}{2}(x^\mu dx^\nu - x^\nu dx^\mu)\\
&\qquad + \frac{i}{2}(\bar{\theta}\gamma^{\mu\nu}\gamma_\rho\theta)\Big(dx^\rho + \frac{i}{6}(\bar{\theta}\gamma^\rho d\theta)\Big),\\
&\tilde{L}_\Sigma^\alpha = d\phi^\alpha + (\gamma_\rho\theta)^\alpha\Big(dx^\rho + \frac{i}{3}(\bar{\theta}\gamma^\rho d\theta)\Big), \quad \tilde{L}^5 = 0,\\
&\tilde{L}_B = d\phi + i(\bar{\theta}\gamma_5)_\alpha d\phi^\alpha + \frac{i}{2}(\bar{\theta}\gamma_5\gamma_\rho\theta)\Big(dx^\rho + \frac{i}{6}(\bar{\theta}\gamma^\rho d\theta)\Big)
\end{aligned} \quad (19)$$

and verify the corresponding MC equations

$$d\tilde{L}^{\mu} = i\bar{\tilde{L}}\gamma^{\mu}\tilde{L} - \tilde{L}_M^{\mu\nu}\tilde{L}_{\nu}, \qquad d\tilde{L}_M^{\mu\nu} = -\tilde{L}_M^{\mu\rho}\eta_{\rho\sigma}\tilde{L}_M^{\sigma\nu},$$
$$d\tilde{L}_Z^{\mu\nu} = \tilde{L}^{\mu}\tilde{L}^{\nu} + i\bar{\tilde{L}}\gamma^{\mu\nu}\tilde{L}_{\Sigma} - \tilde{L}_M^{\mu\rho}\eta_{\rho\sigma}\tilde{L}_Z^{\sigma\nu} - \tilde{L}_Z^{\mu\rho}\eta_{\rho\sigma}\tilde{L}_M^{\sigma\nu},$$
$$d\tilde{L}^{\alpha} = (\gamma_5\tilde{L})^{\alpha}\tilde{L}^5 - \tfrac{1}{4}\tilde{L}_M^{\mu\nu}(\gamma_{\mu\nu}\tilde{L})^{\alpha},$$
$$d\tilde{L}_{\Sigma}^{\alpha} = (\gamma_{\mu}\tilde{L})^{\alpha}\tilde{L}^{\mu} - (\gamma_5\tilde{L}_{\Sigma})^{\alpha}\tilde{L}^5 - \tfrac{1}{4}\tilde{L}_M^{\mu\nu}(\gamma_{\mu\nu}\tilde{L}_{\Sigma})^{\alpha},$$
$$d\tilde{L}_B = i\bar{\tilde{L}}\gamma_5\tilde{L}_{\Sigma}, \qquad d\tilde{L}^5 = 0. \tag{20}$$

These MC equations provide a dual formulation of the superMaxwell algebra introduced in the previous section.

The massless superparticle action invariant under the superMaxwell group is

$$\mathcal{L} = \frac{\pi_{\mu}^2}{2e} + \mathcal{L}^{I*}; \qquad \mathcal{L}^I = \tfrac{1}{2}f_{\mu\nu}\tilde{L}_Z^{\mu\nu} + i\lambda_{\alpha}\tilde{L}_{\Sigma}^{\alpha} + D\tilde{L}_B, \tag{21}$$

where $\pi_{\mu} = \dot{x}_{\mu} + i\bar{\theta}\gamma_{\mu}\dot{\theta}$ is the pullback of $\tilde{L}_{\mu}$ to the world line and e describes the einbein. Here $f_{\mu\nu}$, λ_{α}, D are dynamical variables transforming as Lorentz tensor, Majorana spinor and scalar, respectively. The interaction Lagrangian can be written explicitly as

$$\mathcal{L}^{I*} = \tfrac{1}{2}f_{\mu\nu}\dot{\phi}^{\mu\nu} + i\tilde{\lambda}_{\alpha}\dot{\phi}^{\alpha} + D\dot{\phi} + \pi^{\mu}A_{\mu} + \dot{\theta}^{\alpha}\tilde{A}_{\alpha}, \tag{22}$$

where

$$\tilde{\lambda}_{\alpha} = \lambda_{\alpha} + D(\bar{\theta}\gamma_5)_{\alpha} + \tfrac{1}{2}f_{\mu\nu}(\bar{\theta}\gamma^{\mu\nu})_{\alpha} \tag{23}$$

and the $U(1)$ SUSY gauge potentials are

$$\tilde{A}_{\alpha} = i(\bar{\theta}\gamma^{\mu})_{\alpha}\Big[-\tfrac{1}{2}f_{\mu\nu}x^{\nu} + i\Big(\tfrac{2}{3}\tilde{\lambda} - \tfrac{1}{8}\bar{\theta}\gamma_{\rho\sigma}f^{\rho\sigma} - \tfrac{1}{4}D\bar{\theta}\gamma_5\Big)\gamma_{\mu}\theta\Big], \tag{24}$$
$$A_{\mu} = -\tfrac{1}{2}f_{\mu\nu}x^{\nu} + i\Big(\tilde{\lambda} - \tfrac{1}{4}\bar{\theta}\gamma_{\rho\sigma}f^{\rho\sigma} - \tfrac{1}{2}D\bar{\theta}\gamma_5\Big)\gamma_{\mu}\theta.$$

The variation of $\mathcal{L}$ with respect to $(\phi^{\mu\nu}, \phi^{\alpha}, \phi)$ gives

$$\dot{f}_{\mu\nu} = \dot{\tilde{\lambda}}_{\alpha} = \dot{D} = 0; \tag{25}$$

i.e., the $U(1)$ superpotentials (24) are functions of the superspace coordinates $(x^{\mu}, \theta^{\alpha})$ and the variables $(f_{\mu\nu}, \tilde{\lambda}_{\alpha}, D)$ which take constant values on shell. The variation of $\mathcal{L}$ with respect to $(f_{\mu\nu}, \tilde{\lambda}_{\alpha}, D)$ gives the equations for the variables $(\phi^{\mu\nu}, \phi^{\alpha}, \phi)$

$$(\tilde{L}_Z^{\mu\nu})^* = (\tilde{L}_{\Sigma}^{\alpha})^* = (\tilde{L}_B)^* = 0. \tag{26}$$

The variation of $\mathcal{L}$ with respect to e puts the momenta π_{μ} on mass shell with vanishing mass

$$\pi^2 = 0. \tag{27}$$

Finally, the variation of $\mathcal{L}$ with respect to $(x^{\mu}, \theta^{\alpha})$ gives, using (24) and (25), the superparticle equations of motion in superspace,

$$\frac{d}{d\tau}\Big(\frac{\pi_{\mu}}{e}\Big) = \pi^{\nu}F_{\mu\nu} + \dot{\theta}^{\beta}F_{\mu\beta}, \tag{28}$$

$$2i(\dot{\bar{\theta}}\gamma^{\mu})_{\alpha}\Big(\frac{\pi_{\mu}}{e}\Big) = \pi^{\nu}F_{\nu\alpha}, \tag{29}$$

where the superfield strength using the differential operator $D_{\alpha} = \partial_{\alpha} + i(\bar{\theta}\gamma^{\mu})_{\alpha}\partial_{\mu}$ are

$$F_{\mu\nu} = (\partial_{\mu}A_{\nu} - \partial_{\nu}A_{\mu}) = f_{\mu\nu}, \qquad F_{\mu\alpha} = (\partial_{\mu}\tilde{A}_{\alpha} - D_{\alpha}A_{\mu}) = i(\lambda\gamma_{\mu})_{\alpha}, \tag{30}$$

and the superspace constraints following from (24)

$$F_{\alpha\beta} = (D_{\alpha}\tilde{A}_{\beta} + D_{\beta}\tilde{A}_{\alpha}) - 2i(C\gamma^{\mu})_{\alpha\beta}A_{\mu} = 0 \tag{31}$$

have been used in (29). The sector of our model covered by $(x^{\mu}, p_{\mu}, \theta^{\alpha}, \zeta_{\alpha}, f_{\mu\nu}, \tilde{\lambda}_{\alpha}, D)$ describes therefore a massless superparticle minimally coupled to the super $U(1)$ gauge field. Identifying the interaction term $\mathcal{L}^I = \mathcal{A}$ in (21) with the EM one-form superpotential, the two-superform field strength $\mathcal{F} = d\mathcal{A}$ is, after using the MC Eqs. (20),

$$\mathcal{F} = d\mathcal{A} = \tfrac{1}{2}f_{\mu\nu}L^{\mu}L^{\nu} + i\lambda_{\alpha}(\gamma_{\mu}L)^{\alpha}L^{\mu} + \cdots, \tag{32}$$

where the $\cdots$ terms are linear in the one forms $L_B, L_{\Sigma}^{\alpha}, L_Z^{\mu\nu}$ which vanish on shell. The field strength components are the ones given in (30) and (31).

Our model describes the coupling to a particular choice of $U(1)$ gauge superfield strength $W_{\alpha}(x, \theta)$ in (3), which satisfies the standard superspace constraints for the SUSY gauge theories [13],

$$F_{\alpha\beta} = 0, \qquad F_{\mu\alpha} = W_{\beta}(\gamma_{\mu})^{\beta}{}_{\alpha},$$
$$D_{\alpha}W_{\beta} = -\frac{i}{2}(C\gamma^{\mu\nu})_{\alpha\beta}F_{\mu\nu}, \qquad \partial_{\mu}W_{\beta}(\gamma^{\mu})^{\beta}{}_{\alpha} = 0. \tag{33}$$

It is known (see, e.g., [14]) that the coupling of the $N = 1$ superparticle to the gauge superfield strength $W_{\alpha}(x, \theta)$ satisfying the constraints (33) leads to a κ-invariant interaction. Actually our system is not only invariant under the global Maxwell supersymmetries but also invariant under τ reparametrization and the κ symmetries.

Conclusions.—In this Letter we found supersymmetric extensions of the Maxwell algebra and proposed a κ invariant superparticle model (21) with the superMaxwell symmetries. It couples minimally to a constant $U(1)$ gauge superfield strength satisfying the superspace constraints [see (33)]. It gives a new geometric framework for a superspace filled with a uniform SUSY gauge field by generalizing the known nonsupersymmetric one with Maxwell symmetries. Because supersymmetries have critical importance in current fundamental interaction theories (e.g., string or M theory), we hope such a generalization will be useful in this context, in particular, in the interpretation of fermionic backgrounds.

The superMaxwell algebra is realized if we regard the variables $(f_{\mu\nu}, \tilde{\lambda}_{\alpha}, D)$ as dynamical ones. In the

Hamiltonian formulation of our model (21) they become the generators ($Z_{\mu\nu}$, Σ_α, B) of the superMaxwell symmetries. Note that by taking a fixed solution for ($f_{\mu\nu}$, $\tilde{\lambda}_\alpha$, D) the superMaxwell symmetry is spontaneously broken to smaller ones similarly as in the bosonic case [2]. The evolution of the coordinates ($\phi^{\mu\nu}$, ϕ^α, ϕ) are described by Eq. (26) with their solutions determined by the trajectories in the "physical" subspace (x_μ, θ_α, $f_{\mu\nu}$, $\tilde{\lambda}_\alpha$, D). It will be interesting to find some physical interpretation for the new coordinates ($\phi^{\mu\nu}$, ϕ^α, ϕ) and their dynamical roles. For the bosonic Maxwell case it has been suggested [7] that $\phi^{\mu\nu}$ describes the magnetic moment of a distribution of charged particles with center-of-mass position x^μ.

The superMaxwell algebra $\mathcal{G}$ introduced in this Letter is a minimal superextension of the Maxwell algebra. It can be considered as an enlargement of the Green algebra [15] by adding the tensorial central charges $Z_{\mu\nu}$. In the Green algebra the spinorial generators Σ_α are central [compare with (11)]. We have considered also its central extension $\tilde{\mathcal{G}}$ and the enlargement $\mathcal{G}_5$ by means of the chiral generator B_5. The superMaxwell algebra $\mathcal{G}$ can be embedded into larger superalgebras, in particular, in the known Bergshoeff-Sezgin (BS) p-brane algebra [16]. Thus one can introduce a corresponding BS-invariant superparticle model with the interaction Lagrangian generalizing (22) and gauge superpotentials A_μ^{BS}, A_α^{BS} depending in a unique way on the BS supergroup coordinates. Using the coset with Lorentz stability group we find that the corresponding superfield strength F^{BS}'s do not satisfy the superspace constraints (33); i.e., the BS superparticle dynamics is not κ symmetric. The origin of the noninvariance is the appearance of $Z_{\mu\nu}$ in the $\{Q, Q\}$ anticommutator resulting in $F_{\alpha\beta} \neq 0$ which violates the SUSY constraint (33) [cf. (32)]. We note also that Soroka and Soroka proposed in [5,17] a nonstandard supersymmetrization of Maxwell algebra, without the translation generators in the basic anticommutator $\{Q, Q\}$; moreover in [17] there is presented some superextension of k-deformed Maxwell algebra ($k > 0$ of [8]).

Our geometric scheme introduces additional degrees of freedom, describing uniform gauge field strengths in space and superspace leading to uniform constant energy density. These global degrees of freedom are dynamical; i.e., our model provides a framework in which the cosmological constant could be considered as a dynamical quantity. Recently, many papers propose new types of dynamics to explain the dark energy phenomenon (see, e.g., [18]) as well as the dynamical role of the cosmological constant (see, e.g., [19,20]). Because at present these issues are of fundamental importance, the developments in this Letter should find some important applications.

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