Channeling of the Energy and Momentum during Energetic-Ion-Driven Instabilities in Fusion Plasmas

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New features of instabilities driven by energetic ions are revealed. It is found that these instabilities can affect plasma heating and rotation by channeling the energy and momentum of the energetic ions to the region where the destabilized waves are damped. Because of the energy channeling, the plasma core may not be heated by the energetic ions even when these ions have a very peaked radial distribution. It is likely that this new phenomenon can explain experiments on the spherical torus NSTX where a broadening of the temperature profile and even a drop of the temperature at the plasma center with increasing injected power were observed during Alfvén instabilities [D. Stutman *et al.*, Phys. Rev. Lett. **102**, 115002 (2009)]. The momentum channeling can lead to plasma rotation and frequency chirping due to the Doppler shift varying in time.

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A surprising result was reported recently: increasing the power of the neutral beam injection (NBI) by a factor of 3 in experiments on the spherical torus NSTX did not increase the plasma temperature in the central region and even resulted in its drop when high-frequency global Alfvén eigenmodes (GAE) (in the range of 0.5–1.1 MHz) were destabilized [1,2]. Numerical simulation by the code TRANSP has shown that, in order to reproduce the observed temperature profile in the discharge with the highest injection power ($\mathcal{P} = 6$ MW), one has to assume that the electron heat conductivity has unusual features: it must be strongly enhanced (by a factor of 40) in the central region (at $r/a \sim 0.2$, where r is the radial coordinate and a is the plasma radius) and strongly suppressed (by a factor of 20) at the periphery; see Fig. 1 in Refs. [1,2]. In particular, the calculated heat conductivity coefficient χ_e reached 200 m^2/s at $r/a \sim 0.2$. This exceeds the Bohm diffusion coefficient, $D_B = 125$ m^2/s, and well exceeds (by 2 or 3 orders) the magnitude that can be obtained by using a theory proposed for the explanation of the thermal crashes that occurred during a low-frequency NBI-driven Alfvén instability in the stellarator Wendelstein 7-AS [3]. Moreover, no physical mechanism of the suppression of thermal conductivity by waves is known. Thus, the mentioned NSTX experiments remain a mystery. It is of large importance that these experiments and preceding experiments on Wendelstein 7-AS [3,4] indicate that fast-ion-driven instabilities can strongly affect the plasma performance in tokamaks and stellarators. In this Letter, we reveal mechanisms of the influence of these instabilities on the plasma. Our idea is based on the fact that typically the regions of the wave emission by energetic ions and the wave absorption by the plasma do not coincide. Because of this, the waves can channel energy and momentum from one region to another region. This channeling can lead, in particular, to cooling the plasma core.

Using quasilinear theory equations averaged over the particle bounce or transit time (τ_b) (see, e.g., Ref. [5]), we can write

$$\frac{\partial F}{\partial t} = \frac{1}{\tau_b}\sum_{m,n} \hat{\Pi}\, \tau_b \mathcal{D}\hat{\Pi} F + C^{\rm col}, \qquad (1)$$

where $F = F(\mathcal{E}, \mu, r, t)$ is the unperturbed electron distribution function, $\mathcal{E}$ is the particle energy, μ is the magnetic moment, $\hat{\Pi}^c = \partial/\partial\mathcal{E} + k_\vartheta/(M\omega\omega_B)\partial/\partial r$, ω is the mode frequency, ω_B is the electron gyrofrequency, M is the electron mass, $k_\vartheta = m/r$, $\hat{\Pi}^t$ can be obtained from $\hat{\Pi}^c$ by replacing m with nq, m and n are the mode numbers, q is the safety factor ($m \approx nq$ when the longitudinal wave number $k_\parallel$ is small), the superscripts "c" and "t" refer to circulating and trapped particles, respectively, $\mathcal{D} = \pi e^2 \sum_s |\mathcal{J}|^2 \delta(\omega - m\langle d\vartheta/dt\rangle + n\langle d\varphi/dt\rangle - s\omega_b)$, $\delta(x)$ is the Dirac delta function, ϑ is the poloidal angle, φ is the toroidal angle, brackets denote bounce averaging, $\mathcal{J}$ is proportional to wave amplitudes and depends on particle orbits in the equilibrium magnetic field, $\omega_b = 2\pi/\tau_b$, and $C^{\rm col}$ is the collision term. The perturbations are taken in the form $\tilde{X} = \hat{X}(r)\exp(-i\omega t - in\varphi + im\vartheta)$, and a right-handed coordinate system is used.

Calculation of the second moment of Eq. (1) yields the following equation for the electron temperature T_e:

$$\frac{3}{2} n_e \frac{\partial T_e}{\partial t} + \frac{1}{r}\frac{\partial}{\partial r} r q_e = Q_e^w + Q^{\rm col}, \qquad (2)$$

where n_e is the electron density, $Q^{\rm col}$ is the energy exchange of electrons and other particles through Coulomb collisions, Q_e^w describes the absorption of the wave energy by electrons,

$$Q_e^w = -\sum_{m,n} \int \overline{d^3 v}\, \mathcal{D}\hat{\Pi} F, \qquad (3)$$

the bar over d^3v means flux surface averaging, q_e is the electron heat flux given by

$$q_e = -\sum_{m,n} \frac{k_\vartheta}{M\omega_B\omega} \int \overline{d^3v} \mathcal{E}\mathcal{D}\hat{\Pi} F - \frac{3}{2} T_e \Gamma_e, \qquad (4)$$

and Γ_e is the particle flux across the magnetic field.

One can see that the Q_e^w term well exceeds the $\nabla \cdot \mathbf{q}_e$ term in Eq. (2) when $\omega \gg \omega_{*e}$, where ω_{*e} is the electron diamagnetic drift frequency. This implies that the main effect of high-frequency waves is plasma heating, whereas wave-induced transport processes play a minor role in establishing the temperature. The plasma region heated by the waves can be located away from the region where the waves receive the energy of the energetic ions. Therefore, the destabilized modes can channel the energy of the energetic ions from one region to another region. For instance, this can be the case when beam particles with a peaked radial distribution destabilize high-frequency GAE modes, whereas the main mechanism of the wave damping is the continuum damping at the periphery; see Fig. 1. Of course, this effect is significant provided that the power received by the waves from the beam ions is considerable. If the waves receive a major part of the power injected into the plasma core, the energy channeling will lead to a decrease of the temperature at the plasma center and an increase at the periphery. In other words, the energy channeling plays an important role when, first, the magnitude of $2\gamma_\alpha W$ (γ_α is local contribution of the energetic ions to the instability growth rate, W is the wave energy density) is comparable to the injected power density, and, second, the energy range of the resonance wave-particle interaction is wide, from the energy close to the birth energy ($\mathcal{E}_0$) to $\mathcal{E} \ll \mathcal{E}_0$. The slowing down process caused by the waves is accompanied by the radial motion of the ion. However, the ion displacement is very small at high ω, as follows

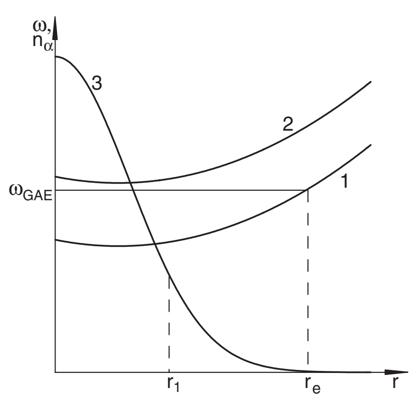

FIG. 1. GAE frequency (horizontal line), Alfvén continuum branches with the mode numbers m, n and $m + 1, n$ (curves 1, 2), and the radial profile of the beam ions (curve 3). This sketch demonstrates the energy and momentum channeling by a GAE mode: the mode receives the energy and momentum of the beam ions mainly inside the region $r < r_1$ but gives the energy and momentum to electrons due to continuum damping at $r \sim r_e$.

from the characteristics of the quasilinear equation:

$$dr^2 = \frac{2m}{M_\alpha \omega_{B\alpha} \omega} d\mathcal{E}, \qquad (5)$$

where superscript α labels energetic particles. Note that Eq. (5) does not depend on the particle mass, but depends on the electric charge. Therefore, both the electrons accelerated due to the wave damping and slowing down energetic ions move outwards (inwards) when $m < 0$ ($m > 0$).

The lack of the local balance between the momentum received by the waves from the energetic ions and the momentum absorbed by the electrons leads to plasma rotation. Using equations of motion for the electrons, ions, and energetic ions and assuming that the waves interact only with the electrons and energetic ions, we obtain the following equation for the frequency of the toroidal rotation Ω_φ:

$$M_i n_i R \dot{\Omega}_\varphi = \sum_{\sigma=e,\alpha} f_{\sigma\varphi} + \frac{R}{r} \frac{\partial}{\partial r} r \eta_* \frac{\partial \Omega_\varphi}{\partial r} + \frac{B_\vartheta}{c} j_r, \qquad (6)$$

where $\mathbf{f}_\sigma = -2\gamma_\sigma \mathbf{k} W/\omega$ is the force acting on σ particles ($\sigma = \alpha, e$) from the waves, γ_e is the local damping rate, $\mathbf{k}$ is the wave number, $\eta_* = M_i n_i \eta$, η is the plasma viscosity, R is the major radius of the torus, $\mathbf{j}$ is the plasma current, B is the magnetic field, and the subscript "i" labels ion quantities. The last term in Eq. (6) and in the corresponding equation for the poloidal rotation frequency, Ω_ϑ, can be neglected because, as one can show, j_r is much less, by a factor of c^2/v_A^2 (v_A is the Alfvén velocity), than the partial currents $j_{\sigma r} = c f_{\sigma\vartheta}/B$. Because of viscosity, steady-state rotation is possible. The characteristic viscosity time is $\tau^{\text{vis}} = (\Delta r)^2/\eta$, where Δr is the scale of the process. Taking this into account, we can evaluate the plasma steady-state rotation velocity $\mathbf{u}$ as

$$u_\vartheta = \frac{f_{\alpha\vartheta} \tau^{\text{vis}}}{M_i n_i}, \qquad u_\varphi \sim u_{i\vartheta} k_\varphi / k_\vartheta. \qquad (7)$$

One can see from here that the rotation velocity can be significant. Let us assume that the energetic ions emit the waves at the radius r_α, whereas the electrons absorb the waves at r_e, with $r_\alpha < r_e$. Then, using the boundary conditions $u_\vartheta(a) = 0$, $\partial u_{i\vartheta}/\partial r|_{r=0} = 0$, we can obtain the picture shown in Fig. 2. We observe that there is sheared rotation in the region $r_\alpha < r < r_e$ and rigid rotation in the region $0 < r < r_\alpha$. The direction of rotation is reversed when $r_\alpha > r_e$. The considered rotation has an important consequence: it leads to frequency chirping due to the growth of the Doppler shift, $\Delta\omega(t) = \mathbf{k} \cdot \mathbf{u}(t) = m\Omega_\vartheta - n\Omega_\varphi$, after the beginning of the instability. One can see that the effect is strongest when $r_\alpha < r_e$. In this case, $\text{sgn} u_\vartheta = \text{sgn} f_{\alpha\vartheta} = -\text{sgn} k_\vartheta$ and $\text{sgn} u_\varphi = \text{sgn} f_{\alpha\varphi} = -\text{sgn} k_\varphi$. Therefore, $\Delta\omega < 0$; i.e., frequency chirping down takes place. Note that even if the velocity of the poloidal rotation were considerably reduced, which may be the case in

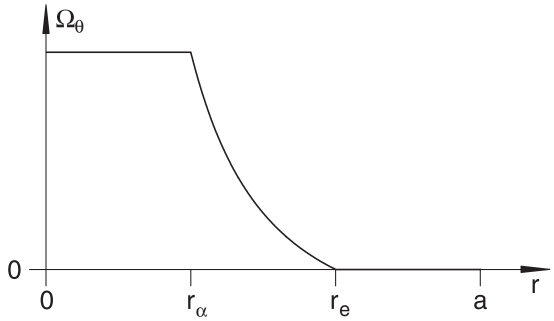

FIG. 2. Sketch of the plasma rotation frequency during fast-ion-driven instabilities.

tokamaks, the Doppler shift due to $u_\vartheta \neq 0$ can be considerable for the modes located close to the magnetic axis. The Doppler shift due to toroidal rotation can also be important, especially in toroidal plasmas with small aspect ratio.

Let us consider Eq. (4) for the heat flux. We observe that the flux is determined by convection (the first term in $\hat{\Pi}$) and heat conductivity. The ratio of the diffusivity term to the convective term in Eq. (4) is small (unless the wave spectrum is symmetric, in which case the convective term vanishes) when $\omega \gg \omega_{*e}$. The convection plays an important role in the region with considerable gradients of the mode energy. The heat conductivity coefficient following from Eq. (4) is given by (details will be published elsewhere)

$$\chi_e^c \approx \sum_{m,n} \frac{1}{4\epsilon^2} \frac{k_\vartheta^2}{k_\parallel^2} \left(\frac{\omega}{k_\parallel v_e}\right)^4 \frac{c^2}{k_\parallel v_e} \frac{|\tilde{E}_\parallel|^2}{B_0^2}, \tag{8}$$

$$\chi_e^t \approx \sum_{m,n} \frac{n^2 q^2 c^2}{r^2 k_\parallel^3 v_e} \frac{|\tilde{E}_\parallel|^2}{B^2} \Lambda, \tag{9}$$

where $\tilde{E}_\parallel$ is the perturbed longitudinal electric field, $v_e = \sqrt{T_e/M_e}$, $\epsilon = r/R$, $\Lambda = \ln(\alpha + \sqrt{\alpha^2 - 1})$, and $\alpha = 2k_\parallel v_e \sqrt{\epsilon}/\omega$.

Now we proceed to analysis of the NSTX experiment [1]. Let us first evaluate χ_e. Equations (8) and (9) cannot be used because $\tilde{E}_\parallel$ is not known from the experiment. Instead, the line-averaged fluctuation amplitude of the electron density, $\langle \tilde{n}_e \rangle / \langle n_e \rangle = 1.5 \times 10^{-4}$, is known. Therefore, we have to express $\tilde{E}_\parallel$ through $\tilde{n}$. Note that $\tilde{E}_\parallel = 0$ in the ideal MHD approximation. However, in reality it never vanishes. As one can see from equation

$$\tilde{E}_\parallel = -k_\perp k_\parallel \rho_s^2 \tilde{E}_\perp \tag{10}$$

($k_\perp$ is the wave number across the magnetic field, $\rho_s = \sqrt{T_e/M_i}$), $\tilde{E}_\parallel$ is considerable for the high-frequency MHD modes having $k_\parallel qR \gg 1$, which was the case in the NSTX experiments. For the Alfvén waves $\tilde{n}_e \approx \tilde{n}_i$; the magnitude $\tilde{n}_i$ is connected with the radial ion displacement ξ_r by the incompressibility condition, $\tilde{n} + n_0' \xi_r = 0$; ξ_r is connected with the perturbed poloidal electric field by the equation $c\tilde{E}_\vartheta + i\omega B \xi_r = 0$. Using these equations and Eq. (10), we obtain

$$\tilde{E}_\parallel = \frac{i\omega k_\parallel}{c k_\vartheta} k_\perp^2 \rho_s^2 B L \frac{\tilde{n}_e}{n_0}, \tag{11}$$

where L is the characteristic inhomogeneity length of the plasma density. Assuming that $\tilde{n}_e$ in the region of the instability exceeds $\langle \tilde{n}_e \rangle$ by a factor of 3 and taking into account that the modes with $n = 2, 3$ dominated in the experiment, we obtain that $\chi_e^t \sim 1\ \mathrm{m}^2/\mathrm{s}$ and $\chi_e^c \ll 1\ \mathrm{m}^2/\mathrm{s}$ (the estimates are made for $r/a = 0.2$, where χ_e calculated in Ref. [1] has a maximum). This is much less than the heat conductivity coefficient calculated by TRANSP, $\chi_e^{\max} = 200\ \mathrm{m}^2/\mathrm{s}$ [1]. This is not surprising: because $\omega \gg \omega_{*e}$, the energy channeling and/or heat convection rather than the conductivity play the main role. The code ignoring these processes finds χ_e, which well overestimates the real heat conductivity coefficient.

Let us see if the energy channeling could indeed be important in the NSTX experiments. For this to be the case, a large fraction of the NBI power in the core region should be transferred to waves. The NBI power density in NSTX was $P_{\mathrm{inj}}(r/a < 0.5) \sim 1\ \mathrm{MW/m}^3$ when $\mathcal{P} = 6$ MW. The power density emitted by the beam ions is

$$P_\alpha = 2\gamma_\alpha W = 2\omega \frac{\gamma_\alpha}{\omega} \frac{B^2}{4\pi} \frac{\tilde{B}^2}{B^2}, \tag{12}$$

where $\tilde{B}/B$ can be evaluated as $\tilde{B}/B \sim k_\parallel L \tilde{n}_e / n_e \sim 5 \times 10^{-3}$. Using this estimate and $B = 0.45$ T, $\omega = 1$ MHz, we obtain that $P_\alpha \sim P_{\mathrm{inj}}$ when $\gamma_\alpha/\omega = 2 \times 10^{-2}$, which is quite reasonable (see, e.g., [6]). Of course, an energetic ion can be slowed down due to interaction with waves only when the resonance region is suitable for this. The resonant velocities of the beam ions are determined by $\omega = [k_\parallel \pm s/(qR)] v_\parallel$, with s an integer, the $|s| > 1$ resonances contributing considerably due to large orbit width of the beam ions [7]. Taking $|s| \leq 4$, $\omega = 1$ MHz, and $v_0/v_A = 2.9$ (v_0 is the birth velocity), we obtain nine resonance velocities in the region $(0.2\text{–}0.9) v_0/\chi$, χ is the particle pitch angle. This means that the average distance between the resonances is $\Delta v^{\mathrm{res}} \lesssim 0.1 v_0$. The presence of many modes with the frequencies from 0.5 to 1.1 MHz, which were observed experimentally, provides many more resonances. The resonance width in the case of Alfvén waves can be evaluated as $\delta v^{\mathrm{res}} = 4\sqrt{eE_\parallel / k_\parallel M_\alpha}$ [3]. We infer that the waves can receive a large fraction of the injected ion energy.

The estimates above were made in the assumption that the radial position of the beam ions does not change during the slowing down, which agrees with the experimental result showing no radial redistribution of the beam ions [1]. Let us see if this agrees with our theory. Using Eq. (5), we obtain that a 90-keV ion is displaced negligibly during slowing down to $\mathcal{E} = 0$; for instance, a particle born at $r/a = 0.2$ is displaced by 0.1 cm.

Now we can propose a simple model to see the role of the energy channeling in the NSTX high-power experiments. We take into account that high-frequency GAEs are weak or absent at $\mathcal{P} = 2$ MW, whereas they are strong at $\mathcal{P} = 6$ MW. This gives us grounds to assume that the plasma is heated by the beam ions through Coulomb collisions at $\mathcal{P} = 2$ MW, whereas it is heated mainly by the waves at $\mathcal{P} = 6$ MW. In addition, because $\omega \gg \omega_*$, we assume that the electron transport is the same at $\mathcal{P} = 2$ MW and $\mathcal{P} = 6$ MW. Then, summing Eq. (2) and a corresponding equation for the ions and taking $T_e = T_i$, we obtain an energy balance equation, which has the following steady-state solution:

$$T(r) = T(a) + 2\int_r^a \frac{dr'}{r'\kappa}\int_0^{r'} dr'' r''(Q^b + Q^w), \qquad (13)$$

where $\kappa = n_e(\chi_e + \chi_i)$, Q^b describes the plasma heating due to beam slowing down by the Coulomb collisions, Q^w is the heating by the waves, $\int d^3rQ^w = \nu\mathcal{P}$, $\int d^3rQ^b = (1-\nu)\mathcal{P}$, ν is the fraction of the injected power received by the waves from the beam. We assume that $Q^b \propto \exp(-12r^2/a^2)$, which is in qualitative agreement with Fig. 2 of Ref. [1]. In order to know ν and Q^w, a detailed modeling of the NSTX experiments is required, which is beyond the scope of this Letter. Instead, we model the plasma heating by the waves by assuming that $Q^w \propto \exp[-\alpha_w(r-r_e)^2/a^2]$, in which case ν, α_w, and r_e/a are adjustable parameters. Note that the region of the wave damping cannot be very narrow. The matter is that, first, the frequency of the mode with the numbers (m, n) intersects several coupled continuum branches with the numbers $(m+j, n)$, where $j = 1, 2, \ldots,$ and, second, the kinetic Alfvén waves generated at the intersections have finite damping length. Bearing this in mind, we calculated $T(r)$ for different $\mathcal{P}$; see Fig. 3. The obtained picture agrees with the experimentally observed one (Fig. 1 of Ref. [1]), from which the code TRANSP obtained χ_e strongly enhanced in the center and suppressed at the periphery for $\mathcal{P} > 2$ MW. On the other hand, our estimates of χ_e and convection indicate that the wave-induced electron transport is too weak and, moreover, no way is seen to explain the reduction of χ_e at the periphery unless the experimental conditions at the periphery were different in different discharges. Therefore, the energy channeling is the only mechanism which can lead to drop of $T(0)$ and the increase of $T(r \sim 0.5a)$ at $\mathcal{P} = 6$ MW.

In summary, it is found that energetic-ion-driven instabilities can channel the energy and momentum of the energetic ions from the birth region of these ions to the region where the destabilized waves are damped, the radial redistribution of the energetic ions being very weakly affected when $\omega \gg \omega_*$. When the radial profile of the energetic ions is peaked but the destabilized modes are damped at the periphery, the energy channeling can strongly deteriorate the plasma performance by decreasing the heating of the plasma core. It is likely that the energy channeling played a key role in the NSTX discharges with high injection power. It is shown that the plasma heating by the waves with $\omega \gg \omega_*$ always exceeds the plasma cooling through the thermal conduction and convection caused by the waves. It is concluded that the momentum channeling leads to plasma rotation and concomitant frequency chirping. It follows from our analysis that a key factor responsible for the influence of fast-ion-driven instabilities on the plasma is the lack of the local balance between the wave emission and absorption. Depending on the magnitude of the imbalance along the radius, the instabilities have either small or large influence on the plasma.

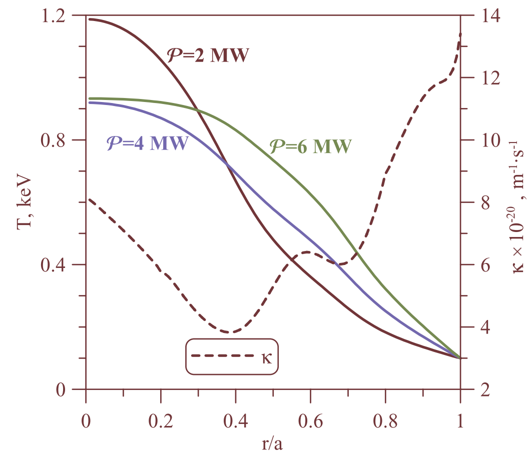

FIG. 3 (color online). The calculated plasma temperature (solid lines) for various magnitudes of the injected power and $\kappa(r)$ in NSTX. $T(r)$ is described by Eq. (13) with $r_e/a = 0.6$ and $\alpha_w = 10$; $\nu = 0$ for $\mathcal{P} = 2$ MW, $\nu = 0.82$ for $\mathcal{P} = 4$ MW, $\nu = 0.98$ for $\mathcal{P} = 6$ MW. It was assumed that χ_e does not depend on $\mathcal{P}$ and is equal to that given by Fig. 1 in Ref. [1] for 2 MW, $\chi_i = \chi^{NC}$ in Fig. 1 in Ref. [1], $n_e(r)$ of Ref. [2].

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