Precise Determination of the Strong Coupling Constant at NNLO in QCD from the Three-Jet Rate in Electron–Positron Annihilation at LEP

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We present the first determination of the strong coupling constant from the three-jet rate in e^+e^- annihilation at LEP, based on a next-to-next-to-leading-order (NNLO) perturbative QCD prediction. More precisely, we extract $\alpha_s(M_Z)$ by fitting perturbative QCD predictions at $O(\alpha_s^3)$ to data from the ALEPH experiment at LEP. Over a large range of the jet-resolution parameter $y_{\rm cut}$, this observable is characterized by small nonperturbative corrections and an excellent stability under renormalization scale variation. We find $\alpha_s(M_Z) = 0.1175 \pm 0.0020({\rm expt}) \pm 0.0015({\rm theor})$, which is more accurate than the values of $\alpha_s(M_Z)$ from e^+e^- event-shape data currently used in the world average.

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Jet observables in electron-positron annihilation play an outstanding role in studying the dynamics of the strong interactions [1], described by the theory of Quantum Chromodynamics (QCD, [2]). In particular, jet rates and related event-shape observables have been extensively used for the determination of the QCD coupling constant α_s (see [3,4] for a review), mostly based on data obtained at the e^+e^- colliders PETRA, LEP, and SLC at center-of-mass energies from 14 to 209 GeV. Jets are defined using a jet algorithm, which describes how to recombine the particles in an event to form the jets. A jet algorithm consists of two ingredients: a distance measure and a recombination procedure. The distance measure is computed for each pair of particles to select the pair with the smallest separation in momentum space. If the separation is below a predefined resolution parameter $y_{\rm cut}$, the pair is combined according to the recombination procedure. The JADE algorithm [5] uses the pair invariant mass as distance measure. Several improved jet algorithms have been proposed for e^+e^- collisions: Durham [6], Geneva [7], and Cambridge [8]. The Durham algorithm has been the most widely used by experiments at LEP [9–12] and SLD [13], as well as in the reanalysis of earlier data at lower energies from JADE [14].

The Durham jet algorithm clusters utilizes the distance measure

$$y_{ij,D} = \frac{2\min(E_i^2, E_j^2)(1-\cos\theta_{ij})}{E_{\rm vis}^2} \quad (1)$$

for each pair (i, j) of particles; $E_{\rm vis}$ denotes the energy sum of all particles in the final state. The pair with the lowest $y_{ij,D}$ is replaced by a pseudoparticle whose four momentum is given by the sum of the four momenta of particles i and j ("E" recombination scheme). This procedure is repeated as long as pairs with invariant mass below the predefined resolution parameter $y_{ij,D} < y_{\rm cut}$ are found. Once the clustering is terminated, the remaining (pseudo-)particles are the jets. In experimental jet measurements, one studies the jet rates, i.e., jet cross sections normalized to the total hadronic cross section, as function of the jet-resolution parameter $y_{\rm cut}$.

The theoretical prediction of jet cross sections is made within perturbative QCD, where the same jet algorithm is applied to the final state partons. The QCD description of jet production is either based on a fixed-order calculation or a parton shower. The fixed-order approach uses exact parton-level matrix elements including higher order corrections where available and/or analytical resummation of large logarithmic corrections for a given jet multiplicity. On the other hand, the parton shower starts with the leading-order matrix element for two-jet production and generates higher multiplicities in an iterative manner, thereby accounting only for the leading logarithmic terms from parton-level processes with higher multiplicity. In multipurpose event generator programs [15–17], such parton showers are complemented by phenomenological models which describe the transition from partons to hadrons. These programs provide a satisfactory description of multijet production rates but, since they generally contain many tunable phenomenological parameters, their predictive power is limited. Nevertheless, in order to compare parton-level predictions with experimental hadronic data, these event generators are vital to estimate the effects due to hadronization and resonance decays.

Until recently, fixed-order calculations were available up to next-to-next-to-leading-order (NNLO) for two jets [18–20] and up to next-to-leading-order (NLO) for three [21–23] and four jets [24–27]. For five and more jets, only leading-order calculations are available [28–30]. For jets involving massive quarks, NLO results are available for three-jet final states [31]. The recent calculations of the α_s^3 corrections (NNLO) for three-jet production [32–35] have already led to precise α_s determinations [36–40], using event-shape observables measured by ALEPH and JADE. However, some of the event-shape variables still suffer from a poor convergence of the perturbative expansion even at NNLO. Furthermore, the usage of event generators, which have been tuned to LEP data, for the determination of the hadronization corrections may lead to a bias in the α_s measurements for some of the event shapes [40]. A comparison of different variables showed that jet broadening variables are most affected by missing higher orders and a potential hadronization bias, while the differential two-jet rate Y_3 is most robust against these effects, and strongly motivates the present study of the three-jet rate.

In this Letter, we describe a determination of the strong coupling constant from the three-jet rate measured by ALEPH [41] at LEP. We use the NNLO predictions as presented in [33]. There it was shown that: (i) For large values of $y_{\rm cut}$, $y_{\rm cut} > 10^{-2}$, the NNLO corrections turn out to be very small, while they become substantial for medium and low values of $y_{\rm cut}$; (ii) The maximum of the jet rate is shifted towards higher values of $y_{\rm cut}$ compared to NLO and is in better agreement with the experimental observations; (iii) The theoretical uncertainty is lowered considerably compared to NLO, especially in the region $10^{-1} > y_{\rm cut} > 10^{-2}$ relevant for precision phenomenology where the theory error is below two percent relative uncertainty; (iv) Finally, in this $y_{\rm cut}$ region, the parton-level predictions at NNLO are already very close to the experimental measurements, indicating the need for only small hadronization corrections.

These findings motivate a dedicated analysis of the three-jet rate, leading to a precise measurement of α_s. Our analysis closely follows the procedure described in [36,40]. The ALEPH data [41] at LEP are based on the reconstructed momenta and energies of charged and neutral particles. The measurements have been corrected for detector effects; i.e., the final distributions correspond to the so-called particle (or hadron) level, and for initial state photonic radiation. In the simulation of the detector response to particles, a bias is introduced by the choice of the physics event generator. This leads to a systematic uncertainty on the three-jet rate of about 1.5% for the relevant $y_{\rm cut}$ range. Further experimental systematic effects are estimated by a variation of the track- and event-selection cuts as advocated in [41], giving an additional small systematic uncertainty of about 1%.

We construct the perturbative expansion up to $O(\alpha_s^3)$ as described in [40], with the coefficients obtained from [33]. These are valid for massless quarks. We take into account bottom mass effects up to NLO [31], for a pole b-quark mass of $M_b = 4.5$ GeV. The latter is varied by ± 0.5 GeV in order to estimate the impact of the b-quark mass uncertainty on the value of the strong coupling. For the normalization to the total hadronic cross section $\sigma_{\rm had}$, we follow

TABLE I. Results of $\alpha_s(M_Z)$ extracted from the three-jet rate measured by ALEPH at LEP1. The uncertainty contributions are given for the statistical error (stat.), the uncertainty related to the choice of the generator for the simulation of the detector response (det.), the quadratic sum of all other experimental systematic uncertainties arising from track and event-selection cut variations (exp.), the hadronization uncertainty obtained by the maximum difference between either PYTHIA, HERWIG, or ARIADNE (had.), the uncertainty on the b-quark mass correction procedure (mass) and the uncertainty for missing higher orders (pert.) estimated by a variation of the renormalization scale.

$\ln(y_{\rm cut})$	$\alpha_s(M_Z)$	stat.	det.	exp.	had.	mass	pert.	total
−5.1	0.1110	0.0004	0.0013	0.0008	0.0003	0.0004	0.0020	0.0025
−4.9	0.1124	0.0004	0.0015	0.0007	0.0003	0.0003	0.0013	0.0022
−4.7	0.1147	0.0004	0.0015	0.0008	0.0004	0.0003	0.0012	0.0022
−4.5	0.1153	0.0004	0.0015	0.0008	0.0005	0.0003	0.0006	0.0019
−4.3	0.1159	0.0004	0.0016	0.0009	0.0005	0.0003	0.0010	0.0022
−4.1	0.1170	0.0004	0.0016	0.0009	0.0005	0.0003	0.0012	0.0023
−3.9	0.1175	0.0004	0.0016	0.0011	0.0006	0.0002	0.0014	0.0025
−3.7	0.1179	0.0004	0.0016	0.0011	0.0006	0.0002	0.0016	0.0026
−3.5	0.1183	0.0004	0.0015	0.0009	0.0006	0.0002	0.0018	0.0026
−3.3	0.1184	0.0004	0.0015	0.0011	0.0008	0.0002	0.0019	0.0029
−3.1	0.1179	0.0004	0.0016	0.0013	0.0010	0.0002	0.0021	0.0031
−2.9	0.1177	0.0004	0.0019	0.0013	0.0010	0.0002	0.0021	0.0033
−2.7	0.1180	0.0004	0.0020	0.0013	0.0013	0.0001	0.0020	0.0034
−2.5	0.1169	0.0005	0.0021	0.0015	0.0013	0.0001	0.0021	0.0036
−2.3	0.1166	0.0005	0.0019	0.0018	0.0014	0.0001	0.0021	0.0037
−2.1	0.1166	0.0006	0.0020	0.0020	0.0015	0.0001	0.0020	0.0038
−1.9	0.1191	0.0008	0.0021	0.0019	0.0014	0.0002	0.0016	0.0036
−1.7	0.1173	0.0010	0.0015	0.0023	0.0016	0.0001	0.0019	0.0038
−1.5	0.1175	0.0016	0.0005	0.0029	0.0014	0.0001	0.0017	0.0040
−1.3	0.1159	0.0037	0.0014	0.0029	0.0018	0.0004	0.0011	0.0054

the procedure adopted in [40], which is based on a N^3LO calculation [$O(\alpha_s^3)$ in QCD] for σ_{had} [42], including mass corrections for the b quark up to $O(\alpha_s)$ and the leading mass terms to $O(\alpha_s^2)$. Weak corrections to the three-jet rate were computed very recently [43]. They are at the one permille level for $Q = M_Z$ and are neglected here.

The nominal value for the renormalization scale $x_\mu = \mu/Q$ is unity. It is varied between $0.5 < x_\mu < 2$ in order to assess the systematic uncertainty related to yet unknown higher order corrections. No attempt is made to combine the NNLO predictions with resummation calculations. At present, the resummation of the three-jet rate [6] is only fully consistent at leading logarithmic level [44], and resummation effects only become numerically relevant over fixed-order NNLO for $\ln y_{cut} \lesssim -4.5$ (as can be seen from the Y_3 transition parameter distribution [45]), which is below our region of interest.

In order to compare the perturbative parton-level theoretical prediction with the hadronic data, it is necessary to apply a correction for hadronization and resonance decays. This bin-by-bin correction is computed with the PYTHIA [15], HERWIG [16], and ARIADNE [17] Monte Carlo generators, all tuned to global hadronic observables at M_Z [46]. The parton level is defined by the quarks and gluons present at the end of the parton shower in PYTHIA and HERWIG and the partons resulting from the color dipole radiation in ARIADNE. Our central values for the strong coupling constant are obtained with hadronization corrections from PYTHIA, which are at the level of 5%. We define the systematic uncertainty on $\alpha_s(M_Z)$ due to these hadronization corrections as the biggest deviation observed when using any of the other generators. Motivated by the observations in [40], we verified that the shapes of the Monte Carlo parton-level predictions are in fair agreement with those at NNLO, for reasonable choices of the strong coupling. Furthermore, the ratios of these predictions are relatively flat over the relevant y_{cut} range, giving further confidence in the reliability of the hadronization corrections.

The corrected ALEPH measurements for the three-jet rate are compared to the theoretical calculation at particle level. Values for $\alpha_s(M_Z)$ are obtained by a least-squares fit, performed separately for each y_{cut} value in the range listed in Table I (for the data at the Z peak), together with the uncertainties as described above. These results are also displayed in Fig. 1. We observe a nice stability of the results, within their total uncertainties, down to resolution parameters of $\ln y_{cut} \approx -4.5$. Beyond that value, we find a fall off of $\alpha_s(M_Z)$, most likely related to the onset of large logarithmic corrections from higher perturbative orders, which are not accounted for in our perturbative prediction.

As final result, we quote our measurement for $y_{cut} = 0.02$, which represents an optimal compromise between minimal systematic uncertainty and stability. We find

$$\alpha_s(M_Z) = 0.1175 \pm 0.0020(\text{expt}) \pm 0.0015(\text{theor})$$

where the first uncertainty includes (in quadrature) the contributions from statistics, detector corrections, and experimental selection cuts, and the second error is the quadratic sum of b-quark mass and renormalization scale uncertainties (cf. Table I). We also performed similar measurements for the LEP2 energies between 133 and 206 GeV, where we find consistent values for $\alpha_s(M_Z)$, but with considerably larger statistical uncertainties. Combining the errors in quadrature yields $\alpha_s(M_Z) = 0.1175 \pm 0.0025$ which is in excellent agreement with the latest world average value [4] of $\alpha_s(M_Z) = 0.1184 \pm 0.0007$ that is based on a number of measurements from τ decay, lattice gauge theory, Upsilon decay, deep-inelastic scattering, and e^+e^- data. As expected, our theoretical uncertainty is smaller than that obtained from fits of event-shape distributions, and even smaller than the experimental error, which is dominated by the model-dependence of the detector corrections. Our result is also more precise than the two extractions of α_s from e^+e^- event-shape data [39,40] currently used in the world average [4].

In this Letter, we reported on the first determination of the strong coupling constant from the three-jet rate in e^+e^- annihilation at LEP, based on a NNLO perturbative QCD prediction. We find a precise value of $\alpha_s(M_Z)$ with an uncertainty of 2%, consistent with the world average. This verifies the expectations that the three-jet rate is an excellent observable for this kind of analysis, thanks to the good behavior of its perturbative and nonperturbative contributions over a sizable range of jet-resolution parameters.

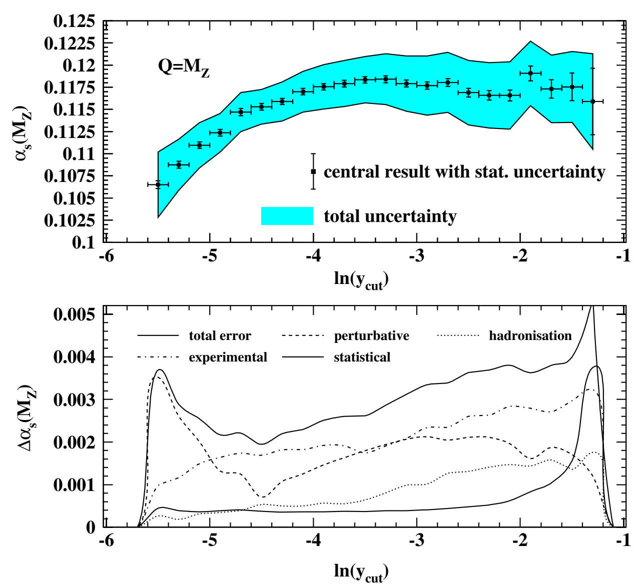

FIG. 1 (color online). Determinations of $\alpha_s(M_Z)$ from the three-jet rate, measured by ALEPH at the Z peak, for several values of the jet-resolution parameter y_{cut}. The error bars show the statistical uncertainty, whereas the shaded band indicates the total error, including the systematic uncertainty. The various contributions to the latter are displayed in the lower plot.

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