Electron Teleportation via Majorana Bound States in a Mesoscopic Superconductor

Liang Fu
Department of Physics, Harvard University, Cambridge, Massachusetts 02138, USA
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Zero-energy Majorana bound states in superconductors have been proposed to be potential building blocks of a topological quantum computer, because quantum information can be encoded nonlocally in the fermion occupation of a pair of spatially separated Majorana bound states. However, despite intensive efforts, nonlocal signatures of Majorana bound states have not been found in charge transport. In this work, we predict a striking nonlocal phase-coherent electron transfer process by virtue of tunneling in and out of a pair of Majorana bound states. This teleportation phenomenon only exists in a mesoscopic superconductor because of an all-important but previously overlooked charging energy. We propose an experimental setup to detect this phenomenon in a superconductor–quantum-spin-Hall-insulator–-magnetic-insulator hybrid system.

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Majorana bound states are localized zero-energy excitations of a superconductor [1,2]. An isolated Majorana bound state is an equal superposition of electron and hole excitations and therefore not a fermionic state. Instead, two spatially separated Majorana bound states together make one zero-energy fermion level [1,3] which can be either occupied or empty. This defines a two-level system which can store quantum information nonlocally, as needed to realize topological quantum computation [4,5]. While several schemes have been recently proposed to detect the existence of individual Majorana bound states [6–12], experimental signatures of the nonlocal fermion occupation of these states remain to be found.

In this work, we predict a nonlocal electron transfer process due to Majorana bound states in a mesosopic superconductor: an electron which is injected into one Majorana bound state can go out from another one far apart maintaining phase coherence. Strikingly, the transmission phase shift is independent of the distance "traveled." In such a sense, we call this phenomenon "electron teleportation." It occurs because of the nonlocal fermion occupation of Majorana bound states and the finite charging energy of a mesoscopic superconductor. The all-important role of charging energy in the study of Majorana fermions has not been recognized before. We propose a realistic scheme to detect the teleportation phenomena in a superconductor–quantum-spin-Hall-insulator–magnetic-insulator hybrid system, which have been recently shown to host Majorana bound states [13,14].

In a macroscopic s-wave superconductor, charge e excitations have a pairing energy gap, whereas charge $2e$ excitations cost zero energy. Therefore the ground state manifold consists of states with an even number of electrons only, as shown in Fig. 1(a). The BCS wave function of the ground state with a definite overall superconducting phase $\phi \in [0, 2\pi]$ is a linear superposition of states with $2N$ electrons. Now consider that a pair of zero-energy Majorana bound states are present at positions R_1 and R_2 in the superconductor, and all other quasiparticle excitations have a finite gap greater than an energy scale Δ. We shall show later how this situation can be realized in a device consisting of an s-wave superconductor and the recently discovered quantum spin Hall (QSH) insulator HgTe quantum well [15,16]. The two Majorana operators γ_1 and γ_2 are defined by

$$\gamma_{1,2} \equiv \int dx e^{-i\phi/2} \xi_{1,2}(x) c^\dagger(x) + e^{i\phi/2} \xi^*_{1,2}(x) c(x). \quad (1)$$

Here $\xi_{1,2}(x)$ are bound state wave functions centered at $R_{1,2}$. We assume that the distance between the two Majorana bound states is much larger than the coherence length—a necessary condition for the notion of nonlocality to be meaningful. A single fermionic operator can then be defined $d^\dagger \equiv (\gamma_1 + i\gamma_2)/2$, which accommodates an extra

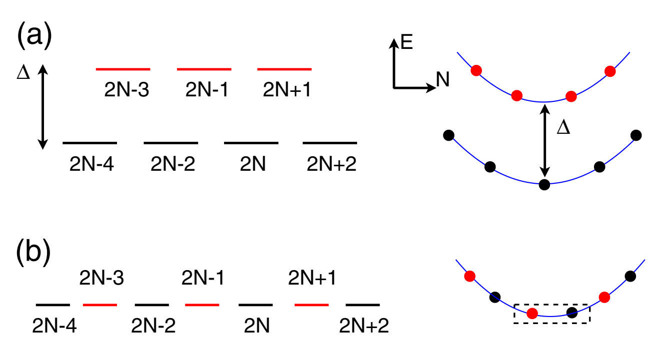

FIG. 1 (color online). Energy spectrum of a superconductor as a function of total number of electrons. States with an even and an odd number of electrons are marked in black and red (gray), respectively. Panels (a) and (b) correspond to superconductors without and with a pair of zero-energy Majorana bound states. Figures on the left and right correspond to superconductors without and with charging energy.

fermion excitation without any energy cost. Now the ground states of the superconductor have two sectors $|e\rangle$ and $|o\rangle$ defined by $d|e\rangle = 0$, $|o\rangle = d^\dagger|e\rangle$, which have an even and odd number of electrons, respectively, as shown in Fig. 1(b),

$$|e, \phi\rangle = \sum_{n=2N} a_n e^{i\phi N}|2N\rangle,$$
$$|o, \phi\rangle = \sum_{n=2N+1} a_n e^{i\phi(N+1/2)}|2N+1\rangle, \quad (2)$$

where a_n is real and slowly varying at large n. Equation (2) says that the fermion occupation of the d level (empty or occupied) $d^\dagger d = (i\gamma_1\gamma_2 + 1)/2$ is fixed by the total number of electrons in the superconductor mod 2:

$$i\gamma_1\gamma_2 = (-1)^n. \quad (3)$$

Equation (3) imposes constraint on the Hilbert space. Equivalently, Eq. (3) implements a gauge transformation $\gamma_j \to -\gamma_j$, $\phi \to \phi + 2\pi$, which is a gauge symmetry in the definition of Majorana operators Eq. (1).

If the superconductor under consideration is of mesoscopic size and connected to ground by a capacitor, the energy spectrum has an additional term due to the finite charging energy:

$$U_c(n) = (ne - Q_0)^2/2C, \qquad n = 0, \pm 1, \pm 2, \ldots, \quad (4)$$

where Q_0 is the gate charge determined by the gate voltage V_g across the capacitor. As a result, states with different n are no longer degenerate. In this work we will consider the regime $U \equiv e^2/C < \Delta$, which can always be satisfied by increasing the size of the superconductor. The low-energy spectrum ($E < U$) then depends crucially on whether Majorana bound states are absent or present. In the former, only states with an even number of electrons are accessible at low energy, which leads to an even-odd effect in tunneling experiments on mesoscopic superconductors [17]. In contrast, if Majorana bound states are present, both even and odd states appear on equal footing in the low-energy spectrum. In this case, when Q_0/e is adjusted to half-integers, an energy-level degeneracy can be achieved between two states that differ by charge e instead of $2e$ as in a usual superconductor with Coulomb blockade. This two-level system shown in Fig. 1 is the main subject of our study.

We now weakly couple the two Majorana bound states to separate normal metal leads. This can be realized in an s-wave superconductor–quantum-spin-Hall-insulator–-magnetic-insulator hybrid system. The QSH insulator used here is a new phase of two-dimensional insulators which have robust edge states [18]. It has been experimentally realized in HgTe quantum wells [15]. The device geometry is shown in Fig. 2: an s-wave superconductor and a magnet are deposited on top of the QSH insulator. Both superconducting proximity effect and Zeeman field of the magnet open up a finite quasiparticle gap for the QSH edge states. However, two zero-energy Majorana

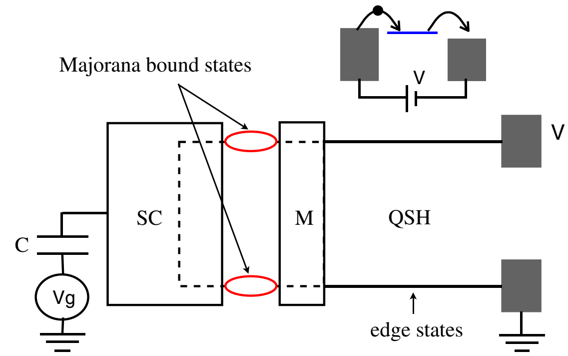

FIG. 2 (color online). The device used to study electron tunneling from leads into two Majorana bound states, consisting of an s-wave superconductor (SC) and a magnetic insulator (M) junction deposited on top of a quantum spin Hall insulator (QSH). By tuning the gate voltage across the capacitor, the superconductor is close to a charge degeneracy point between a total number of electron n_0 and $(n_0 + 1)$. This two-level system corresponds to a resonant level empty or occupied. At a small bias, electron tunneling through two Majorana bound states is equivalent to phase-coherent tunneling through a single resonant level.

bound states $\gamma_{1,2}$ exist at the intersection of the superconductor-magnet interface with the top and bottom edge, respectively [14], conceptually similar to the states localized at the ends of a one-dimensional spinless p-wave superconductor [19]. The edge states in the uncovered part of the quantum spin Hall insulator are naturally used as leads to connect γ_1 and γ_2 to source and drain.

To describe electron tunneling between the lead and the superconductor, we write the electron operator in terms of quasiparticle operators of the superconductor

$$c(x) = e^{-i\phi/2}[\xi_1(x)\gamma_1 + \xi_2(x)\gamma_2 + \cdots]. \quad (5)$$

Since we will only consider small bias voltage $V < U < \Delta$, only zero-energy Majorana operators are important and contributions from other quasiparticle states can be neglected in (5). We now write down the Hamiltonian for the system in Fig. 2: $H = H_L + U_c + H_T$, where $H_L = \sum_{k,j=1,2}\epsilon_j(k)c^\dagger_{j,k}c_{j,k}$ describes the two leads, U_c is the charging energy defined in (4). The effective tunneling Hamiltonian H_T at low energy is given by substituting (5) into the bare tunneling term $t_i c_i^\dagger c$:

$$H_T = \sum_{j=1,2}[\lambda_j c_j^\dagger \gamma_j e^{-i\phi/2} + \lambda_j^* \gamma_j c_j e^{i\phi/2}]. \quad (6)$$

Here c_j annihilates an electron in lead j and $\lambda_j \propto \xi_j(R_j)$ is the tunneling matrix element. As we emphasized earlier, $\xi_1(x)$ and $\xi_2(x)$ have essentially zero wave function overlap so that no coupling between $c_1(c_2)$ and $\gamma_2(\gamma_1)$ exists. The operator $e^{\pm i\phi/2}$ in H_T increases or decreases the total charge of the superconductor by one charge unit $[n, e^{\pm i\phi/2}] = \pm e^{\pm i\phi/2}$, and the Majorana operator $\gamma_{1,2}$ changes the parity of electron number in the superconductor. The "naive" Hilbert space of H is simply the direct product of electron number eigenstate $|n\rangle$ and the two states of d level ($|e\rangle$ and $|o\rangle$), but it is redundant. Instead, the physical Hilbert space only consists of those states $|n = 2N; e\rangle$ and $|n = 2N + 1; o\rangle$ that satisfy the gauge constraint (3).

When the source is biased at a small voltage V, current flows to drain by electron tunneling in and out of the superconductor. Since charging energy U_c favors states with a fixed number of charge in the superconductor, only two charge states $|n_0\rangle$ and $|n_0 + 1\rangle$ give dominant contribution to the current for $V < U$, which is similar to tunneling through a quantum dot in the Coulomb blockade regime. To a good approximation, we can then truncate the Hilbert space keeping only these two states, which we label by $s_z = \pm 1$. H then becomes

$$\tilde{H} = H_L + \frac{\delta}{2} s_z + \sum_{j=1,2} [\lambda_j c_j^\dagger \gamma_j s_- + \lambda_j^* \gamma_j c_j s_+]. \quad (7)$$

Here δ is the energy difference between $|n_0\rangle$ and $|n_0 + 1\rangle$ and depends on the gate voltage. The gauge symmetry (3) then becomes $i\gamma_1\gamma_2 s_z = (-1)^{n_0}$. The key to solving the tunneling problem (7) is to combine Majorana and spin operators into a singe fermion operator f.

$$\begin{aligned} \gamma_1 s^+ &\to f^+, & \gamma_1 s^- &\to f, \\ \gamma_2 s^+ &\to i(-1)^{n_0} f^+, & \gamma_2 s^- &\to i(-1)^{n_0+1} f. \end{aligned} \quad (8)$$

We have checked that this transformation preserves all commutation relations

$$\{\gamma_j s^+, \gamma_j s^-\} = 1, \qquad \{\gamma_i s^+, \gamma_j s^+\} = \{\gamma_i s^-, \gamma_j s^-\} = 0,$$
$$\{\gamma_1 s^+, \gamma_2 s^-\} = \gamma_1\gamma_2 s_z = i(-1)^{n_0+1}, \quad (9)$$

where the gauge constraint is used in the last equation. Conceptually, it is not surprising that the transformation (8) works: after all, the two charge states $|n_0\rangle$ and $|n_0 + 1\rangle$ differ by one electron. Using (8), we rewrite the Hamiltonian $\tilde{H}$ in terms of the fermion operator f:

$$\tilde{H} = H_L + \delta\left(f^\dagger f - \frac{1}{2}\right) + (\lambda_1 c_1^\dagger f + \text{H.c.}) + (-1)^{n_0}(-i\lambda_2 c_2^\dagger f + \text{H.c.}). \quad (10)$$

Equations (6)–(8) and (10) are the main results of this work, and to the best of our knowledge they have not been reported before. Equation (10) says that electron tunneling in and out of two spatially separated Majorana bound states is equivalent to resonant tunneling through a single level, as shown schematically in Fig. 2. Since resonant tunneling is a coherent process, we conclude that an incident electron at $E < U$ tunnels into one Majorana bound state and comes out from its partner far apart still maintaining phase coherence. Strikingly, the magnitude and phase of the transmission amplitude—which we call t—is independent of their distance. In this sense, we call such a nonlocal electron transfer process "teleportation."

The phase coherence over a long distance shown here is in fact a direct consequence of Majorana bound states. Conceptually it can be best understood from electron's Green function $G^{e(o)}(x, t; y, 0) \equiv \langle c(x, t)c^\dagger(y, 0)\rangle_{e(o)}$ defined in the even and odd ground state sector $|e\rangle$ and $|o\rangle$, respectively [20]. Using (5), we find

$$G^{e,o}(x, t \to \infty; y, 0) = \mp i\xi_2^*(y)\xi_1(x) \sim O(1) \quad (11)$$

is finite for $x \sim R_1$, $y \sim R_2$. The long-time limit corresponds to the low-bias regime we are interested in. The fact that (11) is independent of $|R_1 - R_2|$ is most unusual, as first pointed out in Ref. [21].

We now show that, interestingly, the phase of transmission amplitude t depends on the gate charge Q_0 in a surprising way, and therefore it is sensitive to the fermion occupation ($|e\rangle$ versus $|o\rangle$) of the Majorana bound state pair. Consider tuning gate voltage to make Q_0 change by one charge unit. The number of electrons in the superconductor ground state will correspondingly change by one. Although the excitation energy spectrum U_c comes back to itself, we find

$$t \to -t, \quad \text{when } Q_0 \to Q_0 \pm e; \quad (12)$$

i.e., the transmission phase shift changes by π. This behavior is related to the change of fermion number parity in the ground state. The property (12) is evident from the $(-1)^{n_0}$ factor in $\tilde{H}$, which is valid in the two-level approximation. Using second-order perturbation theory in the weak tunneling limit, one can easily show that this $(-1)^{n_0}$ factor comes from the $\pm$ sign in Eq. (11). In general, we can prove (12) using the following symmetry of the full Hamiltonian H and the gauge constraint (3):

$$UH(Q_0, \lambda_2)U^{-1} = H(Q_0 + e, -\lambda_2),$$
$$Ui\gamma_1\gamma_2(-1)^n U^{-1} = i\gamma_1\gamma_2(-1)^n, \qquad U \equiv \gamma_2 e^{-i\phi/2}.$$

To detect the phase coherence of the electron teleportation described above, we consider the interferometer setup in Fig. 3: a point contact is introduced to partially scatter an incident electron at the top edge directly to the bottom edge, and partially transmit it to the superconductor which

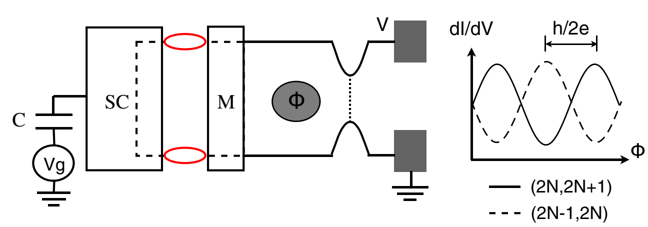

FIG. 3 (color online). Left: An interferometer that probes the phase-coherent electron teleportation via two Majorana bound states. Right: Schematic plot of zero bias differential conductance as a function of magnetic flux at two successive charge degeneracy points $(2N - 1, 2N)$ and $(2N, 2N + 1)$. The $h/2e$ shift in conductance peak signals the change of fermion number parity in the superconductor.

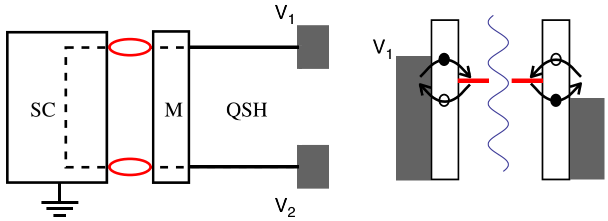

FIG. 4 (color online). Compared with Fig. 2, the superconductor here is grounded without charging energy. Charge transfer between the lead and the superconductor is conducted by local Andreev reflection that transfers charge $2e$.

subsequently comes out at the bottom edge. Interference between the two paths with a magnetic flux Φ enclosed leads to a Φ-dependent differential conductance dI/dV, which is h/e-periodic. If the direct scattering probability is large, interference visibility is maximum at the charge degeneracy point when electron tunneling through Majorana bound states is on resonance. We schematically plot the Φ dependence of dI/dV (at zero bias and zero temperature) for two successive half-integer charges Q_0 in Fig. 3. The sign reversal of t discussed in (12) leads to a $h/2e$ shift in the interference pattern.

Discussion.—It is instructive to compare our study of tunneling into Majorana bound states in the $V < U$ regime with previous studies which do not include the charging energy U [6–8]. Instead of using a "floating" superconductor as in Fig. 2, these works consider a grounded superconductor in tunneling contact with two leads each having an independent bias voltage. Such a three-terminal device can be realized in a geometry shown in Fig. 4. In this setup, transferring two electrons to the superconducting condensate does not cost energy. Therefore, when the two Majorana bound states are sufficiently far apart, an incident electron from a lead can be Andreev reflected as a hole to the same lead, but will never appear in the other lead. In other words, no electron teleportation happens there. Indeed, Bolech and Demler have shown [6] that (a) the conductance of each lead is $2e^2/h$ at zero bias and zero temperature, which is a sign of charge $2e$ transfer by Andreev reflection, and (b) the two leads have independent currents without any correlation (no teleportation). The same results were also obtained using scattering approach within Bogoliubov–de Gennes formalism [8]. In the two-terminal device we studied (Fig. 2), charge $2e$ transfer is suppressed by charging energy and the conductance is at most e^2/h because of single electron tunneling.

Finally, we discuss the feasibility of experiments using the quantum spin Hall insulator HgTe quantum well. A good candidate for the superconductor in our proposed setup is indium with $T_c = 3.4$ K, which is currently used as electrodes to contact HgTe [22]. The relevant parameters for these materials have been estimated [8,14]. Assuming a proximity-induced gap $\Delta \sim 0.1$ meV, the penetration length of Majorana bound states is about 3 μm. So if the top and bottom Majorana bound states in Fig. 2 are 30 μm apart, direct tunneling between them is negligible. If charging energy of the superconductor can be optimized to be comparable to Δ, the resonant-tunneling model for electron teleportation described above is valid below 1 K. The level broadening Γ from coupling to leads sets the temperature scale for detecting the phase coherence.

In summary, we reveal a striking nonlocal electron transport phenomenon through Majorana bound states in a finite-sized superconductor with charging energy. Most interestingly, the transmission phase shift detects the state of a qubit made of two spatially separated Majorana bound states. In a future work [23], we will propose a generalized scheme to electrically detect the internal states of multiple Majorana bound states, with an emphasis on implementing topological quantum computation.

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