Composite Acoustic Medium with Simultaneously Negative Density and Modulus

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We fabricated an acoustic composite structure consisting of a periodic array of interspaced membranes and side holes. Experimental data on the transmission, effective density, and phase velocity are presented. The system exhibits two critical frequencies, ω_{SH} and ω_c. Our metamaterial is double negative and transparent for frequencies lower than ω_{SH}. For the frequencies $\omega_{SH} < \omega < \omega_c$, the medium is opaque and only the density is negative. For the frequencies above ω_c, the system is double positive and transparent. The present medium exhibits a very wide double negative spectral range that opens the possibility of the application of metamaterials for "white lights."

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Metamaterials with negative electromagnetic constitutive parameters give new propagation characteristics for electromagnetic waves [1–9]. Negative permittivity occurs naturally due to the plasma oscillation [2], but negative permeability became available only by Pendry's inventive spirit [3]. A composite structure consisting of the elements of negative permittivity (metal wire) and negative permeability (split ring resonator) resulted in simultaneously negative values of ϵ and μ [double negativity (DNG)] [4,5]. Since light and sound share many important wave characteristics, it is expected that acoustic DNG materials, with simultaneously negative modulus and density, can also be constructed. DNG materials exhibit a negative phase velocity, which means that the wave crests move towards the source [1–6]. Intense effort to develop acoustic metamaterials has resulted in several theoretical and experimental advances [10–18]. It was theoretically demonstrated that an acoustic DNG structure can be achieved by mixing two structures having independently negative bulk modulus and negative mass density [19]. Acoustic media with negative modulus were constructed using elements such as Helmholtz resonators or side holes [14,16]. However, acoustic composite DNG structure has not been fabricated because a structure for the negative density was not available. Very recently, an acoustic medium with a negative density was fabricated using an array of thin membranes [17]. Here we present a composite structure consisting of elements of negative modulus (the side hole) and negative density (the thin membrane).

Figures 1(a) and 1(b) are schematics of the structures of the negative density material based on an array of thin membranes [17] and the negative modulus material consisting of the side holes [16], respectively. Sections of only three unit cells are shown for clarity. Figure 1(c) shows the new composite structure, consisting of interspaced membranes and side holes. The length ($d = 70$ mm) and the inner diameter (32.3 mm) of the unit cells for the structures 1(a)–1(c) are identical. In the structure 1(c), both a membrane and a side hole are placed in each unit cell. The side hole is identical to that of structure 1(b), and the membrane is identical in size and tension to that of structure 1(a). We measured acoustic wave propagation in the metamaterials using the experimental setup shown in Fig. 1(d). The

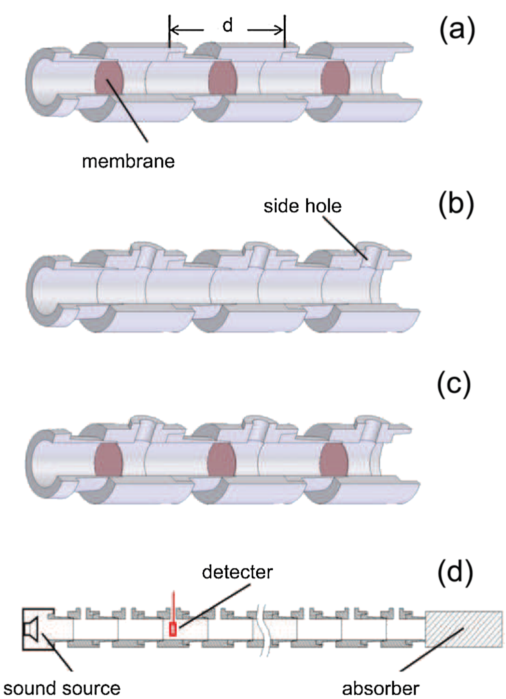

FIG. 1 (color online). (a) A cutoff view of the negative density structure with an array of thin membranes. (b) Negative modulus structure with an array of side holes. (c) The composite structure consisting of interspaced membranes and side holes, which exhibits acoustic DNG. (d) Experimental setup for the measurements of wave characteristics.

speaker at the left transmits acoustic waves to propagate along the structure. The absorber at the right absorbs acoustic energy, making the reflections negligibly small, so that the system behaves as if it extends to infinity. This removes concerns about the effect of finite number of cells used in the experiment, as well as the interference effect from the reflected waves. The absorber is of similar construction as described in Refs. [16,17]. Pressure was measured using a miniature microphone, which was placed into the cell through the side hole, and was moved from one cell to the next to collect the pressure data as a function of time and position.

The negative density material in Fig. 1(a) is a tube with an array of thin tight membranes inside, which exhibits negative density below a cutoff frequency ω_c [17]. The effective density is a function of frequency,

$$\rho_{\text{eff}} = \rho'(1 - \omega_c^2/\omega^2), \tag{1}$$

where ρ' is the average density of the fluid loaded with the membranes ($\rho' \sim 1.34\ \text{kg/m}^3$), which is obtained using the directly measured density of the membrane ($1.0 \times 10^{-5}\ \text{kg/m}^2$). The structure was opaque for the sounds below the plasma frequency, $f_c = 735$ Hz ($f = \omega/2\pi$). The transmission data are shown with open circles in Fig. 2(a).

The structure shown in Fig. 1(b) was reported to exhibit the effective modulus [16] given by

$$B_{\text{eff}} = B(1 - \omega_{\text{SH}}^2/\omega^2)^{-1}. \tag{2}$$

The cutoff frequency was $f_{\text{SH}} = 450$ Hz. The transmission data of this structure are shown in Fig. 2(a) with open triangles. Below ω_{SH}, the material has a negative modulus and is opaque, but above ω_{SH} the modulus is positive and the material is transparent. Experimental transmission characteristics of the structure 1(c) are shown in Fig. 2(b). The transmission for a given frequency was determined by taking the ratio of the pressure amplitudes in the metamaterial at two positions 1.3 m apart (at $z = 0$ and at $z = 1.3$ m). From Fig. 2(b) one can see that there is a frequency gap in the range $\omega_{\text{SH}} < \omega < \omega_c$, and that for the frequencies below and above this gap the medium is transparent. The finite values of the transmission data in the frequency gap represent the noise level of our instruments. It is interesting that, below ω_{SH}, both of the original structures 1(a) and 1(b) are opaque, but as the two structures are combined into the composite structure 1(c), the resulting medium becomes transparent. The same behavior was observed in the electromagnetic DNG composite material by Smith *et al.* and also predicted theoretically for acoustic

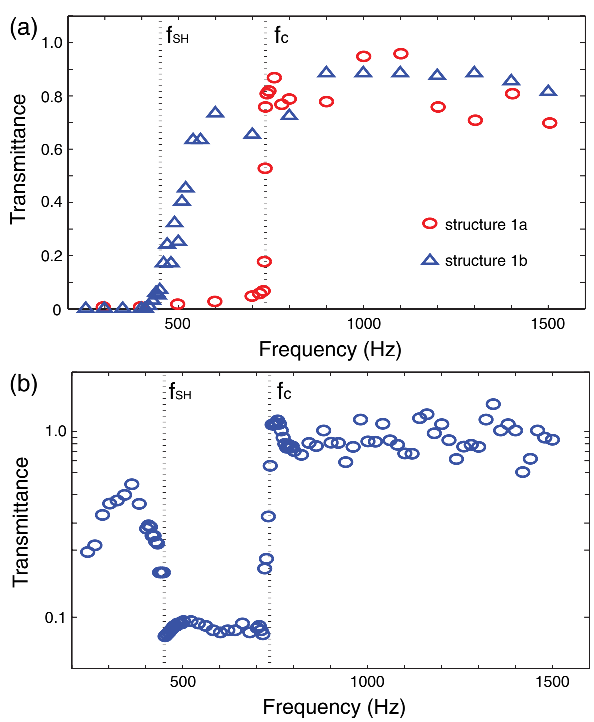

FIG. 2 (color online). (a) Transmission data for the SNG materials of structures 1(a) and 1(b). The structures have cutoff frequencies f_c for 1(a) and f_{SH} for 1(b). (b) Transmission for the composite structure 1(c), with a frequency gap, and passbands below and above the gap.

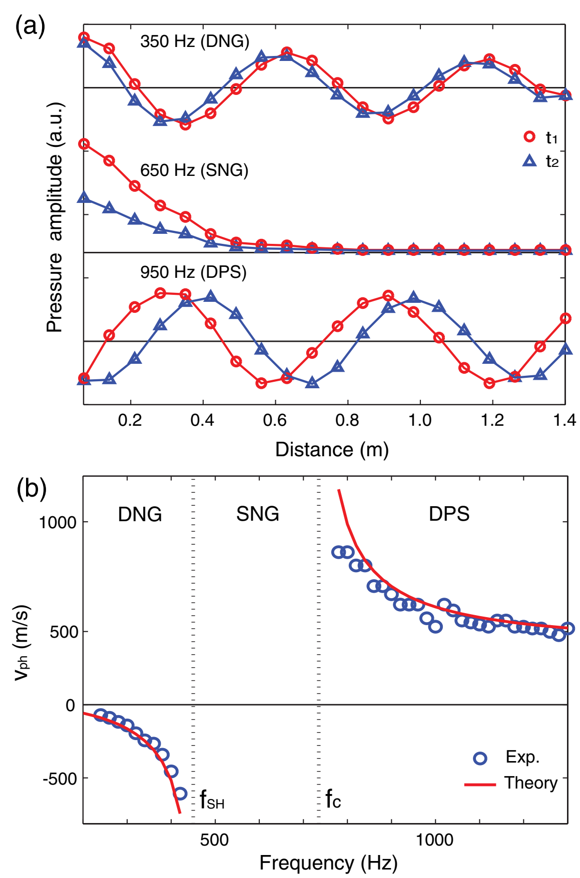

FIG. 3 (color online). (a) Visualization of the three typical waves ($t_2 = t_1 + \Delta t$). At 350 Hz the wave propagated backwards, at 650 Hz the wave was evanescent, and at 950 Hz the wave traveled forward. (b) Phase velocity as a function of frequency.

metamaterials [4,19]. Our composite structure is transparent above ω_c, where the two original structures are both transparent. For the frequency range $\omega_{SH} < \omega < \omega_c$, where only one of the two original structures is transparent, the composite structure 1(c) is opaque. It is surprising that the boundaries of the frequency ranges in the structure 1(c) match precisely the critical frequencies for the original structures 1(a) and 1(b). This indicates that each element of the composite structure functions on its own, independent of the presence of the other. To carefully examine this mutual independence, we start with a hypothesis: The effective density and the effective modulus of the composite structure follow Eqs. (1) and (2) simultaneously. According to this hypothesis, the system is double negative and transparent for frequencies lower than ω_{SH}. For the frequency range $\omega_{SH} < \omega < \omega_c$, the medium is opaque because only the density is negative [single negative (SNG)]. For the frequencies above ω_c, the system is double positive (DPS) and transparent. This explains the transmission data shown in Fig. 2(b). We note, however, that the hypothesis is not intuitively obvious: Eqs. (1) and (2) are characteristics of the two independent single negative materials, and there is no obvious reason for the composite material to follow them simultaneously. In this sense, to find the reason why this hypothesis is valid may be a further task for theoreticians. In this Letter, we report the following experimental facts, which demonstrate the validity of the hypothesis. Fact (1), the acoustic waves below ω_{SH} have negative phase velocities. Fact (2), the range of negative density exhibited by the structure 1(a) remained exactly the same in structure 1(c), despite the presence of the additional side holes.

Typical motions of acoustic waves in DNG, SNG, and DPS ranges are shown in Fig. 3(a), for the frequencies 350, 650, and 950 Hz, respectively. To show the motion, two consecutive "snapshots" in $\Delta t = 0.25$ ms interval are taken for each wave. The pressure data were directly from the microphone inside each cell; thus, the waves shown are visualizations of actual acoustic waves in the tube. The lines connecting data points are to guide the eye. The propagation characteristics of the three waves are different. For the frequency of 350 Hz, the wave propagated backwards. The phase velocity was negative, $v_{ph} = -204$ m/s. For the frequency of 650 Hz, the wave decayed and did not propagate through the metamaterial. For the frequency of 950 Hz, the phase velocity was positive, $v_{ph} = 547$ m/s. Obtained this way, v_{ph} data for the whole frequencies are shown in Fig. 3(b), together with the theoretical curves from the calculation given below. We note that the phase velocities are all negative for the frequencies below ω_{SH}, which provides fact (1).

The wave profile at 350 Hz in Fig. 3(a) shows the sinusoidal wave profile slightly decaying with the distance in the metamaterial. Magnitudes of the real and imaginary components of the wave number can be estimated from the wavelength and decay rate of the wave. The real component ($\sim$10 m^{-1}) is about 1 order of magnitude larger than the imaginary component ($\sim$0.6 m^{-1}). This indicates that the negative phase velocity is mainly from DNG rather than from the effect of dissipation because it is well established that, when negative phase velocity is from the effect of dissipation, the real and imaginary components of the wave vector are the same in magnitude [20].

Figure 4 shows the result of direct measurement of the effective density. We used the same method described in Ref. [17] to directly determine the motion of the fluid relative to the pressure gradient. The method in essence is detecting the motion of the laser beam reflected from the membrane to observe the displacement of the fluid in the tube. The effective density can be obtained from the ratio of the pressure gradient and the acceleration of the fluid caused by the pressure gradient. In case the density is negative, it is expected that $\cos\phi < 0$, where ϕ is the phase difference between the fluid displacement and the pressure gradient. The data shown in Figs. 4(a) and 4(b) show sharp transitions of the observed values of $\cos\phi$ from negative to positive at the same frequency 735 Hz. The ranges for the negative density are the same for structures 1(a) and 1(c). This provides fact (2). It is noted that the density for the structure 1(c) is negative in the passband below ω_{SH}, where the phase velocity is negative. We do not have experimental data on whether the modulus is negative or positive in the frequency range below ω_{SH}. However, if the modulus were positive, the medium would be single negative and opaque. Since structure 1(c) is transparent, it is clear that our metamaterial is double negative below ω_{SH}.

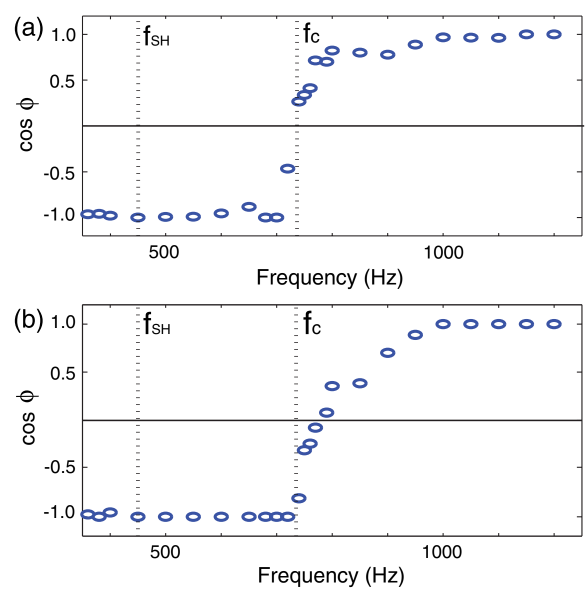

FIG. 4 (color online). (a) The $\cos\phi$ values for the SNG structure 1(a). (b) The $\cos\phi$ values for the DNG structure 1(c). The range for the negative density ($\cos\phi < 0$) are the same for structures 1(a) and 1(c), even though additional side holes are present in 1(c).

From Eqs. (1) and (2), the continuity equation and Newton's equation can be written

$$\nabla \cdot \vec{u} = \frac{1}{B_{\rm eff}} \frac{\partial p}{\partial t}, \qquad -\nabla p = \rho_{\rm eff} \frac{\partial \vec{u}}{\partial t}, \tag{3}$$

where p and $\vec{u}$ are the pressure and the particle velocity inside the tube, respectively. From these equations, the Helmholtz equation is obtained,

$$\frac{\partial^2 p}{\partial z^2} + k^2 p = 0, \tag{4}$$

where the wave number k and the phase velocity $v_{\rm ph}$ are given by

$$k = \sqrt{\frac{\omega^2 \rho_{\rm eff}}{B_{\rm eff}}} = \omega \sqrt{\frac{\rho'}{B}} \sqrt{1 - \frac{\omega_{\rm SH}^2}{\omega^2}} \sqrt{1 - \frac{\omega_c^2}{\omega^2}}, \tag{5}$$

$$v_{\rm ph} = \sqrt{\frac{B}{\rho'}} \sqrt{\frac{1}{1 - \omega_{\rm SH}^2/\omega^2}} \sqrt{\frac{1}{1 - \omega_c^2/\omega^2}}. \tag{6}$$

The theoretical curve for the phase velocity from Eq. (6) is drawn in Fig. 3(b). It is negative for the frequencies below $\omega_{\rm SH}$ and is positive for the frequencies above ω_c. The experimental data agree excellently with the theoretical curve.

In conclusion, we fabricated a DNG acoustic metamaterial consisting of membranes and side holes. Negative phase velocities were observed for the frequencies below $\omega_{\rm SH}$. Because of the lumped nature, we do not expect that the frequency range of the DNG extends down to zero frequency. However, the range of the double negativity in the present medium spans at least from 240 to 450 Hz, which is more than 40% in the scale of $\omega_{\rm SH}$, and thus is very wide compared to other DNG materials. Such a wide spectral range achieved in the present structure opens the possibility of application of DNG materials for "white lights."

We expect the present material to be useful for new acoustic applications and devices, such as superlensing and cloaking [21–27]. In addition, our metamaterial, following Smith *et al.*'s electromagnetic example [4], provides an acoustic case of composite materials that exhibit double negativity. These examples suggest the possibility of double negativity in other composite structures as well.

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