Gaudio *et al.* Reply: In our Letter [1], we investigated the origin of the rounded behavior of the nonclassical rotational inertia as function of temperature and its dependence on the ^{3}He impurities. We generalized the theory of Kotsubo and Williams [2], initially proposed for superfluid (SF) ^{4}He films adsorbed in porous materials, to the specific case of unannealed ^{4}He solid. In particular, we proposed that a two-dimensional Berezinskii-Kosterlitz-Thouless (BKT) SF transition occurs on the premelted liquid film on the grain boundaries, where the dimension of the grain provides a finite size scale for the BKT transition, accounting therefore for the rounded temperature behavior. To illustrate these effects, we concentrated in our Letter on a single grain model, whereas the role of the random network of the 2D superfluid surfaces of the grain boundaries, beyond the scopes of our Letter, was not explicitly addressed.

In their Comment [3], Yucesoy *et al.*, using numerical quantum Monte Carlo simulations, address the issue of how a three-dimensional (3D) grain surface connectivity can modify these results. They first consider an XY model on a single two-dimensional (2D) plaquette of size l to simulate the physics of a single finite size grain. Their numerical results agree very nicely with our analytical approach, with a rounded behavior of the SF density as a function of temperature. As a test, they also consider the XY model on a cubic $N \times N \times N$ three-dimensional system, where they, not surprisingly, find a sharp 3D XY transition. To simulate the connectivity network of the grain surface, they finally consider the XY model on the facets of $N \times N \times N$ cubes, the size of each being l (see Fig. 1). They introduce some degree of disorder by randomly eliminating 70% of the plaquettes. By means of their numerical simulations, they find that the superfluid density in this latter geometry displays a sharp 3D-like transition. They claim that this latter geometry would mimic a realistic 3D network of connected 2D grain surfaces, so they conclude that the results from a single finite size grain do not apply in realistic systems where the connectivity between the grain surfaces provides a three-dimensional environment.

We have, however, strong perplexities about the representative of their chosen geometry to simulate realistic systems. We note, for instance, that, in the absence of disorder, the geometry considered in the Comment can be viewed as obtained by the superposition of a periodical array of N planes in the x direction, as well as in the y and z directions (see Fig. 1). The main drawbacks of such geometry are (i) the size of each cube (meant to simulate a grain) is fixed, (ii) the angle between different plaquettes is also fixed, and (iii) the coupling J connecting different plaquettes is the same as within each plaquette. All of these features favor the formation of 3D coherent correlations. Note also that in this geometry, the length l does not act anymore as finite size scale for the 2D BKT in the plane which is limited instead by the scale $l \times N$. The additional randomness introduced in the Comment (random removing of 70% of plaquettes) is quite unphysical (since it assumes the disappearing of some grain surfaces) and in any case does not affect the above features (i)–(iii), enforcing a 3D character. In our opinion, a more realistic description of the 3D network of two-dimensional superfluid grain surfaces should take into account instead the randomness of the *size* of the grains and of the *orientation* of the grain boundaries. Such effects question thus the assumptions (i)–(ii) without affecting the three dimensionality of the network. Also, the Josephson coupling between different grains in realistic systems is expected to be smaller than on the grain surface. This effect should be taken into account by using a smaller constant $J' < J$ at the plaquette edge than within a plaquette, preserving finite size effects. Further investigations, with more realistic geometries and realistic kinds of disorder, are thus needed in our opinion to assess this issue. We suggest, for instance, to investigate similar effects on the 3D network defined by the surfaces of Voronoi cells of a random distribution of points with given density.

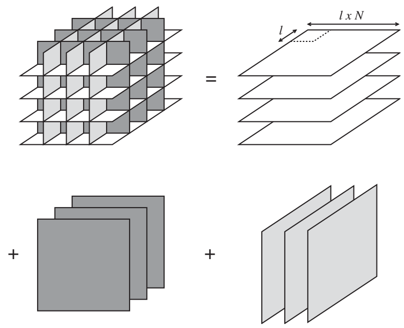

FIG. 1. 3D network of 2D plaquette in the absence of disorder, as chosen by Yucesoy *et al.* in the Comment.

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