Comment on "Finite-Size Berezinskii-Kosterlitz-Thouless Transition at Grain Boundaries in Solid ^{4}He and the Role of ^{3}He Impurities"

Gaudio *et al.* [1] seek to explain the strong rounding of the transition in the nonclassical rotational inertia (NCRI) observed in torsion oscillator experiments on solid ^{4}He. They propose that this rounding is the result of a finite-size Berezinskii-Kosterlitz-Thouless (BKT) transition [2] with the superfluid component residing on grain boundaries in polycrystalline samples. A grain size of approximately 130 Å is required to reproduce the shape of the NCRI signal. This theory, however, is developed for disconnected grains and does not account for the three-dimensional connectivity of the system. Isolated grains of this size within the sample would make unobservably small contributions to the moment of inertia of the macroscopic torsion oscillator. The theory of superflow along grain boundaries in solid ^{4}He is essentially equivalent to that of superfluid films on the surfaces of a porous material, and a correct understanding of both systems requires that the two-dimensional superfluid surfaces are interconnected. A theory of the crossover from finite-size BKT behavior to the critical behavior of the three-dimensional *XY* model was developed in Ref. [3] and predicts a sharp transition rather than the rounded transition characteristic of a finite system.

In order to distinguish between the theoretical pictures presented in Refs. [1,3], we have carried out simulations, using the worm algorithm, of the superfluid density of the *XY* model in three different geometries. The first geometry is an $\ell \times \ell$ square lattice torus with $\ell = 16$, similar to the finite-size spherical geometry analyzed in [1]. The second geometry has *XY* spins on the sites of a simple cubic lattice and serves as a model for the sharp transition in bulk ^{4}He. The third geometry is a diluted lattice of $\ell \times \ell$ plaquettes of spins, again with $\ell = 16$. The plaquettes form the faces of a cubic lattice. On each plaquette, the spins are arranged on a square lattice and have four neighbors while spins along edges and at corners connecting plaquettes may have more neighbors. The lattice of plaquettes is randomly diluted so that only 30% of the possible plaquettes are present. Since the site percolation threshold for the 2D square lattice is 59%, only small planar regions are present although the system is three-dimensionally still connected. This plaquette geometry is an approximation to a disordered connected system of grain boundaries. For most simulations, we consider a system of 32^3 plaquettes. The results are shown in Fig. 1. The $\ell \times \ell$ torus (●) displays a rounded behavior typical for all finite-size systems. The simple cubic lattice (▲) displays the expected sharp 3D *XY* transition. The most interesting curve is for the diluted plaquette geometry (×). For this geometry, we see a sharp transition, not at all like the behavior of a single isolated plaquette. In fact, this transition is slightly sharper than for the simple cubic lattice because of the initial two-dimensional BKT behavior away from the transition temperature. We have also explored other plaquette sizes both with and without disorder and in no case do we see a rounded transition similar to the experimental signature.

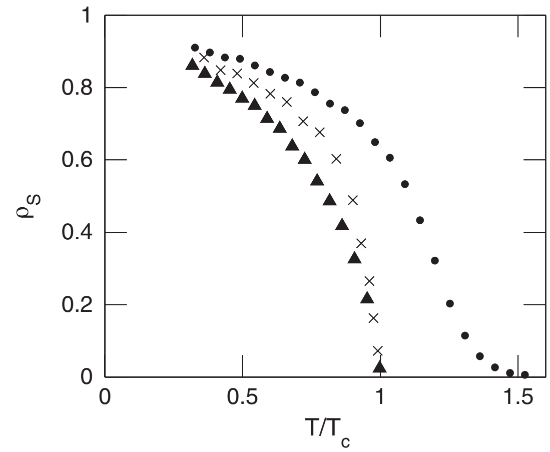

FIG. 1. Superfluid density ρ_s versus temperature T scaled to the critical temperature, T_c, for the *XY* model on three lattices: 3D cubic lattice (▲), diluted 3D cubic lattice of 16×16 plaquettes (×), and 16×16 torus (●).

Our main conclusion disagrees with Ref. [1]. We find a sharp superfluid transition for a connected system of grain boundaries unlike the rounded transition predicted for the same system in Ref. [1]. The explanation of the strongly rounded NCRI signature seen in supersolid experiments remains a mystery.

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[1] S. Gaudio *et al.*, Phys. Rev. Lett. **101**, 075301 (2008).
[2] V. Kotsubo and G. A. Williams, Phys. Rev. Lett. **53**, 691 (1984); Phys. Rev. B **33**, 6106 (1986).
[3] J. Machta and R. A. Guyer, Phys. Rev. Lett. **60**, 2054 (1988); T. Minoguchi and Y. Nagaoka, Jpn. J. Appl. Phys. **26**, L327 (1987); F. Gallet and G. A. Williams, Phys. Rev. B **39**, 4673 (1989).

