Fast and Robust Laser Cooling of Trapped Systems

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We present a robust and fast laser cooling scheme suitable for trapped ions, atoms, or cantilevers. Based on quantum interference, generated by a special laser configuration, it is able to rapidly cool the system such that the final phonon occupation vanishes to zeroth order in the Lamb-Dicke parameter in contrast to existing cooling schemes. Furthermore, it is robust under conditions of fluctuating laser intensity and frequency, thus making it a viable candidate for experimental applications.

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Introduction.—Laser cooling is a crucial ingredient for probing quantum properties of matter [1]. It is a key factor in a wide variety of experiments ranging from Bose-Einstein condensates and quantum computing to quantum simulation with atoms and ions. Variants of cooling schemes range from Doppler cooling for free particles [2] and its partner, sideband cooling for bound particles [3], to dark-state cooling schemes for free [4] and bound particles [5].

At present, sideband cooling is the method of choice for trapped ions. It is a necessary requirement for efficient cooling that motional sidebands with frequency ν can be resolved, i.e., the tunable effective linewidth [6] of the optical transition $\Gamma_{\rm eff} \ll \nu$. Cooling is then achieved by the red sideband transition which excites the atom electronically while at the same time annihilating a phonon to ensure energy conservation. This transition rate must be higher than that on the carrier and blue sideband transitions as these can heat the system either through recoil after spontaneous decay (carrier) or coherent generation of a phonon (blue sideband). Addressing the red sideband transition requires that the Rabi frequency Ω of the laser satisfies $\Omega \ll \nu$. One method to further suppress the carrier and blue sideband transitions employs destructive interference exhibited, for example, in dark states [5]. For this reason, electromagnetically induced transparency (EIT) [7] has become an inspiration for a variety of proposed laser cooling schemes for trapped ions such as EIT cooling [8], Stark-shift cooling [9], and some others [10]. In EIT cooling, interference eliminates the carrier transition to improve cooling performance while in the Stark-shift scheme this is achieved by a carefully tuned Rabi frequency [11].

In EIT cooling [8] the existence of a dark state allows final temperatures below $\hbar\Gamma/k_B$. This is achieved in an electronic three level scheme subject to Raman lasers with strong blue single-photon detuning that couple both the ground state and a metastable state to an excited dissipative state. Among the dressed states of the system is one dark state that cancels the carrier transition. Well chosen parameters can center the red sideband transition under a peak of the Fano-like absorption spectrum, while constraining the blue sideband to a region with negligible absorption, thus achieving low final temperatures. The final state of the system is then of the form

$$\rho^{(\rm EIT)} = \left(|{\rm dark}\rangle\langle{\rm dark}| \otimes \sum_n a_n |n\rangle\langle n|\right) + O(\eta^2), \qquad (1)$$

where $|{\rm dark}\rangle$ refers to the electronic degrees of freedom (d.o.f.) and $|n\rangle$ are the number states of the motional d.o.f. where the final mean phonon number is of order $(\Gamma/4|\Delta|)^2$ [8].

In Stark-shift cooling [9], on the other hand, one laser drives transitions between the ground and a metastable state and another pair of resonant Raman lasers couples a superposition of both to the excited state. This first laser generates Rabi oscillations between the dark state and the orthogonal bright state. If the coupling strength is correctly tuned, the oscillations will also involve neighboring mechanical levels so that states $|{\rm dark}\rangle|n\rangle$ and $|{\rm bright}\rangle|n-1\rangle$ are coupled while carrier transitions are eliminated. Both, EIT and Stark-shift cooling achieve a final temperature that is, in leading order, independent of the Lamb-Dicke parameter η.

Here we demonstrate that the two schemes can be combined, leading to a new interference effect that is then exploited to achieve a qualitative and quantitative improvement of the cooling dynamics and its stability, thus outperforming its constituent schemes. The new scheme, robust cooling, is based on the existence of a joint dark state in EIT and Stark-shift cooling. This in turn allows us to tune the quantum interference to cancel both the carrier and the blue sideband transitions, and hence heating. As a result the final state is of the form

$$\rho = |{\rm dark}\rangle\langle{\rm dark}| \otimes |0\rangle\langle 0| + O(\eta^2), \qquad (2)$$

with a final phonon number that vanishes in leading order in the Lamb-Dicke parameter. To the best of our knowledge this is the first scheme that is able to achieve this for trapped particles.

Remarkably, we will also demonstrate that our scheme exhibits a significant improvement with regard to the stability under fluctuating laser intensities. This implies that it remains robust under general experimental conditions, making it a promising candidate for rapid cooling of ions and atoms and also for achieving ground state cooling of cantilevers where limited Q factors make fast and robust cooling essential.

Description of proposal.—The combined scheme, as presented in Fig. 1, involves a trapped ion with an internal electronic structure made up of a ground state and a metastable state $|g_1\rangle$ and $|g_2\rangle$ and an excited dissipative state $|e\rangle$. A harmonic well models the trap potential. It is characterized by its equally spaced levels $|n\rangle$ representing the Fock state of n phonons and the creation and annihilation operators b and $b^\dagger$. Stark-shift and EIT coolings can be regarded as particular instances of this scheme when $\eta_A \to 0$ and $\Omega_B \to 0$, respectively.

The quantum theory of laser cooling of trapped particles was developed in [12,13] and we will follow the notation in [13]. Expanding the Hamiltonian to first order in the Lamb-Dicke parameters, yields three contributions. First the trap potential and the internal d.o.f.:

$$H_0 = \nu b^\dagger b - \Delta|e\rangle\langle e|, \tag{3}$$

where Δ, the detuning of both lasers, is chosen to be equal. Next, the interaction of the lasers with the ion due to the EIT part and the Stark-shift part, respectively:

$$H_{\mathrm{EIT}} = \Omega_A(\sigma_x^{g_1,e} + \sigma_x^{g_2,e}) + \eta_A\Omega_A(\sigma_y^{g_1,e} - \sigma_y^{g_2,e})(b + b^\dagger), \tag{4}$$

$$H_{\mathrm{SSh}} = \Omega_B\sigma_x^{g_1,g_2} + \eta_B\Omega_B\sigma_y^{g_1,g_2}(b + b^\dagger), \tag{5}$$

with $\sigma_x^{(m,n)} = (|m\rangle\langle n| + \mathrm{H.c.})$ and $\sigma_y^{(m,n)} = (-i|m\rangle\langle n| + \mathrm{H.c.})$.

The physics of the scheme can be understood by extending the analysis of the steady state Eq. (2) to the next order

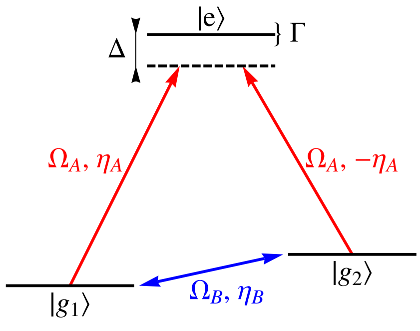

FIG. 1 (color online). A three-level electronic structure made up of $|g_1\rangle$ and $|g_2\rangle$ and an excited state $|e\rangle$ which dissipates energy at rate Γ. The lower levels are coupled to $|e\rangle$ by a pair of Raman beams under a detuning Δ whose Rabi frequencies are Ω_A and which interact mechanically with the ion with Lamb-Dicke parameters η_A of opposite sign (this may be achieved, e.g., by counterpropagating beams). The $|g_i\rangle$ are directly coupled by a Rabi frequency Ω_B and Lamb-Dicke parameter η_B.

in the Lamb-Dicke parameter. Consider

$$|\Psi\rangle = |-\rangle|0\rangle - i\eta_A|+\rangle|1\rangle + O(\eta^2), \tag{6}$$

where $|\pm\rangle = \frac{1}{\sqrt{2}}(|g_1\rangle \pm |g_2\rangle)$, η accounts for both Lamb-Dicke parameters, and we omit normalization. Under the effect of the EIT couplings [Eq. (4)] both terms of the superposition interfere destructively, and as a result $|\Psi\rangle$ remains invariant. However, $|\Psi\rangle$ is not invariant under the free Hamiltonian Eq. (3), which introduces a relative phase between the two components. For suitably tuned parameters, the Stark-shift coupling part [Eq. (5)] cancels this relative phase. Hence $|\Psi\rangle$ will be an invariant dark state as it does not suffer any spontaneous emission losses either. This is achieved when

$$\frac{\eta_B}{\eta_A} = \left(\frac{\nu}{\Omega_B} + 2\right). \tag{7}$$

From an experimental point of view, it is worth noting that this resonance condition is characterized by the ratio of the Lamb-Dicke parameters. These can be set up at the beginning of the experiment to a high precision.

Losses from the excited level are incorporated in the master equation

$$\frac{d\rho}{dt} = -i\hbar[H_0 + H_{\mathrm{EIT}} + H_{\mathrm{SSh}}, \rho] + \mathcal{L}^d\rho, \tag{8}$$

by a Liouvillian $\mathcal{L}^d$

$$\mathcal{L}^d\rho = \sum_{i=g1,g2}\gamma_{e,i}2\sigma_{i,e}\bar{\rho}_{e,i}\sigma_{e,i} - \rho\sigma_{e,e} - \sigma_{e,e}\rho, \tag{9}$$

where $\sigma_{j,k} = |j\rangle\langle k|$ and $\bar{\rho}_{e,i} = \frac{1}{2}\int_{-1}^{1} dx e^{ik_{e,i}x}\rho e^{-ik_{e,i}x}$.

After expansion up to second order in the Lamb-Dicke parameter and adiabatic elimination of the internal d.o.f., we find the rate equation:

$$\begin{aligned}\frac{d\rho_{\mathrm{ext}}^{n,n}}{dt} &= [(n+1)(A_-\rho_{\mathrm{ext}}^{n+1,n+1} - A_+\rho_{\mathrm{ext}}^{n,n}) \\ &\quad + n(A_+\rho_{\mathrm{ext}}^{n-1,n-1} - A_-\rho_{\mathrm{ext}}^{n,n})],\end{aligned} \tag{10}$$

where $\rho_{\mathrm{ext}}^{n,m}$ is the (n, m) element of the density matrix after the internal degrees of freedom have been traced out. In the spirit of [13] the rates $A_\pm$ can be expressed:

$$A_\pm = 2\,\mathrm{Re}[D + S(\mp\nu)], \tag{11}$$

where D is the diffusion coefficient due to spontaneous emission from the excited electronic states. Here $D = 0$ as the population of the excited states vanishes due to the dark-state nature of the final state. $S(\nu)$ is the fluctuation spectrum of Heisenberg operator $F(t)$:

$$S(\nu) = \frac{1}{2M\nu}\int_0^\infty dt e^{i\nu t}\langle F(t)F(0)\rangle, \tag{12}$$

where, in the Schrödinger picture, $F = F_{\mathrm{EIT}} + F_{\mathrm{SSh}}$ and $F_{\mathrm{EIT}} = \eta_A\Omega_A(\sigma_y^{g_1,e} - \sigma_y^{g_2,e})$ and $F_{\mathrm{SSh}} = \eta_B\Omega_B\sigma_y^{g_1,g_2}$ are the part in the interaction Hamiltonian Eqs. (4) and (5) that multiply $b + b^\dagger$. The average can be calculated using the quantum regression theorem. We may split the computation of the overall heating rate into three parts: the EIT part $A_+^{\rm EIT}$, the Stark-shift part $A_+^{\rm SSh}$, and interference between EIT and the Stark-shift part $A_+^{\rm int}$ for the remaining cases. We find $A_+^{\rm EIT} = [\eta_A(\nu + 2\Omega_B)]^2/\mathcal{D}$, $A_+^{\rm SSh} = (\eta_B\Omega_B)^2/\mathcal{D}$, and $A_+^{\rm int} = -2\eta_A\eta_B(\nu + 2\Omega_B)\Omega_B/\mathcal{D}$, where $2\Omega_A^2\Gamma\mathcal{D} = \Gamma^2(\nu + 2\Omega_B)^2 + [-2\Omega_A^2 + (\nu + 2\Omega_B)(\Delta + \nu + \Omega_B)]^2$. Hence

$$A_+ = [\eta_A(\nu + 2\Omega_B) - \eta_B\Omega_B]^2/\mathcal{D}, \tag{13}$$

which will vanish for Eq. (7), hence yielding vanishing mean phonon number $\langle n\rangle = \frac{A_+}{A_- - A_+} = 0$. To achieve $\langle n\rangle = 0$ the interference between the EIT and Stark-shift cooling is essential, hence emphasizing the importance of the constructive interference of both cooling schemes. Figure 2 shows the numerical results of the approach to the $\langle n\rangle = 0$ point as the Lamb-Dicke parameter tends to zero.

Robustness.—The constructive interference between EIT and Stark-shift contribution is also crucial for understanding the robustness of the scheme under fluctuating parameters. If the Rabi frequencies deviate from Eq. (7) by $\Delta\Omega_{A/B}$, the final population is affected by

$$\langle n\rangle \propto (\Delta\Omega_A)^4(\Delta\Omega_B)^2, \tag{14}$$

instead of second order as is usually the case. As is exemplified in Fig. 3, for a given value of the fluctuations of the laser intensities, the final mean phonon number decreases rapidly as one moves away from the pure Stark-shift or pure EIT regime. This guarantees excellent performance under real experimental conditions, overcoming an important drawback of previous dark-state cooling schemes.

Indeed, the experimental realization of EIT cooling achieved final phonon populations of order 10^{-1} [14]. In contrast, Fig. 3 shows populations of at most 10^{-4} for comparable cooling rates to the EIT cooling. More realistic Rabi frequency fluctuations of about 2% would yield final mean phonon numbers of the order of 10^{-8} indicating considerable potential for experimental progress.

Rate.—The interference structure of this scheme is also essential for the high cooling rate $W = A_- - A_+$. For the resonance condition Eq. (7) we find

$$W = \frac{8\Gamma\eta_A^2\nu^2\Omega_A^2}{[2\Omega_A^2 + (\nu - 2\Omega_B)(\Delta - \nu + \Omega_B)]^2 + \Gamma^2(\nu - 2\Omega_B)^2}. \tag{15}$$

Ω_B is the only Rabi frequency involved in the condition Eq. (7). As a function of Ω_B, Eq. (15) takes the approximate shape of a squared Lorentzian, with a peak close to $\Omega_B = \nu/2$ at which point the cooling rate expression reduces to

$$W = \frac{\Gamma\eta_B^2\nu^2}{8\Omega_A^2}. \tag{16}$$

This is also an optimal point for Stark-shift cooling [9], but it should be noted that the cooling rate of the current proposal is slightly higher than that of Stark-shift cooling thanks to the suppression of carrier *and* blue sideband. Since the internal dynamics should be much faster than the external one for the perturbation approach to work, the analytic result is not valid for $\Omega_A < \eta_A\nu$. Numerical studies show that the cooling rate can approach a rate which is 2 orders of magnitude smaller than the trap frequency and more than an order of magnitude lager than EIT cooling. It is noteworthy that the final state Eq. (6) is pure to first order in the Lamb-Dicke parameter and that therefore a simple unitary rotation exists such that in the final state the number of phonons scales as a fourth power of the Lamb-Dicke parameter.

Implementation.—The way the effective Ω_B and η_B couplings can be implemented is not unique. One option is to use two lasers to create the EIT cooling part and to use magnetic gradients [15] for the Stark-shift part. In this

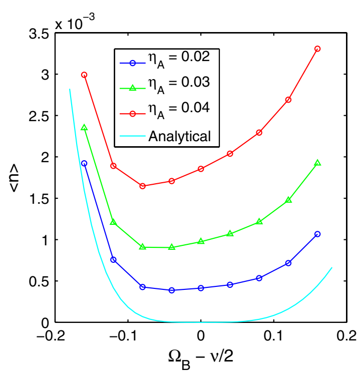

FIG. 2 (color online). The deviation of the analytical from the exact numerical results in the final population is plotted versus the deviation in Ω_B (in units of trap frequency) from the optimal operating point for different Lamb-Dicke parameters. Parameter η_B varies as well so that $\eta_B/\eta_A = 4$, which from Eq. (7) involves an optimal Rabi frequency $\Omega_B = \nu/2$ ($\Delta = 10\nu$, $\Gamma = 10\nu$, $\Omega_A = 0.1\nu$).

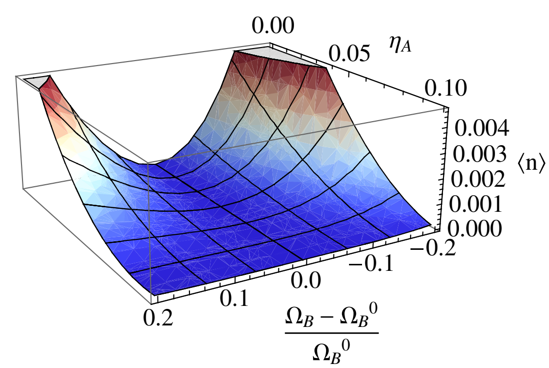

FIG. 3 (color online). Mean photon number $\langle n\rangle$ for $\eta_B = 0.4$ and $\Omega_A = 0.1\nu$ as a function of the Lamb-Dicke parameter η_A and of variations around the optimal Rabi frequency Ω_B^0 ($\Delta = 10\nu$, $\Gamma = 10\nu$). For $\eta_A = 0.1$ we find $\langle n\rangle \leq 10^{-4}$ and also observe that fluctuations have a much weaker effect than in the Stark-shift cooling regime ($\eta_A = 0$) indicating the robustness of this cooling scheme.

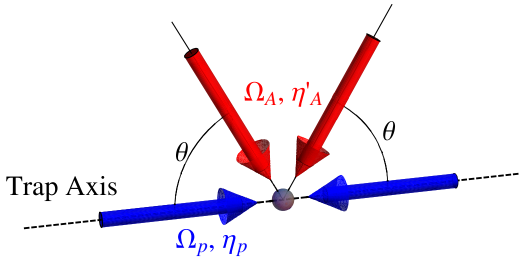

FIG. 4 (color online). Physical realization of Ω_A and Ω_B couplings, for cooling in the trap axis, as two Raman pair of beams characterized by their respective Rabi frequency Ω_A and Ω_p and their Lamb-Dicke parameters η'_A and η_p.

system the magnetic gradients create a coupling of the following type: $\lambda\sigma_z(b + b^\dagger)$, where λ is proportional to the magnetic gradients and the two level system is driven using a microwave, $\Omega_d\sigma_x\cos\omega_d t$, where Ω_d corresponds to the Rabi frequency and ω_d to the angular frequency of the driving wave. After a polaron transformation the resulting Hamiltonian is exactly as in Eq. (5) when the Rabi frequency is replaced by Ω_d and the Lamb-Dicke parameter is replaced by $\frac{\lambda}{\nu}$.

This scheme can be especially useful to cool nanoscale resonators, by using the setup described in [16]. In cantilevers the speed of cooling is crucial due to the finite Q value, which is a central factor limiting the attainable final temperatures at present. The high cooling rate achieved by the described scheme will result in lower final temperatures bringing us closer to the goal of reaching the quantum regime in cantilever systems.

Alternatively, one can also use Raman beams with large single-photon detuning and parameters Ω_p and η_p to couple levels $|g_1\rangle$ and $|g_2\rangle$, as is shown in Fig. 4. A derivation of the correspondence $\eta_B = 2\eta_p$ will be presented elsewhere. For cooling in the trap axis, $\eta_A = \eta'_A\cos\theta$, and for simplicity, $\eta'_A = \eta_p$.

If we choose the optimal point for the cooling rate and fluctuations $\Omega_B = \frac{\nu}{2}$, the condition becomes $\eta_B/\eta_A = 4$. This can be achieved for a layout where $\theta = 60°$, in which case the wavelengths of both pairs are the same. Even if θ is constrained to other nonoptimal values, the robustness of the scheme ensures excellent performance for any geometrical configuration. A proposal of experimental implementation for this scheme would use vacuum chambers with windows at 22.5° and/or 45° from the trap axis, which generally allow for an angle range of about $\pm 10°$. Taking 45° as an operating value, we find $\eta_B/\eta_A = 2\sqrt{2}$. Different optimal sets of parameters can be obtained for this situation depending on whether the cooling rate or the final temperature has to be optimized. For the latter, the condition in Eq. (7) has to be observed, $\Omega_A = 0.6\nu$ and $\Delta \simeq 0$. This will assure an extremely stable cooling rate smaller than EIT cooling but still better than sideband cooling. This final result can be improved depending on the particular values of the transition linewidth $\Gamma_{\rm eff}$. If in turn the cooling rate is to be enhanced, Eq. (7) does not need to be satisfied. In particular, for $\Omega_A \simeq 0.4\nu$, $\Omega_B \simeq 0.45\nu$ with 5% fluctuation and $\Delta \simeq -2\nu$ the population can still be as low as 10^{-3} while still cooling faster than EIT cooling. Taking into account the fact that angles up to 55° are accessible the cooling rate can be improved further by up to 2 orders of magnitude.

Multimode cooling.—Finally, the cooling scheme has also been tested for an ion chain using Monte Carlo simulation [17]. Remarkable results have been obtained and a detailed study will be presented elsewhere.

Conclusion.—We have introduced a robust and fast cooling scheme. Theory predicts that in zeroth order in the Lamb-Dicke parameter and for a readily accessible resonance condition on the Lamb-Dicke parameter this scheme reaches zero temperature. The final temperature is therefore essentially bounded by laser and electrode fluctuations. Beyond the academic interest of proving the existence of such a scheme, its robustness makes it extremely attractive for a broad range of experimental realizations.

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