Direct Measurement of Thermal Fluctuation of High-Q Pendulum

Kazuhiro Agatsuma,[1,*] Takashi Uchiyama,[1] Kazuhiro Yamamoto,[2] Masatake Ohashi,[1] Seiji Kawamura,[3] Shinji Miyoki,[1] Osamu Miyakawa,[1] Souichi Telada,[4] and Kazuaki Kuroda[1]

[1]*Institute for Cosmic Ray Research, University of Tokyo, Kashiwa, Chiba 277-8582, Japan*
[2]*Max Planck Institute for Gravitational Physics (Albert Einstein Institute), Callinstrasse 38, D-30167 Hannover, Germany*
[3]*National Astronomical Observatory of Japan, Mitaka, Tokyo 181-8588, Japan*
[4]*National Institute for Advanced Industrial Science and Technology, Tsukuba, Ibaraki 305-8563, Japan*
(Received 23 August 2009; published 28 January 2010)

We have achieved a direct measurement of the thermal fluctuation of a pendulum in an off-resonant and wide frequency region using a laser interferometric gravitational-wave detector. These measurements have been well identified for over one decade by an agreement with a theoretical prediction, which is derived by a fluctuation-dissipation theorem. Thermal fluctuation was dominated by the contribution of resistances in coil-magnet actuator circuits. When we tuned these resistances, the noise spectrum also changed according to a theoretical prediction. The measured thermal noise level corresponds to a high quality factor on the order of 10^5 of the pendulum.

DOI: 10.1103/PhysRevLett.104.040602 PACS numbers: 05.40.Ca, 04.80.Nn, 07.60.Ly

Introduction.—Mechanical thermal fluctuation has been studied in fields that require ultraprecise measurements, because it gives the fundamental limits of the sensitivity, even in an off-resonant region. A rigid cavity for laser frequency stabilization is limited by the thermal noise [1]; also, a laser interferometric gravitational-wave (GW) detector [2] is one of the most representative types of apparatus. GW detectors consist of mirrors suspended by pendulums to bring them close to a state of free mass. The thermally excited mechanical vibrations of the mirrors themselves and suspensions are kinds of fundamental limits of GW detectors.

Direct observations of the mirror thermal fluctuation have been performed by Numata [3] or Black [4]. They identified the thermal fluctuation using the fluctuation-dissipation theorem (FDT) [5,6], which could evaluate the fluctuation from the dissipation in thermal equilibrium. These experiments could not observe the thermal fluctuation of the pendulum because the seismic noise and the mirror thermal fluctuation had large amplitudes in their experiments. The seismic noise was dominant below 100 Hz. The mirror thermal fluctuation was larger than the pendulum thermal fluctuation above 10 Hz, because they set up a small spot size (radii of 49 and 85 μm of Numata [3] or 160 μm of Black [4] on mirrors) so as to increase the mirror thermal fluctuation.

Identifications of the pendulum thermal noise have ever been limited to a region only in several times of the resonance [7–9]. Our experiment could achieve measurements of the pendulum thermal fluctuation in both off-resonant and in a wideband region of over one decade, which is close to the most expected band of GW detection (around 100 Hz). This experiment was the first to identify the thermal fluctuation above the resonance in a wide frequency region. The seismic noise could be reduced by a quiet environment at 1000 m underground site. The mirror thermal noise was also reduced owing to a large spot size (radii of 4.9 and 8.5 mm, where the electric field fell off to $1/e$) of a laser on a mirror surface. As a manner of identification of the thermal fluctuation of the pendulum, we made use of coil-magnet actuators, which had some merits. One was that the dissipation that occurred in coil circuits was easy to control and to analyze. Another was that all of the parameters were measurable. The third was that they did not cause any modification of the interferometer components with changes of the dissipation in the coil circuits. These merits allowed a high reliability to our experiment.

This Letter describes direct measurements of the thermal fluctuation of the pendulum, and compares it with predicted estimates from the FDT over a wideband from 20 to 400 Hz. We observed shifts in the noise floors of the interferometer when we controlled the dissipation of a pendulum by exchanging the resistances of the coil-magnet actuators. The measured spectra and their shift agreed with a theoretical prediction.

Theoretical background.—Pendulum thermal fluctuation is caused by several kinds of dissipations in the whole pendulum. There have been experiments and studies so far for each type of dissipation: an internal loss in the materials of suspension wires [9,10], clamps of wires [11], residual air [8], and the bobbins or holders of coils [12,13]. We focus on the thermal fluctuation caused by coil-magnet actuators.

In the current interferometric GW detectors that have a Fabry-Perot (FP) cavity in their Michelson arms, coil-magnet actuators are equipped to keep the FP cavity on resonance by controlling the mirror positions. One actuator consists of a piece of magnet glued onto a mirror, and a coil circuit connected to a coil driver. The dissipation in the coil circuits of the actuators produces the pendulum thermal fluctuation because a pendulum is unified with coil circuits, and is in thermal equilibrium with them due to coil-magnet coupling. An oscillation of the pendulum causes eddy currents by electromagnetic induction in the coil circuits. These currents generate Joule heat in the resistances of the circuits. The pendulum thermal fluctuation can be calculated by applying the FDT to the dissipation from this Joule heat. The other evaluation method is to calculate the pendulum fluctuation caused by the thermal current of the resistance [14] via coil-magnet coupling. These two estimates are consistent as long as the thermal equilibrium between the coil circuits and the pendulum is maintained.

As a model of the pendulum, we assume a suspended mirror (test mass of m) and actuators constructed with a pair of coil magnets of N. When a current I flows inside of the coil circuits, a force of $F = I\alpha N$ is applied to the mirror, where α is the coupling factor per one coil-magnet actuator. From an energy conversion as $F\dot{x} = NI^2Z$, a motion of the mirror causes a current of $I = \alpha\dot{x}/Z$ in the coil circuits. That motion equation of the mirror in a frequency domain is written as

$$m(-\omega^2 + \omega_0^2)\tilde{x} + i\frac{N\alpha^2\omega}{Z}\tilde{x} = 0. \quad (1)$$

Here $\omega_0 = 2\pi f_0$, where f_0 is the resonant frequency of the pendulum, $\tilde{x}$ is the displacement of the mirror, and Z is the impedance of the coil circuit, which is the sum of the coil's impedance, and the connected driver output impedance. It is assumed that the magnitude of the impedance in the coil circuit is dominated by its resistance R; it can then be regarded as $Z = R$. From the imaginary part of Eq. (1), the quality factor is defined as

$$Q = \frac{m\omega_0 R}{N\alpha^2}. \quad (2)$$

The fluctuation is characterized by the loss angle $\phi = \omega/(\omega_0 Q)$, which indicates viscous damping caused by an eddy current in the coil circuits. By using the FDT, the fluctuation of a harmonic oscillator, i.e., a pendulum in a higher off-resonant region, is approximately expressed as

$$G = \frac{4k_B T N\alpha^2}{m^2\omega^4 R}. \quad (3)$$

$\sqrt{G}$ is a one-sided power spectrum density, k_B is the Boltzmann constant, and T is the temperature of the suspension and the coil circuits.

Experimental setup.—A GW detector, called CLIO (Cryogenic Laser Interferometer Observatory) [15], was employed as a mirror displacement sensor. CLIO is a prototype for the next Japanese GW telescope project, LCGT (Large Scale Cryogenic Gravitational-Wave Telescope) [16], featuring the use of cryogenic mirrors and a quiet underground site. The main purpose of CLIO is to demonstrate a reduction of the mirror thermal noise by cooling sapphire mirrors. In this Letter, however, CLIO is described as being operated at room temperature. CLIO is located in the Kamioka mine, which is 220 km away from Tokyo, Japan, and lies 1000 m underground from the top of a mountain. This site is suitable for precise measurements of the displacement at below the 100 Hz region, because the seismic noise is smaller than that in an urban area by about 2 orders [17]. Through the progress of noise hunting in CLIO, the background (BG) noise has reached about $1 \times 10^{-18}\ \mathrm{m}/\sqrt{\mathrm{Hz}}$ at 100 Hz. This sensitivity is close to the design sensitivity of $5 \times 10^{-19}\ \mathrm{m}/\sqrt{\mathrm{Hz}}$.

Figure 1 shows a schematic view of CLIO. A laser beam, which has a power of 2 W and a 1064 nm wavelength (Innolight Inc. Mephisto), is used as a light source. It is shaped into TEM00 through a mode cleaner (MC) cavity with a length of 9.5 m, and then enters two 100 m length FP cavities after being divided by a beam splitter. These cavities are arranged in an L shape. The cavities are kept on resonance (we call this state locked) by servo systems, which readout displacement signals of the mirrors using the Pound-Drever-Hall method [18]. The modulation frequencies are 15.8 MHz for the arm cavities and 11.97 MHz for the mode cleaner. A multistage control system [19] is applied for a laser frequency stabilization, which has two cascaded loops of the MC and an inline arm cavity. The inline arm is locked by controlling the frequency of the laser and the perpendicular arm is locked by controlling the position of the front mirror by using coil-magnet actuators. From a feedback signal to the coil-magnet actuators, a differential displacement between two arm cavities, which corresponds to a GW signal, can be obtained.

The four mirrors of the two arm cavities are individually suspended by six-stage pendulums, which include four-stage blade springs and two-stage wire suspensions for isolation from any seismic vibration. The mirrors are made of sapphire, whose weights are 1.8 kg, suspended

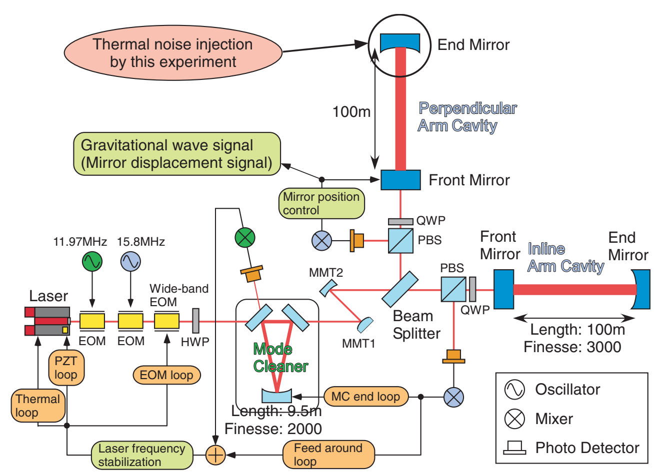

FIG. 1 (color online). Schematic view of CLIO and its control system; CLIO is a so-called locked Fabry-Perot interferometer, which has 100 m Fabry-Perot cavities as Michelson arms and a mode cleaner. The optics, like the lenses, the Faraday-isolators, and some wave plates are omitted in this figure. Abbreviated words mean: EOM, electro-optic modulator; HWP, half wave plate; QWP, quarter wave plate; MMT, mode matching telescope; and PBS, polarized beam splitter.

by Bolfur [20] wires at the last stage with the resonant frequency of 0.8 Hz. Coil-magnet actuators are set for two mirrors in a perpendicular cavity. In order to observe the thermal fluctuation of the pendulum, we injected sufficient thermal noise to the end mirror (see Fig. 1), so that its fluctuation could dominate, by controlling the dissipation at the coil circuits. On the other hand, the actuator of the front mirror is used to keep the cavity locked.

The coil-magnet actuator at the end mirror consists of two Nd-Fe-B magnets, which are glued onto the mirror and of two copper coils approached to magnets in the horizontal direction (see Fig. 2). The Nd-Fe-B magnets have a cylindrical shape. This diameter is 2 mm and the length is 10 mm. A copper wire of 0.5 mm diameter is used for solenoidal coils, and is wound by 15 turns times 6 layers around a ceramic bobbins, whose diameter is 16.5 mm. In order to verify our measurement of the thermal fluctuation, the resistance R of Eq. (3) is changed as a variable parameter. R is the sum of the coil's impedance and a connected circuit impedance. Although a coil-driver circuit is usually used as the connected circuit, a relay circuit including resistances is used to change R in this case. The relay circuit is switched to three kinds of resistances, which are named "SHORT," "2-OHM," and "OPEN." The sum of the resistances of the coil circuits and a relay circuit were measured as 0.72 Ω in SHORT, and 2.77 Ω in 2-OHM. OPEN means that the end of the relay circuit is opened, and then R becomes infinity.

It is necessary to know the quantities in Eq. (3) to estimate the thermal fluctuation. In our experiment, N is 2, m 1.8 kg, f_0 0.8 Hz and T 3.0×10^2 K. The coupling factor α is an actuator force applied for the mirror per driver output current. That is yielded from measured actuator response, a resistance of volt-current conversion in the coil driver, and the weight of a test mass. The actuator response is a transfer function from the driver input voltage to the mirror displacement. The coil-magnet actuator response at the front mirror is measured using a Michelson interferometer constructed with front mirrors and the BS. The driver output current is obtained by a resistance of volt-current conversion, R_c ($R_c = 50\ \Omega$ in CLIO). Using a measured actuator response at 100 Hz as A_{100} [21], the coupling factor per one coil-magnet actuator is $\alpha = A_{100} R_c m (2\pi \times 100)^2 / N$. The response of the end mirror actuator is measured by a calibrated actuator of the front mirror. Only during this calibration, the relay circuit is exchanged to a driver circuit. The value of α at the end mirror is 6.9×10^{-3} N/A.

In this experiment, other dissipations of pendulums are adequately suppressed below the additional dissipation of the coil circuits. Coil bobbins are made of a Macor (ceramic) which has an electrical conductivity of $10^{-13}\ \Omega^{-1}\,\mathrm{m}^{-1}$. The pendulum is housed in a vacuum chamber, whose vacuum level is 10^{-3} Pa, so as to reduce any dissipation from residual air. The suspension thermal noise as a part of the design sensitivity, which comes from wire-material dissipations of the internal damping [10], was calculated to be below the level of the BG noise. The thermal noise from the coil-magnet actuator at the front mirror is under the BG noise, because the output impedance of the coil driver is $10^4\ \Omega$ at 100 Hz and α is about ten times smaller than that of the end mirror.

Results and discussion.—Figure 3 shows measured spectra of the mirror displacements when we changed the resistances of coil circuits, and also shows other noises of the detector. The noise levels from 20 to 400 Hz are shifted by switching the dissipation. The measured values agree with the sums of theoretical estimates from Eq. (3) and the BG model. When the OPEN was chosen, a BG noise of

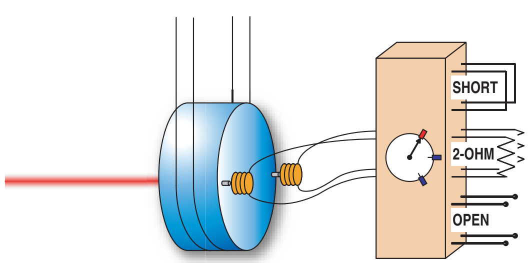

FIG. 2 (color online). Schematic view of our experiment. The pendulum is composed of a sapphire mirror suspended by Bolfur wires. Two coil-magnet actuators are set. The resistances of the coil circuits are changed by switching a relay circuit to SHORT, 2-OHM, and OPEN. Therefore, we can control the injected thermal noise using this relay circuit.

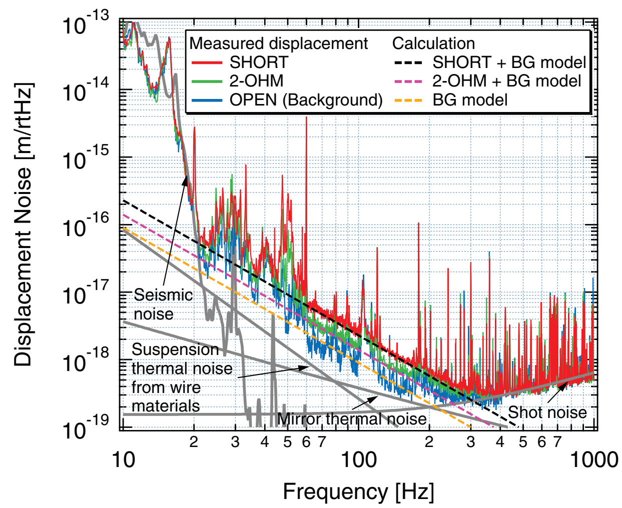

FIG. 3 (color). Measured spectra of the pendulum thermal fluctuation and their theoretically expected lines. The solid lines of red, green, and blue denote the conditions of the SHORT, 2-OHM, and OPEN, respectively. The spectrum at OPEN corresponds to the background (BG) noise of the interferometer. The doted lines indicate the sum of the theoretical calculation of the thermal fluctuation and the BG model. The sensitivity is limited by the shot noise above 400 Hz, and by the seismic noise below 20 Hz. The mirror thermal noise and the suspension thermal noise (from material dissipation) were calculated to be under the BG noise.

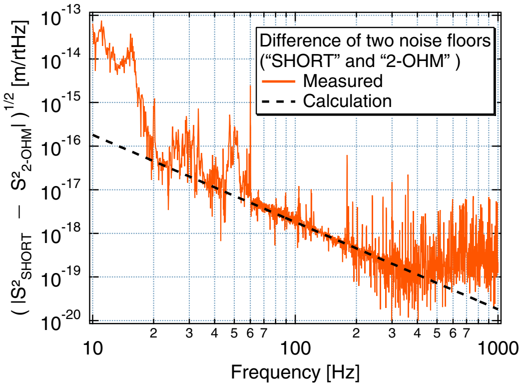

FIG. 4 (color online). Comparison of the pendulum thermal fluctuation with a theoretical calculation by removing the BG noise; The solid orange line shows the thermal fluctuation difference of the SHORT from the 2-OHM. The dashed line shows the theoretical calculation corresponding to this difference.

about 1×10^{-18} m/$\sqrt{\mathrm{Hz}}$ at 100 Hz was measured, since the injection thermal noise was lower than the BG level, owing to a large amount of resistance R. (After this experiment, the BG noise was reduced and became comparable with the suspension thermal noise of structure damping [22]. In order to confirm whether the observed spectrum is due to thermal noise or not, a cryogenic experiment is in progress.) For simplicity of calculations, the BG noise was modeled as 0.9×10^{-18} m/$\sqrt{\mathrm{Hz}}$ at 100 Hz with a slope of f^{-2}. In the case of SHORT, for instance, the noise level at 100 Hz was predicted to have a value of 2.3×10^{-18} m/$\sqrt{\mathrm{Hz}}$, which includes a pendulum thermal fluctuation of 2.1×10^{-18} m/$\sqrt{\mathrm{Hz}}$ and a value of the BG model. In 2-OHM, the total value of 1.4×10^{-18} m/$\sqrt{\mathrm{Hz}}$ at 100 Hz included a thermal fluctuation of 1.1×10^{-18} m/$\sqrt{\mathrm{Hz}}$ and the BG model. We also estimated Q of the pendulum, which corresponded to these thermal noises from Eq. (2). The calculated Q is 6.7×10^4 in SHORT and 2.6×10^5 in 2-OHM. A Q of 10^5 is as large as that of the pendulums in km-scale interferometers [23,24].

For a more precise and wideband comparison between the measured spectra and theoretical calculations, we show Fig. 4, which is the difference between SHORT and 2-OHM taken as $S_d = (|\, S^2_{\mathrm{SHORT}} - S^2_{2\text{-}\mathrm{OHM}} \,|)^{1/2}$; here, S_{SHORT}, $S_{2\text{-}\mathrm{OHM}}$ indicate the spectra of SHORT, 2-OHM in Fig. 3, respectively. S_d, where the BG noise of two noise floors are canceled, is useful for comparing with the theoretical calculation of the thermal fluctuation. The result indicates a good agreement between the measurement and a calculation from 20 to 400 Hz. Especially, the agreement is better at a region from 60 to 300 Hz. The peak structure around 50 Hz comes from the resonant motion of a coil holder at the end mirror excited by the seismic motion. The noisy structure around 30 Hz comes from the BG noise.

Summary.—We have identified the thermal fluctuation of a pendulum in an off-resonant regime and a wideband range around the GW band. This result was confirmed by a comparison between the measured thermal noises and a calculation from the FDT under tuning the resistance of coil-magnet actuators, which were the main sources of thermal noise. The noise level indicates that the pendulums of CLIO have a high quality factor on the order of 10^5. Coil-magnet actuators are essential tools for the interferometer operation and for position controls in other fields as well, in the present and the next generation. However, the thermal noise of this actuator is a serious problem. This Letter contributes to the evaluation of this thermal noise and to the design of actuators for the goal sensitivity. Especially, that is significant for the next generation GW detectors because their goal sensitivities are more demanding.

This work was supported in part by GCOE for Phys. Sci. Frontier, MEXT, Japan and in part by a JSPS Grant-in-Aid for Scientific Research (No. 18204021).

*agatsuma@icrr.u-tokyo.ac.jp

[1] K. Numata, A. Kemery, and J. Camp, Phys. Rev. Lett. **93**, 250602 (2004).
[2] Details are in *Gravitational Wave Detection*, edited by K. Tsubono, M.-K. Fujimoto, and K. Kuroda (University Academy Press, Tokyo, 1997).
[3] K. Numata *et al.*, Phys. Rev. Lett. **91**, 260602 (2003).
[4] E. D. Black, A. Villar, and K. G. Libbrecht, Phys. Rev. Lett. **93**, 241101 (2004).
[5] H. B. Callen and T. A. Welton, Phys. Rev. **83**, 34 (1951).
[6] H. B. Callen and R. F. Greene, Phys. Rev. **86**, 702 (1952).
[7] C. M. Mow-Lowry *et al.*, Phys. Rev. Lett. **100**, 010801 (2008).
[8] M. Kajima, N. Kusumi, S. Moriwaki, and N. Mio, Phys. Lett. A **264**, 251 (1999).
[9] G. I. González and P. R. Saulson, Phys. Lett. A **201**, 12 (1995).
[10] P. R. Saulson, Phys. Rev. D **42**, 2437 (1990).
[11] D. J. Dawid and S. Kawamura, Rev. Sci. Instrum. **68**, 4600 (1997).
[12] G. Cagnoli *et al.*, Rev. Sci. Instrum. **69**, 2777 (1998).
[13] S. Frasca *et al.*, Phys. Lett. A **252**, 11 (1999).
[14] H. Nyquist, Phys. Rev. **32**, 110 (1928).
[15] S. Miyoki *et al.*, Classical Quantum Gravity **21**, S1173 (2004).
[16] K. Kuroda *et al.*, Prog. Theor. Phys. Suppl. **163**, 54 (2006).
[17] A. Araya *et al.*, Rev. Sci. Instrum. **64**, 1337 (1993).
[18] R. W. P. Drever *et al.*, Appl. Phys. B **31**, 97 (1983).
[19] S. Nagano *et al.*, Rev. Sci. Instrum. **74**, 4176 (2003).
[20] A product of UNITICA, http://www.unitika.co.jp/.
[21] The measurement at 100 Hz is a middle point of our interest and an accurate measurement of the transfer function is possible.
[22] K. Agatsuma *et al.*, arXiv:0911.3090 [Classical Quantum Gravity (to be published)].
[23] B. C. Barish and R. Weiss, Phys. Today **52**, No. 10, 44 (1999).
[24] B. Caron *et al.*, Classical Quantum Gravity **14**, 1461 (1997).