

# Few-Photon Transport in Low-Dimensional Systems: Interaction-Induced Radiation Trapping

Paolo Longo

*Institut für Theoretische Festkörperphysik, Universität Karlsruhe (TH), 76128 Karlsruhe, Germany*

Peter Schmitteckert

*Institut für Nanotechnologie, Forschungszentrum Karlsruhe in der Helmholtz-Gemeinschaft, 76021 Karlsruhe, Germany*

Kurt Busch

*Institut für Theoretische Festkörperphysik and DFG-Center for Functional Nanostructures (CFN), Universität Karlsruhe (TH), 76128 Karlsruhe, Germany*

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We present a detailed analysis of the dynamics of photon transport in waveguiding systems in the presence of a two-level system. In these systems, quantum interference effects generate a strong effective optical nonlinearity on the few-photon level. We clarify the relevant physical mechanisms through an appropriate quantum many-body approach. Based on this, we demonstrate that a single-particle photon-atom bound state with an energy outside the band can be excited via multiparticle scattering processes. We further show that these trapping effects are robust and, therefore, will be useful for the control of photon entanglement in solid-state based quantum-optical systems.

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Over the past years, the conception and development of solid-state based quantum-optical functional elements have received steadily increasing interest [1–3]. As compared to other approaches, solid-state-based systems offer an obvious scalability and handling advantage of the resulting devices as well as the utilization of modified light-matter interactions through judicious designs of the corresponding waveguides' dispersion relations and/or mode profiles.

However, since high-quality samples such as coupled-optical-resonator-waveguide arrays (CROWs) [4,5] have become available only recently, there is limited theoretical work regarding the potential of utilizing modified light-matter interaction in (effectively) low-dimensional quantum-optical systems. The basic underlying problem, i.e., that of a system with discrete levels that is coupled to a continuum of states has attracted attention for a long time [6]. For single photons, quantum interference effects in one-dimensional waveguides with an embedded quantum impurity allow the realization of effective energy-dependent mirrors [7–9]. For two or more photons, this system induces an effective photon-photon interaction and even bound photon-photon states that may be exploited for efficient control of photon-entanglement [10–12]. Except for our work on the one-photon case [9], all of the above calculations have been carried out in the stationary regime. In particular, the more challenging few-photon case has been addressed with sophisticated Bethe-ansatz [10,11] and Lehmann-Symanzik-Zimmermann reduction techniques [12] that allow one to determine the corresponding scattering matrices for such systems. However, these field-theoretical approaches employ linearized dispersion relations without band edges.

In the present Letter, we apply our computational framework of time-domain simulations using Krylov-subspace-based operator-exponential methods [9,13] to the case of few-photon transport through a quantum impurity in a one-dimensional waveguiding system similar to wave packet dynamics in electronic systems [13,14]. This allows us to analyze the scattering of two or more photons at the quantum impurity in a very general way. In particular, for a cosine-type dispersion relation, we are able to confirm the existence of two bound photon-atom states [12]. Furthermore, we show how these states can be excited and controlled through the photon nonlinearity that is induced by the quantum impurity. This elucidates the mechanism through which the quantum impurity can be utilized for controlling photon entanglement. In the field-theoretical approaches discussed above [10–12], the photon-atom bound states are (due to the absence of band edges) energetically shifted to infinity and are thus removed from the physically accessible Hilbert space.

Starting from the well-known Dicke-Hamiltonian [15], we can derive a tight-binding Hamiltonian that describes photon propagation in an effectively one-dimensional waveguide with cosine-type dispersion relation (such as the CROWs of Refs. [4,5]) that is coupled to a quantum impurity as [9]

$$\hat{H} = -J \sum_{x=1}^{N-1} (a_x^\dagger a_{x+1} + a_{x+1}^\dagger a_x) + \frac{\Omega}{2} \sigma_z + V(a_{x_0} \sigma_+ + a_{x_0}^\dagger \sigma_-). \quad (1)$$

Here,  $a_x^\dagger$  and  $a_x$  denote, respectively, bosonic (photon) creation and annihilation operators at lattice site  $x$  and  $J$  denotes the corresponding hopping element. The quantum

impurity is modeled as a two-level system (TLS) with transition frequency  $\omega_0 = \Omega/\hbar$  that is located at lattice site  $x_0$  and couples with a coupling element  $V$  to the modes of the photonic band. When measuring energies from the center of the band, the corresponding dispersion relation is  $\hbar\omega_k = -2J \cos(ka)$ , where  $a$  denotes the lattice constant and  $k$  stands for a wave number that lies within the first Brillouin zone. Finally, the TLS is described through the Pauli operators  $\sigma_z$  and  $\sigma_{\pm} = \sigma_x \pm i\sigma_y$ .

While being physically intuitive, the above Hamiltonian (1) does not allow for the most transparent discussion of the underlying physics. Instead, we find it most useful to reformulate the problem in terms of the Hamiltonian

$$\hat{H} = -J \sum_{x=1}^{N-1} (a_x^\dagger a_{x+1} + a_{x+1}^\dagger a_x) + \Omega b^\dagger b + V(a_{x_0} b^\dagger + a_{x_0}^\dagger b) + Ub^\dagger b(b^\dagger b - 1), \quad (2)$$

where we have replaced the TLS by an additional bosonic lattice site. More precisely, we have replaced the Pauli operators of the TLS by appropriate combinations of bosonic creation and annihilation operators,  $b^\dagger$  and  $b$ . The ground and excited states of the TLS correspond, respectively, to none and a single boson on this additional site (TLS site). Unphysical multiple occupancies of the TLS site have been addressed through the addition of the last term on the right-hand side (rhs) of (2). This term ensures that once the TLS site is occupied, i.e., the TLS is in its excited state, adding a further boson to the TLS site requires the energy  $U > 0$ . Thus, Hamiltonians (1) and (2) are equivalent in the limit  $U \rightarrow \infty$  and this is the only case we consider in this work. The  $U$  term induces inelastic scattering that allows us to discuss the physically relevant processes. For actual numerical calculations, we use Hamiltonian (1).

With this reformulation several issues become apparent. Quantum interference processes associated with the coupling between TLS and the waveguide modes induce an effective interaction between photons as described by the nonlinear term  $Ub^\dagger b(b^\dagger b - 1)$ . While this effective few-photon optical nonlinearity is spatially localized to the immediate vicinity of the TLS site, this system nevertheless represents a true quantum-mechanical many-particle problem. For instance, Hamiltonian (2) looks very similar to a bosonic version of the celebrated single-impurity Anderson model [16] that describes magnetic impurities in metals. Therefore, it is suggestive to apply methods that have been developed for correlated quantum systems to the Hamiltonians (1) and (2) [9–12]. From Hamiltonian (2), it becomes apparent that the TLS will induce correlations between two or more photons. This raises the fascinating question to what extent the TLS can be utilized to engineer this entanglement and what role the photon-atom bound states play in this (note that photon-atom bound states have been discussed in a different context before [17]).

To address this question, we have to go beyond stationary calculations that determine the scattering matrices of photons in plane wave states for linearized dispersion relations where the photon-atom bound states are physically inaccessible [10–12]. To do so, we employ our computational framework which we have described in detail elsewhere [9]. This framework allows us to analyze both the dynamics of multiphoton wave packets that interact with the TLS and the dynamics of the TLS itself. Furthermore, it takes into account all aspects introduced by the finite-bandwidth dispersion relation. First, we would like to note that on energetic grounds a single photon cannot excite the photon-atom bound states described above and, therefore, these states are of no relevance in single-photon scattering calculations from a TLS [8,9]. In other words, the TLS (partially) absorbs an incoming single photon and a decomposition of the system's initial state into the (polaritonic) single-particle eigenstates of the Hamiltonian (1) does not involve the bound photon-atom states. Thus, the excited TLS will eventually decay into its ground state. However, our reformulated Hamiltonian (2) suggests that, by virtue of the nonlinear interaction term, the bound states can, in principle, be energetically reached via multiphoton processes. In Fig. 1, we demonstrate that this is indeed possible: A two-photon wave packet interacts with the TLS and a sizable fraction of the photon population becomes trapped at the TLS site. In other words, once the TLS is appreciably excited by one of the incoming photons, the remaining photon sees a modified (saturated)

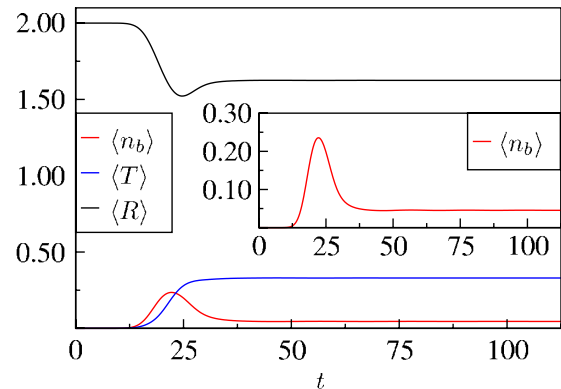


FIG. 1 (color online). Time evolution (in units of  $\hbar/J$ ) of transmission  $\langle T \rangle$ , reflection  $\langle R \rangle$ , and impurity occupation  $\langle n_b \rangle = \langle b^\dagger b \rangle = \langle \sigma_z + 1/2 \rangle$  for a two-photon wave packet that scatters at a TLS. The TLS has a transition energy  $\Omega = \sqrt{2}J$  and couples with coupling strength  $V = J$  to the central lattice site  $x_0 = 100a$  of a tight-binding lattice with total extent  $L = 199a$  and hopping element  $J$ . The photons are described via boson-symmetric wave packets that are constructed from single-particle Gaussian wave functions of width  $s = 6a$  with wave number  $k = 3\pi/4a$  and initial center  $x_c = 70a$  (see text and Ref. [9] for further details). All calculations are stopped at times not exceeding the transit time, i.e., the time the wave packet needs to pass through the waveguide, thus avoiding artificial reflections from the system's boundaries.

TLS and is thus (partially) scattered into the hitherto unreachable bound photon-atom states via multiparticle scattering processes. After the scattering is complete, the bound photon-atom states are again decoupled from the continuum (such as is the case for the scattering of a single photon discussed above) and, thus, cannot decay. These bound states are of a polaritonic nature; i.e., they are multimoded dressed eigenstates of (1) and (2) with complex wave numbers solely induced by the existence of the waveguide's finite bandwidth. This implies that a fraction of the radiation remains trapped at the TLS site in form of a partial occupation of the TLS.

In order to verify the role of the multiparticle processes, we display in Fig. 2 the time evolution of the TLS' excited-state occupation for the scattering of multiphoton wave packets with different particle numbers. The increase in the trapped photon population with the number of photons implies a corresponding increase in the rate at which radiation is scattered into the bound states. The strength of this interaction further depends on the detuning of the TLS relative to the photon frequency as well as on the strength of the coupling matrix element  $V$  between TLS and the waveguide modes. In Fig. 3, we depict the corresponding dependence of the trapped photon population at the TLS for a fixed photon wave number  $k = 3\pi/4a$ . Consistent with our above interpretation, trapping is most pronounced for zero detuning  $\delta = \Omega - \hbar\omega_k$  (recall that  $\hbar\omega_{k=3\pi/4a} = \sqrt{2}J$ ). Furthermore, proximity of the TLS resonance frequency to the band edge (or cutoff frequency of the waveguide) is clearly advantageous for realizing efficient trapping: for frequencies near a band edge the multiparticle scattering mechanism has to provide less additional energy for exciting the energetically closest bound state. Less intuitive is the fact that there exists an

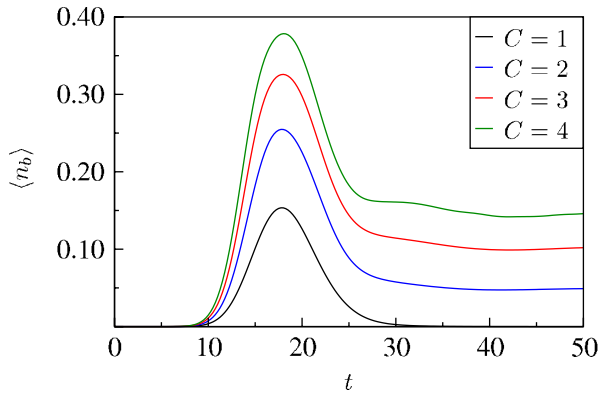


FIG. 2 (color online). Time evolution (in units of  $\hbar/J$ ) of the impurity occupation  $\langle n_b \rangle$  for initial multiphoton states with different photon numbers  $C$  that are constructed analogous to the two-photon states in Fig. 1. The corresponding system parameters are  $L = 99a$ ,  $x_0 = 50a$ ,  $\Omega = \sqrt{2}J$ , and  $V = J$ . The photon parameters are  $x_c = 25a$ ,  $s = 5a$ , and  $k = 3\pi/4a$ . The results for photon numbers  $C = 3$  and  $C = 4$  have been obtained with a time-dependent density matrix renormalization group (DMRG) technique as described in Ref. [13].

optimal coupling strength  $V_{\text{opt}} \sim J$  between TLS and waveguide modes for which maximal trapping occurs. We have confirmed these findings for a number of different dispersion relations. For instance, we have extended Hamiltonian (2) to include a next-nearest-neighbor hopping term  $J^{(2)} \neq 0$  that allows us to significantly modify the cos-type dispersion relation of the tight-binding model (not shown). In addition, we have found analogous behavior for strictly linear dispersion relations with cutoff at finite energies (not shown).

The above results suggest a certain robustness of the trapping effect which we have further analyzed by qualitatively considering losses. This is accomplished by coupling the TLS to a second waveguide that can deexcite the TLS into modes other than those of the original waveguide. The incorporation of this “loss channel” into the Hamiltonian (1) thus proceeds by adding two additional terms analogous to, respectively, the first (hopping term  $J'$ ) and third term (coupling term  $V'$ ) of the rhs of (1). Clearly, the hopping term  $J'$  has to be chosen such that the energies of the bound photon-atom states that are associated with the first waveguide and the TLS alone lie in the band of the second waveguide. In Fig. 4, we display the time evolution of the TLS' excited-state occupation for different coupling strengths  $V'$  of the TLS to such a “broadband loss waveguide.” The trapping effect persists even for rather strong coupling to the loss channel. If we, for instance, interpret the coupling to the loss waveguide as a (admittedly crude) model for fabrication tolerances that in a quasi-one-dimensional system couple strictly guided modes to a continuum of radiative modes, we are led to speculate that the trapping effect would be observable in experimentally accessible systems.

Finally, we have analyzed the possibility of tuning the trapped photon population at the TLS site. To do so, we have prepared two identical single-photon wave packets on

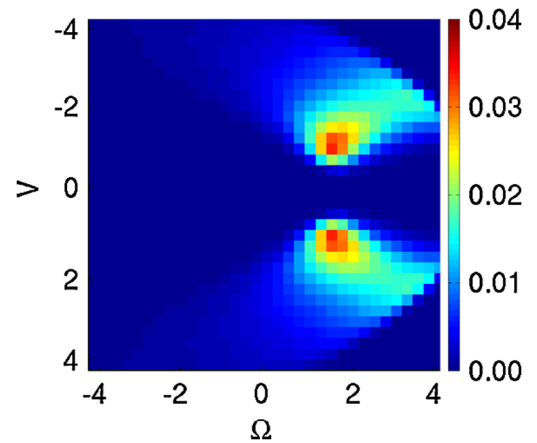


FIG. 3 (color online). Impurity occupation  $\langle n_b \rangle$  in the long-time limit (see Fig. 1) after scattering of two-photon states for different system parameters  $V$  (in units of  $J$ ) and  $\Omega$  (in units of  $J$ ). The fixed parameters are:  $L = 199a$ ,  $x_0 = 100a$ ,  $x_c = 70a$ ,  $s = 12a$ , and  $k = 3\pi/4a$ .

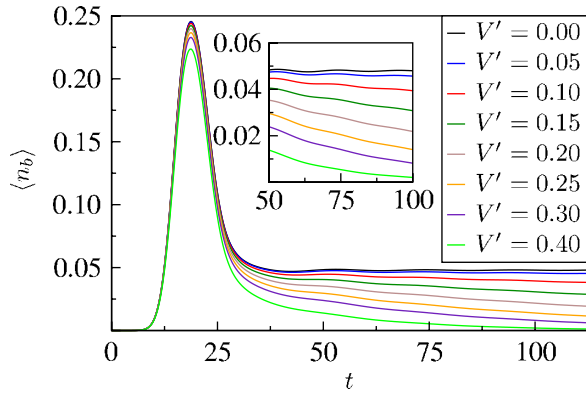


FIG. 4 (color online). Time evolution (in units of  $\hbar/J$ ) of the impurity occupation  $\langle n_b \rangle$  for initial two-photon states (see Fig. 1) where a broadband loss waveguide has been introduced. The corresponding system parameters are (loss waveguide's parameters are primed):  $L = L' = 399a$ ,  $x_0 = x'_0 = 200a$ ,  $\Omega = \sqrt{2}J$ ,  $V = J$ , and  $J' = 2J$ . The strength of the coupling  $V'$  (in units of  $J$ ) to the loss channel is varied. The photon parameters are  $x_c = 175a$ ,  $s = 6a$ , and  $k = 3\pi/4a$ .

different sides of the TLS and have launched them towards the TLS. By changing their initial relative separation to the TLS, we can exert some control over the multiparticle processes in the Hamiltonian (2) (see Fig. 5). While the dynamics is (expectedly) rather distinct for the different cases, we observe monotonic behavior of the trapped population: maximal trapping occurs for symmetrically launched pulses with zero relative initial distance. For increased distances the trapped population decreases to zero once there is no overlap of the pulses at the TLS site.

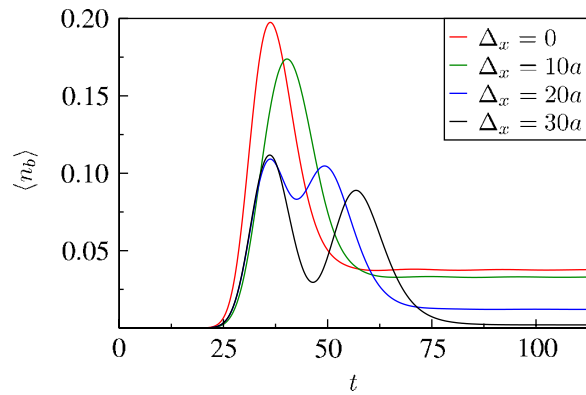


FIG. 5 (color online). Time evolution (in units of  $\hbar/J$ ) of the impurity occupation  $\langle n_b \rangle$  for a system where two single-photon Gaussian wave packets of width  $s = 7a$  with different initial positions ( $x_c^{(1)}$  and  $x_c^{(2)}$ ) are launched from different sides towards the TLS. The corresponding system parameters are  $L = 199a$ ,  $x_0 = 100a$ ,  $\Omega = \sqrt{2}J$ , and  $V = J$ . The photon parameters are  $x_c^{(1)} = 50a$ ,  $k^{(1)} = 3\pi/4a = -k^{(2)}$ , and the initial position  $x_c^{(2)} = 150a + \Delta_x$  is varied.

In conclusion, we have analyzed the dynamics of photon transport in waveguiding systems in the presence of a TLS within the context of a quantum many-body framework. Our reformulation (2) allows us to identify strong multiparticle processes that may be utilized to excite and control photon-atom bound states. In turn, this facilitates trapping of radiation at the TLS. In addition, we have shown that this trapping effect exhibits a certain degree of robustness and can be found in a number of systems. Since few-photon (or low intensity) coherent states are superpositions of a few Fock states only (those that we have discussed in the present work), we expect that the excitation and control of the photon-atom bound states and associated effects will also occur in such situations.

Finally, we would like to emphasize the generality of our approach which is capable of treating systems with arbitrary dispersion relations and atom-field coupling strengths both in real and momentum space. Thus, the trapping of the photon population and its control suggest that such systems may be exploited for engineering photon entanglement as well as for the realization of quantum logic circuits in a number of systems that range from silicon integrated optical elements all the way to superconducting quantum circuits for microwave photons.

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