Prediction of the B_c^* Mass in Full Lattice QCD

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(Received 28 September 2009; published 12 January 2010)

By using the highly improved staggered quark formalism to handle charm, strange, and light valence quarks in full lattice QCD, and NRQCD to handle bottom valence quarks, we are able to determine accurately ratios of the B meson vector-pseudoscalar mass splittings, in particular, $[m(B_c^*) - m(B_c)]/[m(B_s^*) - m(B_s)]$. We find this ratio to be 1.15(15), showing the "light" quark mass dependence of this splitting to be very small. Hence we predict $m(B_c^*) = 6.330(7)(2)(6)$ GeV, where the first two errors are from the lattice calculation and the third from existing experiment. This is the most accurate prediction of a gold-plated hadron mass from lattice QCD to date.

DOI: 10.1103/PhysRevLett.104.022001 PACS numbers: 12.38.Gc, 11.15.Ha, 14.40.Lb, 14.40.Nd

Introduction.—Particle physicists now have long familiarity with the low-lying spectrum of $b\bar{b}$ (Upsilon) and $c\bar{c}$ (psi) mesons but they nevertheless continue to provide a very important testing ground for our understanding of strong interaction physics. The similar $b\bar{c}$ (B_c) system, on the other hand, is largely unexplored territory so predictions of the meson masses are very valuable. These predictions need to be as accurate as possible (and with an error budget) to provide stringent tests of QCD. Lattice QCD is clearly one of the best ways to do this, now that accurate calculations including the full effect of u, d, and s sea quarks inside hadrons are possible [1]. It has already provided successful predictions of the pseudoscalar η_b mass [2] (with a 14 MeV error) and the B_c mass [3] (with a 20 MeV error), both subsequently seen by experiment. Here we give a prediction for the vector B_c^* mass through the mass difference between the B_c^* and the B_c.

Mesons composed of valence heavy (b and c) quarks are relatively simple because they are nonrelativistic systems and a potential model may be expected to work reasonably well (see, for example, [4–6]). This is especially true for the Υ system where $v_b^2 \approx 0.1$ (in units of c^2). It is less true for charmonium where $v_c^2 \approx 0.3$ and so relativistic corrections are much larger there. The ground state hyperfine (vector-pseudoscalar) mass splitting is such a correction, but is given in leading order perturbation theory by a simple formula since the $\vec{S} \cdot \vec{S}$ potential is proportional to $\delta^3(\vec{r})$.

$$\Delta M = \frac{32\pi\alpha_s|\psi(0)|^2}{9m_1m_2}, \tag{1}$$

where m_1 and m_2 are the masses of the quark and antiquark and $\psi(0)$ is the wave function at the origin from the potential model. Using this formula to calculate the splitting will have a systematic error at $\mathcal{O}(v^2)$, i.e., 30% in $c\bar{c}$, 10% in $b\bar{b}$ and 20% in $b\bar{c}$. However, the $c\bar{c}$ hyperfine splitting has been used in the past to fix the effective value of α_s in Eq. (1) and then that 30% error affects all subsequent calculations. A larger problem, perhaps, is the variation in results between different potential models tuned to the spin-independent spectrum. This is because that spectrum does not in practice constrain the wave function at the origin at all strongly. The mass splitting between B_c^* and B_c can vary in the range 40–90 MeV [4–6] between different potentials, which makes it hard to decide a "central value" and error budget.

The reduced mass in the B_c system is roughly one half that of $b\bar{b}$ and 1.5 times that of the $c\bar{c}$. Then $v_b^2 \approx 0.05$ in B_c but $v_c^2 \approx 0.4$–0.5, which makes a nonrelativistic treatment worse in principle than for charmonium. An alternative approach is to treat the B_c as a "heavy-light" system using ideas from HQET but, for example, it is difficult to estimate the light quark mass dependence of the $1/m_Q$ operator giving rise to the hyperfine splitting, limiting again the accuracy in the prediction.

Lattice QCD, on the other hand, can provide very stringent tests of QCD from the hadron spectrum, in which all sources of systematic error can now be tested and quantified [1]. The only parameters are those of QCD itself (a quark mass for every flavor and a coupling constant) and impressively accurate results in agreement with experiment can be produced for the whole range of gold-plated hadron masses known experimentally. Our previous prediction of the B_c mass [3] dates from the relatively early days of full lattice QCD calculations and is now being improved. We have since developed a much more accurate method for handling charm quarks within lattice QCD and that has enabled a determination of the B_c mass with smaller systematic errors [7]. This method also allows an accurate prediction of the B_c^* and we describe that calculation here.

Lattice QCD calculation.—An optimal lattice QCD approach to the B_c is to combine a nonrelativistic method for the b quark with a relativistic one for c. We use lattice NRQCD for the b, developed over many years [8–10] to provide accurate bottomonium spectroscopy [2] by including spin-independent terms through $\mathcal{O}(v_b^4)$ and leading spin-dependent terms with discretization corrections through $\mathcal{O}(a^2)$ (a is the lattice spacing). For the c quark we use Highly Improved Staggered Quarks (HISQ) [11], a fully relativistic discretization of the Dirac action which is accurate enough to handle c quarks, being fully improved through $\mathcal{O}(a^2)$ and with the leading $(m_c a)^4$ errors removed. This enables us to use the same lattice QCD action for c, s and l quarks. We use the notation l for "light" to mean either u or d since we will use everywhere $m_u = m_d = m_l$. Some systematic errors then cancel between different B systems and we can obtain the hyperfine splitting in the B_c (or B_l) as a multiple of the experimentally known splitting [12] in the B_s system.

We work with five ensembles of gluon field configurations provided by the MILC collaboration. These include the full effect of u, d and s sea quarks using the improved staggered (asqtad) formalism (again $m_u = m_d = m_l$). The configurations have large spatial volumes ($>(2.4\ \text{fm})^3$) and are available at multiple values of m_l. We use configurations at three values of a between 0.15 fm and 0.09 fm with parameters as listed in Table I. On each configuration in the ensemble we generate b quark propagators using NRQCD and c, s and l quark propagators using HISQ. The parameters of the valence quarks are given in Table II. The b quark mass is tuned to give the correct Υ mass [14] and the charm, strange and light masses are taken from [15].

The b quark is then combined in turn with each of the other three with appropriate spin matrices to make pseudoscalar or vector mesons. To increase statistics we generate propagators from sources at several different timeslices per configuration (see Table I). We also use a random wall source for the quarks [15], taken as a set of U(1) random numbers at each point on the source time slice. This mimics multiple sources across a time slice when the propagators are paired up, improving statistics further. For the NRQCD propagators, as well as a local source, we also need "smeared" sources [2] chosen to improve the overlap with the ground state in the meson correlator. Exponentially growing noise is a problem in the B system (particularly as the lighter quark mass becomes small) and smearing enables us to extract an accurate ground state energy from the correlator at smaller time separation from the source than otherwise [16]. We use a Gaussian form for the smearing function with two different radii. These various sources for the NRQCD quark must be combined with the random wall described above. In addition the NRQCD quark source must now include the matrix that converts spinless staggered quarks into naive quarks for combination with 2-spin NRQCD quarks in an adaption [7,16] of the standard method of combining heavy quarks with staggered quarks [17].

We fit our 3×3 matrix [18] of B correlators using the standard Bayesian method [19] to a sum of exponentials, including oscillating parity partner states as

$$C_B(i, j; t - t_0) = \sum_{k=0}^{N_{\text{exp}}-1} a_{i,k} a_{j,k}^* e^{-E_k(t-t_0)} + \sum_{k'=0}^{N_{\text{exp}}-1} b_{i,k'} b_{j,k'}^* (-1)^{(t-t_0)} e^{-E'_{k'}(t-t_0)}, \tag{2}$$

where i, j index different smearing radii. We look for

TABLE I. Ensembles (sets) of MILC configurations used with gauge coupling β, size $L^3 \times T$ and sea masses ($\times$ tadpole parameter, u_0) m_l^{asq} and m_s^{asq}. The lattice spacing values in units of r_1 after "smoothing" are given in column 3 [13]. The configurations were generated using the HMD R algorithm, checked for step-size errors [13]. Column 8 gives the number of configurations and time sources per configuration used here. On set 5 only half the number were used for l quarks.

Set	β	r_1/a	$au_0 m_l^{\text{asq}}$	$au_0 m_s^{\text{asq}}$	L/a	T/a	$N_{\text{conf}} \times N_t$
1	6.572	2.152(5)	0.0097	0.0484	16	48	624×2
2	6.586	2.138(4)	0.0194	0.0484	16	48	628×2
3	6.760	2.647(3)	0.005	0.05	24	64	507×2
4	6.760	2.618(3)	0.01	0.05	20	64	589×2
5	7.090	3.699(3)	0.0062	0.031	28	96	556×4

TABLE II. Parameters for the valence quarks. aM_b is the b quark mass in NRQCD, and u_{0L} is the tadpole-improvement parameter used there [2]. We use stability parameter [2] $n = 4$ everywhere. Since NRQCD quarks propagate in one direction in time only we improve statistics by generating propagators both forwards in time (for $T/2$ time units) and backwards in time from each source. Columns 4, 6, and 7 give the charm, strange and light bare quark masses for the HISQ action. $1 + \epsilon$ is the coefficient of the Naik term in the charm case [11].

Set	aM_b	u_{0L}	am_c^{hisq}	$1 + \epsilon$	am_s^{hisq}	am_l^{hisq}
1	3.4	0.8218	0.85	0.66	0.066	0.0132
2	3.4	0.8225	0.85	0.66	0.066	0.0264
3	2.8	0.8362	0.65	0.79	0.0537	0.0067
4	2.8	0.8359	0.66	0.79	0.054 65	0.013 65
5	1.95	0.8541	0.43	0.885	0.0366	0.007 05

stability in the fits and their errors as a function of $N_{\rm exp}$ for ground state energies, E_0.

E_0 is not the meson mass but contains an energy shift due to the nonrelativistic treatment of the b quarks [8–10]. The shift cancels in the mass difference between states with the same NRQCD quark content. Thus the B_q^*-B_q splitting is obtained directly from $\Delta_q = E_0(B_q^*) - E_0(B_q)$. Because errors are strongly correlated between similar quantities calculated on the same ensembles we fit B_q and B_q^* correlators simultaneously to the form above and determine Δ_q directly from the fit. Values are given in Table III for $q = l, s, c$.

The terms from the HISQ action that contribute to the hyperfine splitting are hidden inside the discretization of the Dirac covariant derivative. Because HISQ is a relativistic action, these terms will automatically be correct in the $a \to 0$ limit. The spin-dependent term in the NRQCD action that gives rise to the hyperfine splitting can instead be explicitly pinpointed as the $\vec{\sigma} \cdot \vec{B}$ term [9]. This term has the correct tree-level coefficient to match full QCD at $\mathcal{O}(v_b^4)$ but radiative corrections beyond this have not been included. Hence the normalization of this term, and the normalization of the hyperfine splitting, have an uncertainty of $\mathcal{O}(\alpha_s)$ ($\approx$20%). This uncertainty is part of the NRQCD action and hence the same uncertainty appears regardless of which light quark is combined with the b quark and cancels in ratios of hyperfine splittings. In Table III we also give values for

$$R_c = \frac{\Delta_c}{\Delta_s} = \frac{E_0(B_c^*) - E_0(B_c)}{E_0(B_s^*) - E_0(B_s)}. \qquad (3)$$

and the corresponding quantity, R_l, for u or d quarks. On sets 1–4 R_l is given directly by a joint fit to B_s and B_l correlators. All R_l values agree with 1 within 30% errors.

Figure 1 shows R_c as a function of lattice spacing. There is little dependence on m_l, since neither the B_c nor the B_s contain valence light quarks and we do not expect strong sensitivity to the sea content. Lattice spacing dependence is mild—the line shows an extrapolation to the continuum limit at $a = 0$ that includes a^2 and a^4 terms and allows for linear dependence on sea quark masses. Our continuum result, to be compared to experiment, is $R_c = 1.14(15)$. This, along with the results for R_l show, somewhat surprisingly, that the hyperfine splitting varies hardly at all with the mass of the lighter quark in the B system, up to and including charm.

The result is backed up by the existing experimental results on heavy-light and heavy-strange mesons. In the D system the hyperfine splittings differ by only 2% between the D_s and the D_d. Some of this difference may in fact be a result of coupled-channel effects since the D_d^* is just above threshold for the decay to $D\pi$, whereas the D_s^* has only the OZI-disfavoured decay mode $D_s\pi$ available. The experimental situation is less clear in the B sector since some experimental results favor a B_s^*-B_s splitting very close to the B^*-B (not yet differentiated into charged and neutral modes) and others favor a somewhat larger splitting [12]. We use the PDG average value of 46.1(1.5) MeV [12] for the B_s^*-B_s splitting because, in keeping with our result and indications from the D sector, this is closer to the B^*-B splitting than the PDG fit value of 49.0(1.5) MeV [12].

Our result for R gives 53(7) MeV for the B_c^*-B_c splitting, where the error is statistical only. Additional systematic errors come from relativistic corrections to the $\vec{\sigma} \cdot \vec{B}$ term in the NRQCD action [9]. We can estimate the size of these from the size of v_b^2 in the B_c (0.05) and the B_s ($=(\Lambda_{\rm QCD}/m_b)^2 = 0.01$). The cancellation between them leads to a 4% systematic error. Any mistuning of the b quark mass will cancel in R, and small mistunings of the s and c quark masses lead to a negligible error. Electromagnetic hyperfine effects missing from our calculation should also be negligible (less than 1%).

Conclusions.—Adding our value for the B_c^*-B_c splitting to the experimental mass for the B_c gives the mass of the B_c^* as 6.330(7)(2)(6) GeV. The first two errors are statistics and systematics from the lattice QCD calculation; the third

TABLE III. Mass differences between vector and pseudoscalar B mesons with valence l, s or c quarks on different MILC ensembles. $\Delta_q = E_0(B_q^*) - E_0(B_q)$. R_q is the ratio Δ_q/Δ_s. The last row gives R_c and R_l extrapolated to $a = 0$.

Set	Δ_l	Δ_s	Δ_c	R_l	R_c
1	0.0318(78)	0.0311(37)	0.0324(2)	1.02(27)	1.04(12)
2	0.0374(35)	0.0359(21)	0.0326(3)	1.04(11)	0.908(53)
3	0.0306(54)	0.0287(19)	0.0268(2)	1.06(19)	0.934(62)
4	0.0245(68)	0.0261(27)	0.0271(4)	0.94(26)	1.04(11)
5	0.0177(35)	0.0189(12)	0.0210(2)	0.94(19)	1.111(72)
$a = 0$				1.00(23)	1.14(15)

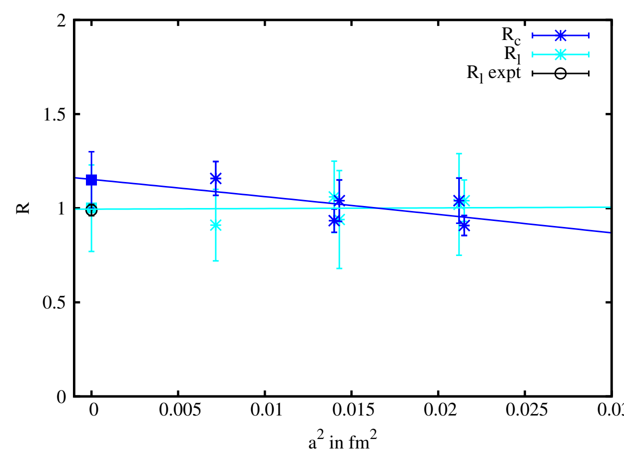

FIG. 1 (color online). The ratio R_c of the B_c and B_s hyperfine splittings [Eq. (3)] as a function of lattice spacing, a, from full lattice QCD. Our continuum extrapolation is also given and the result at $a = 0$. The lighter points and line give the equivalent points for R_l along with the experimental value [12].

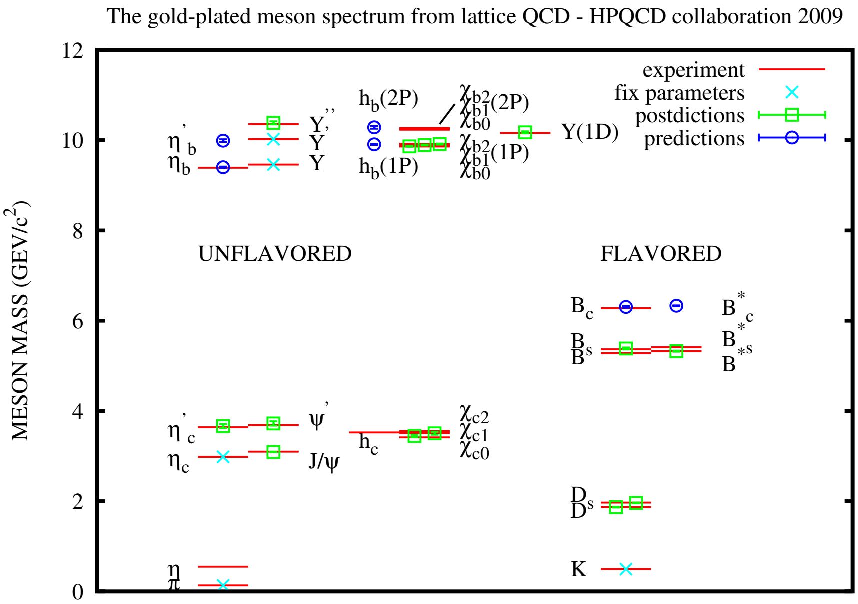

FIG. 2 (color online). The spectrum of "gold-plated" mesons from HPQCD calculations. Results are divided into those used to fix the parameters of QCD (4 quark masses and a coupling constant); those which are postdictions [2,11,15] and those, like the B_c^* described here, which are predictions [2,3]. The calculation of masses of non-gold-plated mesons are also of interest, particularly that of the flavor-singlet η' meson. Calculation of its mass is very hard, but is underway [20].

error is from experiment for the B_c and the B_s^*. The relatively small value of the B_c^*-B_c splitting will make it challenging to find the B_c^* from its decay to $B_c\gamma$.

The absence of strong dependence of the hyperfine splitting on the mass of the lighter quark in the B system is an interesting result, which has implications for other spin-dependent splittings in the B_c system. In HQET language it says that matrix elements of the hyperfine operator are insensitive to the lighter quark mass, up to and including charm. In constituent quark model language, using Eq. (1) for the hyperfine splitting, the result implies that $|\psi(0)|^2$ varies as m_q to cancel the m_q in the denominator [4]. The amplitude $a_{\mathrm{loc},0}$ from the fit in Eq. (2) is proportional to $\psi(0)$ in a nonrelativistic approach. This does show significant dependence on the lighter quark mass in the B. For example, $a_{\mathrm{loc},0}(B_c)/a_{\mathrm{loc},0}(B_s) \approx 2$ [7].

Finally, in Fig. 2 we summarize the current status of the gold-plated meson spectrum as determined from lattice QCD, highlighting those meson masses which have been made as predictions ahead of experiment. The result here is the most accurate prediction to date.

We are grateful to MILC for their configurations, to C. McNeile and J. Koponen for useful discussions and to STFC, MICINN, NSF, and DOE for support. Computing was done on USQCD's Fermilab cluster and at the Ohio Supercomputer Centre.

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