Spinning Black Holes as Particle Accelerators

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It has recently been pointed out that particles falling freely from rest at infinity outside a Kerr black hole can in principle collide with an arbitrarily high center of mass energy in the limiting case of maximal black hole spin. Here we aim to elucidate the mechanism for this fascinating result, and to point out its practical limitations, which imply that ultraenergetic collisions cannot occur near black holes in nature.

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Bañados, Silk, and West (BSW) [1] recently showed that particles falling freely from rest outside a Kerr black hole can collide with an arbitrarily high center of mass energy in the limiting case of maximal black hole spin. They proposed that this might lead to signals from ultrahigh energy collisions, for example of dark matter particles. In this Letter we aim to elucidate the mechanism for this result, and to point out its practical limitations given that extremal black holes do not exist in nature. In particular, we clarify why infinite collision energy can only be attained at the horizon, and with a maximally spinning black hole. We also show that the maximum center of mass energy grows very slowly as the black hole spin approaches its maximal value, so it will not be so high for astrophysical black holes. Finally, we calculate the upper bound for the energy of the ejecta of the collision and find that to be only slightly above the mass of the particles, even in the extremal limit. We use units with $G = c = M = 1$, where M is the black hole mass, and metric signature $(+ - - -)$.

While one can theoretically extract 100% of the rest energy of a mass by lowering it into a nonrotating black hole, and one can extract even more energy using a Penrose process lowering it into a rotating black hole, neither of these possibilities suggests that just by falling in freely from far away, a pair of particles can experience an infinite collision energy in their center of mass frame. If this is indeed possible then although the debris would be red-shifted on the way out, it might still reveal features of the S matrix at arbitrarily high energies. This is surprising since one seems to get an infinite energy boost—despite conservation of energy—from the finite process of falling into the black hole. But this is a misconception, as we will explain, since it takes in fact an infinite time to access the infinite collision energy.

We restrict attention here to orbits in the equatorial plane of a Kerr black hole with spin parameter a. Given the energy E, angular momentum l, and the unit 4-velocity condition, one can solve for the four velocity u at any given (Boyer-Lindquist) radial coordinate r, up to a discrete ambiguity in the sign of $\dot{r}$. Then one can compute the squared center of mass energy for a pair of particles of mass m,

$$E_{\rm cm}^2 = (mu_1 + mu_2)^2 = 2m^2(1 + u_1 \cdot u_2), \tag{1}$$

where the square and dot refer to the local Lorentz metric. For the case that the particles begin at rest at infinity, $E = m$, this yields Eq (8) of Ref. [1],

$$\begin{aligned}(E_{\rm cm}^{\rm Kerr})^2 = \frac{2m^2}{r(r^2-2r+a^2)}[&2a^2(1+r) - 2a(l_2+l_1) \\ &- l_2 l_1(-2+r) + 2(-1+r)r^2 \\ &- \sqrt{2(a-l_2)^2 - l_2^2 r + 2r^2} \\ &\times \sqrt{2(a-l_1)^2 - l_1^2 r + 2r^2}].\end{aligned} \tag{2}$$

The largest collision energy for such particles occurs when they collide at the horizon, carrying the maximum and minimum angular momenta that permit a fall all the way to the horizon. (We have not proved this analytically, but rather by numerical exploration.) These angular momenta correspond to those at which the centrifugal barrier drops just low enough so that there is no turning point for the radial motion. The particles therefore fall on a trajectory that spirals asymptotically into an unstable circular orbit at some critical radius, taking a logarithmically divergent proper time to do so. Another branch of the trajectories begins at this orbit and spirals into the black hole. It is this latter branch on which the maximum collision energy occurs at the horizon.

The location of the critical radius can be found using the effective potential for the radial motion with unit Killing energy per unit mass in the equatorial plane. The proper time derivative of the (Boyer-Lindquist) radial coordinate of orbital motion satisfies

$$\dot{r}^2/2 + V_{\rm eff}(r, l) = 0, \tag{3}$$

where the effective potential is given in terms of the angular momentum l per unit mass by [2]

$$V_{\rm eff} = -\frac{1}{r} + \frac{l^2}{2r^2} - \frac{(l-a)^2}{r^3}. \qquad (4)$$

The critical point we are looking for is defined by

$$V_{\rm eff} = dV_{\rm eff}/dr = 0, \qquad (5)$$

and is found to occur with angular momenta

$$l = l_\pm = \pm 2(1 + \sqrt{1 \mp a}), \qquad (6)$$

and at radius

$$r = r_\pm = 2 \mp a + 2\sqrt{1 \mp a}. \qquad (7)$$

In the nonspinning case this yields $l_\pm = \pm 4$ and $r_\pm = 4$, which lies well separated from the horizon. With these values, (2) gives $E_{\rm cm}^{\rm Kerr} = 2\sqrt{2}m$, while at the horizon these same angular momenta give $E_{\rm cm}^{\rm Kerr} = 2\sqrt{5}m$ [3]. In the maximally spinning case it yields $l_\pm = 2, 2(1+\sqrt{2})$, and $r_\pm = 1, (3+2\sqrt{2})$. The horizon lies at $r_h = (1 + \sqrt{1-a^2})$, so that in the extremal case $a = 1$, the critical radius $r_+ = 1$ coincides with the horizon.

In the maximally spinning case the corotating critical orbit is thus asymptotically tangent to the horizon. Its 4-velocity therefore tends to the null direction generating the horizon, since any other direction in the horizon is spacelike. In other words, the particle is moving at the speed of light, so the center of mass energy with any particle not on this horizon generator is infinite. This makes clear why infinite collision energy can only be attained at the horizon, and with a maximally spinning black hole. Since the particle never crosses the horizon, an infinite proper time passes for the particle as it spirals asymptotically onto the horizon. The nature of the divergence can be seen from the radial equation $\dot{r} = \sqrt{-2V_{\rm eff}(r)}$. At the critical orbit radius $r_\pm$ the effective potential has a maximum and vanishes; hence, nearby it is a negative quadratic function, $V_{\rm eff} = -r_\pm^{-3}(r - r_\pm)^2 + \cdots$. Therefore $\dot{r} \propto (r - r_\pm)$, so the proper time diverges logarithmically as $r_\pm$ is approached.

As BSW pointed out, for a black hole with spin parameter a less than M there will be an upper bound to the energy. What is somewhat surprising is how slowly the largest collision energy grows as the maximally spinning case is approached. We can estimate the maximal energy with the help of Eq. (2). In terms of the small parameter $\epsilon = 1 - a$, we find that the maximal collision energy per unit mass, i.e., the relative gamma factor, is approximately

$$\frac{E_{\rm cm}^{\rm max}}{m} \sim 4.06\epsilon^{-1/4} + O(\epsilon^{1/4}). \qquad (8)$$

In particular, for $\epsilon = 0.1, 0.01, 0.001, 0.0001$ the numerical result is, respectively, 6.90, 12.5, 22.6, 40.5. For an astrophysical black hole, accretion processes prohibit any spin factor greater than $a = 0.998$ as a theoretical upper limit [4], and MHD simulations [5] suggest the smaller upper limit of $a \lesssim 0.95$. These imply upper bounds of around 20 and 10, respectively, for the maximum collision gamma factor. Hence it seems that, even neglecting the effects of gravitational radiation [6], hyper-relativistic collision energies will not be realized in nature.

Note that the essential ingredient in the large collision energy is that one of the particles has the maximum angular momentum l_+. Above we indicated the result if the other particle has the minimum angular momentum l_- and the collision occurs at the horizon. If instead the collision occurs at r_+ (the critical unstable corotating circular orbit) the coefficient 4.06 of $\epsilon^{-1/4}$ in the leading approximation is replaced by 3.70. If the collision occurs at the horizon, but the second particle has zero angular momentum, it is replaced by 2.20. And if the collision occurs at r_+, and the second particle has zero angular momentum, it is replaced by 2.00. These examples illustrate that, if the collision energy is to be much larger for a spinning black hole than in the nonspinning case, the key is for one of the particles to carry the maximum angular momentum that can be captured. The angular momentum of the other particle is not really constrained, nor is the location of the collision, as long as it lies at or inside the critical radius r_+.

Finally, another point made by BSW is that although the collision energy can be arbitrarily large in the extremal limit, the energy of any collision products ejected to infinity will be redshifted. We can obtain an upper bound for the ejecta energy from a collision at the horizon as follows. In this limiting case one of the particles has a 4-momentum vector k that is (asymptotically) tangent to the horizon generator, while the other particle, has 4-momentum p. In order not to fall into the black hole, the 4 momentum of an ejecta particle must also be tangent to the horizon generator, so is λk for some scalar λ. (This is the marginal case. To escape, a particle must start strictly outside the horizon.) If the remaining reaction products have total 4-momentum p', then

$$p + k = p' + \lambda k, \qquad (9)$$

hence

$$p' = p + (1 - \lambda)k. \qquad (10)$$

Now since p, p', and k are all future pointing vectors, $p' \cdot p > 0$ and $k \cdot p > 0$, hence $\lambda - 1 < p \cdot p / k \cdot p$. If the mass of each of the initial particles is m, we have

$$\frac{p \cdot p}{k \cdot p} = \frac{m^2}{(E_{\rm cm}^2/2 - m^2)}. \qquad (11)$$

Thus the ejecta particle can have Killing energy at most twice that of k, i.e., at most $2m$. As the collision energy increases, this ejecta energy drops to something just slightly above m.

To summarize, we have examined some practical limitations of using black holes as particle accelerators, as proposed in Ref. [1]. Infinite center of mass energies for the colliding particles can only be attained when the black hole is exactly extremal and only at infinite time and on the horizon of the black hole. Additionally, the upper bound on the collision energy for an astrophysically realistic black hole is, neglecting radiation, less than 10 times the mass of the particles. The energy of the ejecta of the collision is no more than twice the particle rest mass. In conclusion, intriguing as it may be in principle, the possibility of spinning black holes catalyzing hyper-relativistic particle collisions does not seem realizable in practice.

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Note added.—As the present manuscript was being completed, a Comment by Berti *et al.* [6] appeared which also presents some of the points made here.

Note added in proof.—The possibility of ultrahigh energy collisions catalyzed by a rotating black hole was noticed long ago, in the context of the study of collisional Penrose processes [7,8]. The energy that can be extracted at infinity was analyzed there as well.

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