

Measurement of the Spiral Spectrum of Entangled Two-Photon States

H. Di Lorenzo Pires,* H. C. B. Florijn, and M. P. van Exter

Leiden University, Huygens Laboratory, P.O. Box 9504, 2300 RA Leiden, The Netherlands

(Received 18 September 2009; published 15 January 2010)

We implement an interferometric method to measure the orbital angular momentum (OAM) spectrum of photon pairs generated by spontaneous parametric down-conversion. In contrast with previous experiments, which were all limited by the modal capacity of the detection system, our method operates on the entire down-conversion cone and reveals the complete distribution of the generated OAM. In this geometry, new features can be studied. We show that the phase-matching conditions can be used as a tool to enhance the azimuthal Schmidt number and to flatten the spectral profile, allowing the efficient production of high-quality multidimensional entangled states.

DOI: 10.1103/PhysRevLett.104.020505

PACS numbers: 03.67.Bg, 03.67.Mn, 42.50.Tx

It is now recognized that the use of larger alphabets in quantum information processing brings possible advantages over multiple qubit schemes [1]. However, from the experimental point of view, the generation of quantum states exhibiting controllable and detectable multidimensional entanglement is still in its first stages of development. Of particular interest is the generation of photon pairs entangled in orbital angular momentum (OAM). The discrete nature of the underlying infinite dimensional Hilbert space and the limited sensitivity of photonic states to environmental noise make them strong candidates as carriers of quantum information. Experimentally, two-photon states entangled in OAM can be generated via the nonlinear process of spontaneous parametric down-conversion (SPDC). Since the existence of quantum OAM correlations between photon pairs was first demonstrated [2], an increasing effort is being devoted to manipulate and measure these states. This includes violations of bipartite Bell inequalities [3,4], the enhancement of OAM entanglement via concentration [5], and an implementation of a quantum coin-tossing protocol [6].

In this context, a full characterization of the OAM correlations is essential. The main question we want to address experimentally is the following: what is the precise form of the OAM spectra of down-converted photons? In other words: what are the relative weights P_l of different l modes, where l represents the photon topological winding number? The width of such modal expansion, denominated by Torres *et al.* [7,8] as quantum spiral bandwidth, is directly related to the amount of entanglement. Full knowledge of the OAM spectra allows one to inspect the quality of the entangled state and to determine whether the distribution of l modes is broad enough as compared to the channel capacity, which is essential to some protocols [9].

It is important to note that all previous experiments on OAM analysis [2,10,11] were limited by the modal capacity of the detection geometry and did not measure the true spectrum of the generated two-photon states. For instance, in Ref. [10] an azimuthal Schmidt number of $K_{\text{az}} = 7.3$ was measured, while one would expect

$K_{\text{az}} = 51.7$ based on the experimental parameters. The reason for this discrepancy is that only light within small angular sections around diametrically opposed regions of the SPDC cone were collected. This scheme not only discards most of the wave function, but also conceals the importance of the phase-matching conditions to the OAM spectrum. Furthermore, as we will argue, it is also fundamentally impossible to measure pure OAM correlations by using mode projections with holograms or phase plates and single mode fibers, as this configuration is also sensitive to radial field distributions of source and detectors (related to the mode number p). These are the main reasons why the outcome of previous works could not be directly compared with predictions of the well-known SPDC wave function. It has been recognized in the literature that [12,13] “an experiment aimed at detecting the global OAM of the down-converted photons is a significant challenge that it is yet to be solved.” In this Letter we will present such an experiment. We measure, for the first time, the complete OAM spectrum of the source. Additionally, we show how to efficiently extend the range of entangled modes via phase matching.

We consider SPDC emission along a principle axis of a birefringent nonlinear crystal pumped by a Gaussian beam. In this so-called noncritical phase-matching geometry, transverse walk-off can be neglected. Generalizations to other geometries will be given at the end of this Letter. For noncritical (type I and type II) SPDC the two-photon wave function in momentum representation is well known and assumes the form [14,15]

$$A(\mathbf{q}_s, \mathbf{q}_i) = \tilde{E}(\mathbf{q}_s + \mathbf{q}_i) \text{sinc}\left(\frac{L}{8k} |\mathbf{q}_s - \mathbf{q}_i|^2 + \varphi\right), \quad (1)$$

where $\tilde{E}(\mathbf{q})$ is the angular spectrum of the (Gaussian) pump beam, L is the crystal thickness, $k = 2\pi/\lambda$ is the wave vector of the generated light in the crystal, and $\mathbf{q}_{s,i}$ are the transverse components of the signal and idler wave vectors. Additionally, this function contains the collinear phase mismatch φ , which is also relevant in the forthcoming analysis. We can make use of polar coordinates

$\mathbf{q}_\alpha = (q_\alpha \cos\theta_\alpha, q_\alpha \sin\theta_\alpha)$, with $\alpha = s, i$, in order to separate the azimuthal and radial contributions. The rotation symmetry of Eq. (1) allows one to decompose the two-photon amplitude in the form [16] $A(\mathbf{q}_s, \mathbf{q}_i) = \sum_l \sqrt{P_l} e^{il(\theta_s - \theta_i)} F_l(q_s, q_i)/2\pi$. In order to access exclusively the azimuthal dependence of the field two requirements should be met. First, all the light must be collected by the detection scheme and second, one must use bucket, i.e., mode insensitive, detectors. This assures that the radial dependence will be completely integrated out as $\iint |F_l(q_s, q_i)|^2 q_s q_i dq_s dq_i = 1$. Naturally, this requirement is not satisfied when the state is spatially filtered or when a coherent detection scheme is used. Considering only the azimuthal dependence, the nature of the OAM correlations can thus be expressed in the entangled state $|\psi\rangle = \sum_{l=-\infty}^{+\infty} \sqrt{P_l} |l\rangle_s | -l\rangle_i$ where P_l is the probability of finding a signal photon with orbital angular momentum l and an idler photon with $-l$. It is important to stress once again that this decomposition refers to the whole generated state. The distribution of P_l , which we call the OAM spectrum of the two-photon field, is precisely the quantity we want to measure.

Figure 1 illustrates our experimental setup. It consists basically of a Mach-Zehnder interferometer with an image rotator inserted in one of its arms. It has been theoretically shown that such arrangement can be used to measure the OAM spectrum of an one-photon field in a superposition of l modes [17,18]. We will generalize the concept and argue that it can also be used to reveal the OAM spectrum of entangled two-photon states. Here, the visibility of a Hong-Ou-Mandel (HOM) interference as a function of the angle of rotation, in a collection geometry that does not constrain the photon wave vectors [19], will provide the necessary information.

The effect of the interferometer on the state $|\psi\rangle$ can be summarized as follows: the two photons (generated with orthogonal polarizations in type II SPDC) are separated at a polarizing beam splitter (PBS). The transmitted photon acquires an extra phase $e^{il\theta}$ due to the presence of an image rotator (IR). This homemade device is similar to a Dove

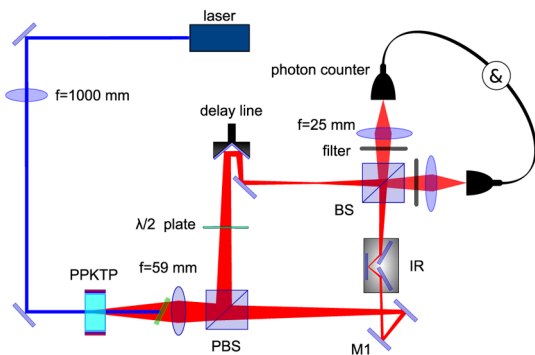


FIG. 1 (color online). Experimental setup used to generate entangled two-photon pairs and to measure its OAM spectrum. See details in the text.

prism, but consists of three mirrors instead. It offers the advantage that it can be more precisely aligned, so that the output beam moves less than $30 \mu\text{m}$ in position and less than $150 \mu\text{rad}$ in angle (see Ref. [10] for more information about our device). It is essential that one photon is reflected an even number of times and the other an odd number of times. If the interferometer is balanced, its output state, postselected on coincidences, is $|\psi_{\text{out}}\rangle \propto \sum_l \sqrt{P_l} (e^{il\theta} - e^{-il\theta}) |l\rangle_1 |l\rangle_2$. Notice that due to the uneven number of reflections, both photons have the same OAM index l at the output. The measured coincidence counts is given by $R_{\text{cc}} \propto \langle \psi_{\text{out}} | \psi_{\text{out}} \rangle$. The corresponding visibility of the HOM-like interference will depend on the rotation angle θ and is given by $V(\theta) = 1 - R_{\text{cc}}^{\tau=0} / R_{\text{cc}}^{\tau=\infty}$, where τ denotes the time delay between the two arms of the interferometer. One can show that

$$V(\theta) = \sum_{l=-\infty}^{+\infty} P_l \cos(2l\theta). \quad (2)$$

By measuring this visibility as a function of θ we can recover the weights P_l via inverse Fourier transform.

Entangled photon pairs are obtained via type II SPDC by pumping a 2-mm thick periodically poled KTP crystal (PPKTP) with a krypton-ion laser beam ($\lambda_p = 413.1 \text{ nm}$, $w_p = 150 \mu\text{m}$). The pump is blocked by an AR-coated GaP wafer. A $f = 59 \text{ mm}$ lens is used to make a $13\times$ magnified image of the SPDC light in the center of the image rotator; this image is then demagnified by $1/19\times$ and focused by two $f = 25 \text{ mm}$ lenses onto the active area of two photon counting modules. The polarization of the photon reflected by the PBS is rotated by a $\lambda/2$ plate in order to allow interference at the second beam splitter (BS). Spectral filters (2 nm at 826.2 nm) are used to select photons close to frequency degeneracy.

The experiment consists of measuring the HOM visibility for various angles θ . We do this by measuring, for each angle, the coincidence counts inside the HOM dip and then the coincidence counts outside the HOM dip, by imposing a time delay $\tau = 1.7 \text{ ps}$. The setup is fully automated. When $\theta = 0^\circ$ we expect a visibility $V = 100\%$, but measure at most $V = 80\%$. This discrepancy occurs because for type II SPDC with 2 nm bandwidth filters, the combined spatial-spectral profile of the photons still contains some “which-path” information. In the presented data we will compensate for it by normalizing the maximum measured visibility to 1. This renormalization is allowed because the combined spatial-spectral labeling involves only the radial coordinates of the field, and not the azimuthal. The use of even narrower band interference filters would eliminate this effect.

Figure 2(a) shows the measured (normalized) visibility obtained with the 2-mm PPKTP at perfect phase matching ($\varphi = 0$). The dots are the experimental results and the curve the theoretical prediction. Notice the excellent agreement between the two curves; no fitting parameters are needed. The curve is obtained by substituting the theoret-

cal probabilities P_l^{th} in Eq. (2). These probabilities can be numerically obtained by performing a Schmidt decomposition [7,16] of the two-photon amplitude of Eq. (1).

Figure 2(b) shows the measured OAM spectrum of the two-photon field. The bars are the experimental values, obtained via a discrete Fourier transform of the visibility in Fig. 2(a), and the dashed curve is the theoretical P_l^{th} . Predictions for the mode distributions are also present in Ref. [7] as a function of the parameter $\bar{w}_p = w_p/\sqrt{\lambda_p L}$. We experimentally confirm the predictions therein, at an experimental value of $\bar{w}_p = 5.2$.

Having the OAM spectral distribution, we can quantify the amount of entanglement present using the azimuthal Schmidt number K_{az} , defined as $K_{\text{az}} = 1/\sum_l P_l^2$. We obtain $K_{\text{az}}^{\text{ex}} = 21.4 \pm 0.5$, while the theoretical value is $K_{\text{az}}^{\text{th}} = 21.6$. In both computations we included l modes up to $|l| = 70$, where $P_l < 10^{-4}$.

It has been predicted in the literature that using pump beams with more complex spatial structures [20] or engineering the transverse structure of periodically poled crystals [21] may lead to an increased Schmidt number. Here we will take an alternative route and show how the manipulation of the phase-matching conditions may lead to the efficient generation of high-quality entangled states.

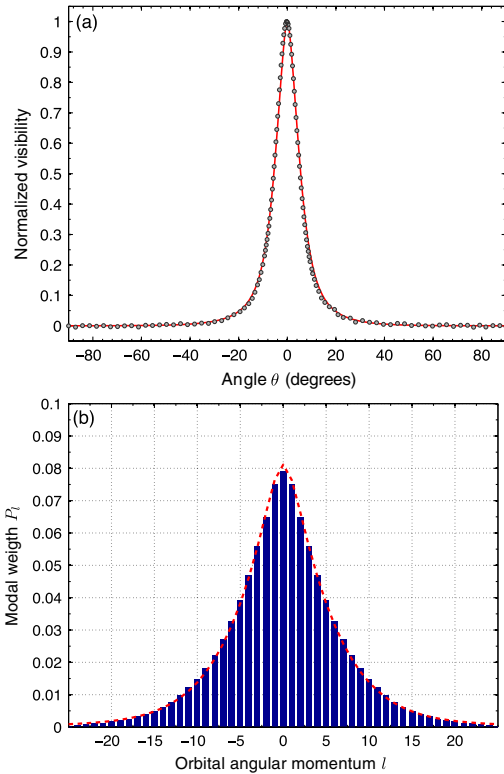


FIG. 2 (color online). (a) Normalized visibility of the HOM interference as a function of the rotation angle for a 2-mm crystal. The dots are experimental results and the curve the theoretical calculation. (b) Corresponding OAM spectrum obtained via Fourier transform. The bars are the experimental results and the curve the theoretical expectation. Notice the excellent agreement, where no fitting parameters are used.

The phase-mismatch parameter φ can be controlled experimentally by simply changing the temperature of the crystal. To further explore this effect we will switch to type I SPDC, because our type II crystals otherwise had to be operated below $T = 0^\circ\text{C}$. We change the PBS by a BS, remove the $\lambda/2$ plate, and change mirror M1 to a piezo-driven mirror. The latter allows us to remove during the measurements the additional interference fringes, on top of the HOM dip, that would otherwise be present (see Ref. [22] for details). All other parameters are unchanged, except for the thickness of the crystal. We now use a 5-mm type I PPKTP. Repeating the measurements at perfect phase matching ($\varphi = 0$) we obtain an azimuthal Schmidt number of $K_{\text{az}}^{\text{ex}} = 13.8 \pm 0.5$, while the theoretical $K_{\text{az}}^{\text{th}} = 13.9$, in agreement with the expected scaling $K_{\text{az}} \propto 1/\sqrt{L}$.

Next, we explore the effect of phase mismatch on the Schmidt number, especially for $\varphi < 0$, where the SPDC rings are open and the pair-generation process is almost twice as efficient. For this crystal $\varphi = 1.04 \times (T - T_0)$, where T_0 is the crystal temperature for perfect phase matching.

Figure 3 shows both the experimental values and the theoretical curve for K_{az} as a function of φ . The error bars are obtained by repeating the experiment, including realignment of the setup. We have thus demonstrated that the phase-matching conditions can be used to boost the Schmidt number. But what is the effect on the OAM spectrum?

Figure 4 shows how the modal decomposition changes for four values of φ . We see that for more negative values of φ the spectral profile tends to flatten, which leads to higher values of K_{az} . In the range considered ($|l| \leq 10$), the quality of the entangled state is such that it virtually eliminates the need for an entanglement concentration protocol [5,23]. This is important because most of the potential applications of quantum entanglement work best for maximally entangled states. Phase mismatch there-

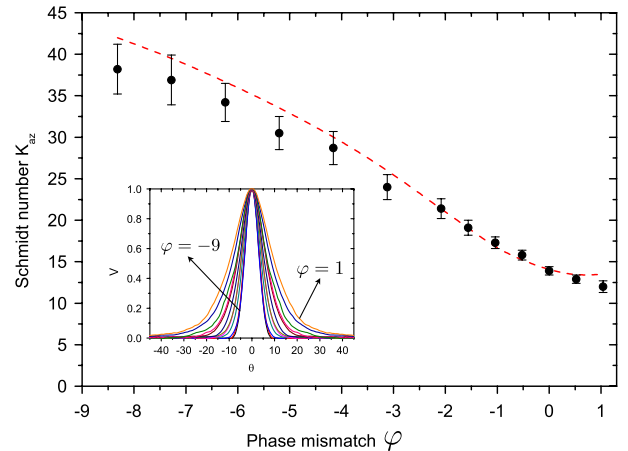


FIG. 3 (color online). Dependence of the azimuthal Schmidt number K_{az} on the phase-mismatch parameter φ for a 5-mm crystal. The inset shows the measured visibilities, where the inner curve (at $\varphi = -9$) corresponds to the highest K_{az} .

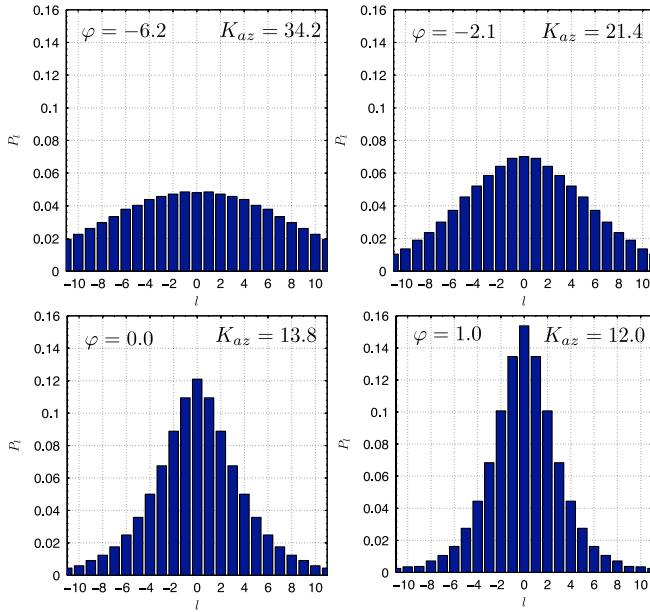


FIG. 4 (color online). The effect of phase mismatch on the modal decomposition P_l . For negative φ the SPDC rings are open and the pair generation is more efficient. By adjusting the phase-matching conditions we can increase the Schmidt number and flatten the spectral profile, producing high-quality entangled states in a larger l range.

fore not only extends the range of useful l modes in practical applications, allowing larger alphabets, but can also increase the conversion efficiency.

Finally, we would like to discuss two possible extensions of our approach, which applied to a rotationally symmetric pump in a noncritical geometry. First, one could use also a pump beam with $l_p \neq 0$, for which the down-converted state assumes the form $\sum_l \sqrt{P_l} |l\rangle |l_p - l\rangle$, where $P_l = P_{l_p - l}$. The complete OAM spectrum can again be determined from the measured visibility $V(\theta) = \sum_l P_l \cos[(l_p - 2l)\theta]$. Alternatively, one could also generate SPDC light in configurations where the OAM is not necessarily conserved [24–26]. If one wants to probe the most general state $\sum_{l,k} \sqrt{C_{l,k}} |l\rangle |k\rangle$, modifications of the setup are necessary. One now first needs to separate the two photons in a PBS before sending each photon to its own (one-photon) Mach-Zehnder interferometer with IR [18]. Measurements of coincidences between the output ports of the two interferometers will now provide the visibility $V(\theta_1, \theta_2)$. A double Fourier transformation is enough to recover $C_{l,k}$.

In conclusion, we have reported measurements of OAM spectra of entangled photon pairs generated by SPDC. This is, to the best of our knowledge, the first experiment where the entire down-converted cone is considered and where the detection geometry does not shape the spectrum nor limits its dimensionality K_{az} . By combining an interferometric technique with bucket detectors our method can access the pure OAM correlations, i.e., without coupling

azimuthal l with radial p modes. Our results can be directly confronted with the theoretical predictions for the generated state in SPDC. We found an excellent agreement between our experimental results and the predicted Schmidt decomposition. Furthermore, we have shown how the phase-matching conditions can be used as a tool to efficiently boost the Schmidt number, flatten the spectral profile, and virtually eliminate the need of entanglement concentration operations. Our experimental implementation can be generalized to measure OAM correlations of pure bipartite states of the most general form.

The authors acknowledge fruitful discussions with C. H. Monken, W. H. Peeters, and J. P. Woerdman. This work has been supported by the “Stichting voor Fundamenteel Onderzoek der Materie (FOM)” and by the EU under the FET-Open Agreement “HIDEAS,” No. FP7-ICT-221906.

*pires@molphys.leidenuniv.nl

- [1] H. Bechmann-Pasquinucci and W. Tittel, Phys. Rev. A **61**, 062308 (2000).
- [2] A. Mair *et al.*, Nature (London) **412**, 313 (2001).
- [3] A. Vaziri, G. Weihs, and A. Zeilinger, Phys. Rev. Lett. **89**, 240401 (2002).
- [4] J. Leach *et al.*, Opt. Express **17**, 8287 (2009).
- [5] A. Vaziri *et al.*, Phys. Rev. Lett. **91**, 227902 (2003).
- [6] G. Molina-Terriza *et al.*, Phys. Rev. Lett. **94**, 040501 (2005).
- [7] J. P. Torres, A. Alexandrescu, and L. Torner, Phys. Rev. A **68**, 050301(R) (2003).
- [8] L. Torner, J. P. Torres, and S. Carrasco, Opt. Express **13**, 873 (2005).
- [9] J. B. Pors *et al.*, Phys. Rev. Lett. **101**, 120502 (2008).
- [10] W. H. Peeters, E. J. K. Verstegen, and M. P. van Exter, Phys. Rev. A **76**, 042302 (2007).
- [11] A. K. Jha *et al.*, Phys. Rev. A **78**, 043810 (2008).
- [12] G. Molina-Terriza, J. P. Torres, and L. Torner, Nature Phys. **3**, 305 (2007).
- [13] C. I. Osorio, G. Molina-Terriza, and J. P. Torres, Phys. Rev. A **77**, 015810 (2008).
- [14] C. K. Hong and L. Mandel, Phys. Rev. A **31**, 2409 (1985).
- [15] C. H. Monken, P. H. Souto Ribeiro, and S. Pádua, Phys. Rev. A **57**, 3123 (1998).
- [16] C. K. Law and J. H. Eberly, Phys. Rev. Lett. **92**, 127903 (2004).
- [17] J. Leach *et al.*, Phys. Rev. Lett. **92**, 013601 (2004).
- [18] R. Zambrini and S. M. Barnett, Phys. Rev. Lett. **96**, 113901 (2006).
- [19] G. A. Barbosa, Phys. Rev. A **79**, 055805 (2009).
- [20] S. P. Walborn *et al.*, Phys. Rev. A **69**, 023811 (2004).
- [21] J. P. Torres *et al.*, Opt. Lett. **29**, 376 (2004).
- [22] T. Yarnall *et al.*, Opt. Express **16**, 7634 (2008).
- [23] C. H. Bennett *et al.*, Phys. Rev. A **53**, 2046 (1996).
- [24] H. H. Arnaut and G. A. Barbosa, Phys. Rev. Lett. **85**, 286 (2000).
- [25] G. Molina-Terriza *et al.*, Phys. Rev. A **72**, 065802 (2005).
- [26] S. Feng and P. Kumar, Phys. Rev. Lett. **101**, 163602 (2008).