# Highly Excited and Exotic Meson Spectrum from Dynamical Lattice QCD

Jozef J. Dudek,[1,2,*] Robert G. Edwards,[1] Michael J. Peardon,[3] David G. Richards,[1] and Christopher E. Thomas[1]

(for the Hadron Spectrum Collaboration)

[1]*Jefferson Laboratory, 12000 Jefferson Avenue, Newport News, Virginia 23606, USA*
[2]*Department of Physics, Old Dominion University, Norfolk, Virginia 23529, USA*
[3]*School of Mathematics, Trinity College, Dublin 2, Ireland*
(Received 13 October 2009; published 31 December 2009)

Using a new quark-field construction algorithm and a large variational basis of operators, we extract a highly excited isovector meson spectrum on dynamical anisotropic lattices. We show how carefully constructed operators can be used to reliably identify the continuum spin of extracted states, overcoming the reduced cubic symmetry of the lattice. Using this method we extract, with confidence, excited states, states with exotic quantum numbers ($0^{+-}$, $1^{-+}$, and $2^{+-}$), and states of high spin, including, for the first time in lattice QCD, spin-four states.

DOI: 10.1103/PhysRevLett.103.262001 PACS numbers: 12.38.Gc, 14.40.Cs

*Introduction.*—The spectroscopy of excited meson states is enjoying a renaissance through the observations of multiple new states in the charmonium sector. This will continue through the forthcoming experimental efforts at GlueX, BES III, and PANDA that will probe the spectroscopy of mesons in both the light and charm sectors. New states demand explanation within QCD and may offer insight into the appropriate degrees-of-freedom of low energy QCD. A particular example is mesons of exotic $J^{PC}$, those states whose quantum numbers cannot be constructed from a quark-antiquark bound state, and whose existence may signal the influence of explicit gluonic degrees of freedom.

Lattice QCD provides an *ab initio* method for the determination of the hadron spectrum. This approach to spectroscopy necessitates methods for measuring the two-point correlation functions of field operators with the selected quantum numbers under investigation. However, it has proven difficult to extract precise information from lattice QCD about excited states, states of high spin, and states with exotic $J^{PC}$. In this letter we will present results using a large basis of composite QCD operators and a variational analysis method which show that such extractions are now possible.

Access to states of spin-two or higher requires operators with spatially separated quark fields. To facilitate this kind of construction, a new quark-field construction algorithm, called ‘‘distillation’’, was developed [1] recently which enables efficient calculations of a broad range of hadron correlation functions, including those with spatially separated quark fields.

In Euclidean space, excited-state contributions to correlation functions decay faster than the ground-state and at large times are swamped by the larger signals of lower states. In improving our ability to extract excited states, better temporal resolution of correlation functions proves extremely helpful. An anisotropic lattice, where the temporal direction is discretized with a finer grid spacing than its spatial counterparts, is one means to provide this resolution while avoiding the increase in computational cost that would come from reducing the spacing in all directions. To this end, a large-scale program has been initiated to generate dynamical anisotropic gauge fields with two light clover quarks and one strange quark [2,3].

In this work, these anisotropic lattices are combined with the distillation technique for the construction of quark-antiquark operators with multiple derivative insertions. Only the connected Wick contractions are computed, giving access to isovector quantum numbers. For this first investigation, the three-flavor degenerate-quark-mass data set is used ($m_\pi = m_K = m_\eta \approx 700\,\mathrm{MeV}$), with lattice spacings $a_s \sim 0.12$ fm, $a_t^{-1} \sim 5.6$ GeV and a spatial lattice extent of $\sim 2$ fm. We will argue later that using a relatively large quark mass in this first study reduces complications due to mesons being able to decay into multimeson states.

*Spin on a cubic lattice.*—Lattice QCD computations consider the theory discretized on a four-dimensional Euclidean hypercubic grid. The reduced three-dimensional rotational symmetry with respect to the continuum introduces complications when one wishes to study particles of a particular spin, since spin no longer identifies irreducible

TABLE I. Continuum spins subduced into lattice irreps $\Lambda$(dim).

| $J$ | Irreps |
|---|---|
| 0 | $A_1(1)$ |
| 1 | $T_1(3)$ |
| 2 | $T_2(3) \oplus E(2)$ |
| 3 | $T_1(3) \oplus T_2(3) \oplus A_2(1)$ |
| 4 | $A_1(1) \oplus T_1(3) \oplus T_2(3) \oplus E(2)$ |

 

representations of the cubic symmetry group [4]. There are five single-cover lattice *irreps* for each parity and charge-conjugation: $A_1$, $T_1$, $T_2$, $E$, $A_2$. The distribution of the various $M$ components of a spin-$J$ meson into the lattice irreps is known as *subduction*, the result of which is displayed in Table I. In the continuum limit, the full rotational symmetry is restored and the components subduced into different irreps will be degenerate, whereas at finite lattice spacing they will be split by an amount scaling with at least one power of the lattice spacing, $a_s$.

This suggests a simple method to assign continuum spins by attempting to identify degeneracies across lattice irreps compatible with the subduction patterns in Table I. Unfortunately the empirical meson spectrum shows a number of approximate degeneracies that may be confused with those originating through discretisation. As an example consider the $\chi_{c0,1,2}$ states in charmonium, split only by a small spin-orbit force. These states would appear in a lattice computation as a single state in each of $A_1^{++}$, $T_1^{++}$, $T_2^{++}$ and $E^{++}$ and could easily be mistaken with a single $J = 4$ state split by discretisation effects. In the high lying part of the calculated spectrum, shown in Figs. 1 and 2, we observe considerable degeneracy that renders spin-identification by this method virtually impossible. In this letter we consider using the additional information embedded in the overlaps of states onto carefully constructed operators at zero momentum.

*Meson operators.*—By using a circular basis for both spatial derivatives and the three-vector-like gamma matrices ($\gamma_i$), we can utilize the SO(3) Clebsch-Gordan coefficients to construct continuum operators of definite spin. For example, with one derivative and a vector gamma matrix we can construct operators of overall spin $J = 0$, 1, 2:

$$(\Gamma \times D^{[1]}_{J_D=1})^{J,M} \equiv \langle 1, m_1; 1, m_2 | J, M \rangle \bar{\tilde{\psi}} \Gamma_{m_1} \overleftrightarrow{D}_{m_2} \tilde{\psi},$$

where repeated $m$ indices are summed. In the distillation framework, the fermion fields, $\tilde{\psi}$ are smeared using a low-rank filtering operator.

In the case of two derivatives we couple into a definite spin before coupling to the gamma matrix:

$$(\Gamma \times D^{[2]}_{J_D})^{J,M} \equiv \langle 1, m_3; J_D, m_D | J, M \rangle \times \langle 1, m_1; 1, m_2 | J_D, m_D \rangle \times \bar{\tilde{\psi}} \Gamma_{m_3} \overleftrightarrow{D}_{m_1} \overleftrightarrow{D}_{m_2} \tilde{\psi}.$$

For three derivatives combining the outermost derivatives together first ensures definite charge conjugation:

$$(\Gamma \times D^{[3]}_{J_{13},J_D})^{J,M} \equiv \langle 1, m_4; J_D, m_D | J, M \rangle \times \langle 1, m_2; J_{13}, m_{13} | J_D, m_D \rangle \times \langle 1, m_1; 1, m_3 | J_{13}, m_{13} \rangle \times \bar{\tilde{\psi}} \Gamma_{m_4} \overleftrightarrow{D}_{m_1} \overleftrightarrow{D}_{m_2} \overleftrightarrow{D}_{m_3} \tilde{\psi}.$$

This scheme can be extended to any desired number of covariant derivatives, which in practical computations are replaced by gauge-covariant finite differences. The gauge links appearing in these differences are stout-smeared to reduce UV fluctuations. To be of any real use in lattice calculations these operators of definite continuum spin, $J$, must be *subduced* into the irreducible representations of the cubic lattice rotation group ($\Lambda = \{A_1, T_1, T_2, E, A_2\}$). Noting that each class of operator is closed under rotations,

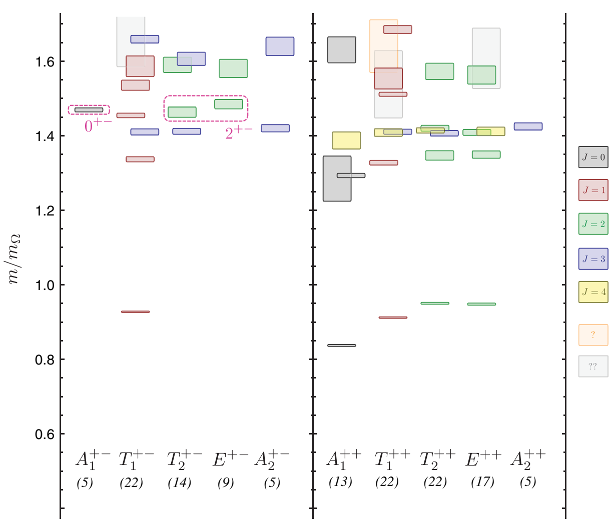


FIG. 1 (color online). Extracted spectrum of states in the $PC = +-$, $++$ channels displayed by lattice irrep. The number of operators in each irrep is given below the irrep label. All masses scaled by the $\Omega$ baryon mass as extracted on this lattice [3]. Boxes represent the extracted mass and one sigma statistical uncertainties. Color coding indicates continuum spin identification. Orange boxes have well-determined masses but undetermined spin. Grey boxes have masses that are not well determined by the variational fitting method. States with exotic quantum numbers $0^{+-}$ and $2^{+-}$ are highlighted.

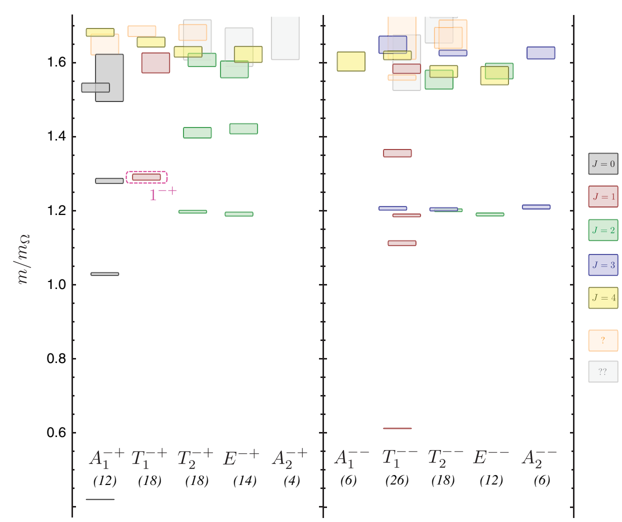


FIG. 2 (color online). As previous but for $PC = -+$, $--$. The lowest lying exotic $1^{-+}$ is highlighted.

the subductions can be performed using known linear combinations of the $M$ components for each $J$:

$$\mathcal{O}^{[J]}_{\Lambda,\lambda} \equiv (\Gamma \times D^{[n_D]}_{\dots})^{J}_{\Lambda,\lambda} = \sum_M \mathcal{S}^{J,M}_{\Lambda,\lambda} (\Gamma \times D^{[n_D]}_{\dots})^{J,M}$$
$$\equiv \sum_M \mathcal{S}^{J,M}_{\Lambda,\lambda} \mathcal{O}^{J,M}, \quad (1)$$

where $\lambda$ is the "row" of the irrep. Note that, although $\mathcal{O}^{[J]}_{\Lambda,\lambda}$ can have an overlap with all spins contained within $\Lambda$, it still carries the memory of the $J$ from which it was subduced, a feature we exploit below.

*Spectral analysis.*—For each lattice irrep $\Lambda^{PC}$ the full matrix of correlators $C_{ij}(t)$ was computed with equivalent rows ($\lambda$) averaged over. The dimension of the matrix is therefore equal to the number of operators constructed in that irrep.

The correlation matrix can be described by a spectral decomposition $C_{ij}(t) = \sum_{\mathfrak{n}} \frac{Z_i^{\mathfrak{n}*} Z_j^{\mathfrak{n}}}{2m_{\mathfrak{n}}} e^{-m_{\mathfrak{n}} t}$ (we only consider zero momentum), where $Z_i^{\mathfrak{n}} = \langle 0|\mathcal{O}_i|\mathfrak{n}\rangle$ encodes the overlap of state $\mathfrak{n}$ onto operator $\mathcal{O}_i$. An optimal method (in the variational sense [5,6]) to extract mass and $Z$ information from the matrix of correlators is by solution of a generalized eigenvalue problem, $C_{ij}(t) v_j^{\mathfrak{n}} = \lambda^{\mathfrak{n}}(t, t_0) C_{ij}(t_0) v_j^{\mathfrak{n}}$, where the eigenvectors $v^{\mathfrak{n}}$ are related to the $Z$ by $Z_j^{\mathfrak{n}} = \sqrt{2m_{\mathfrak{n}}} e^{m_{\mathfrak{n}} t_0/2} v_i^{\mathfrak{n}*} C_{ij}(t_0)$. Our implementation of this approach is described in [7].

The extracted spectrum across lattice irreps, including all operators with up to three derivatives, is shown in Figs. 1 and 2. We give the number of operators in each irrep and the color coding indicates continuum spin assignment suggested by a method we now describe.

Our particular choice of operator construction offers us a method to identify the continuum spin of a state. We take advantage of the fact that, at the lattice spacing we work, we expect lattice operators acting on extended objects such as mesons to behave in a manner reasonably close to the full rotational symmetry. In the continuum our operators are of definite spin such that $\langle 0|\mathcal{O}^{J,M}|J', M'\rangle = Z^{[J]}\delta_{J,J'}\delta_{M,M'}$ and so $\langle 0|\mathcal{O}^{[J]}_{\Lambda,\lambda}|J', M\rangle = \mathcal{S}^{J,M}_{\Lambda,\lambda} Z^{[J]}\delta_{J,J'}$. $Z^{[J]}$ is a single number of dynamical origin describing the overlap of the state of spin $J$ onto the operator used. We form a correlator in a given irrep $\Lambda$ and average over equivalent rows, $\lambda$,

$$\frac{1}{\dim(\Lambda)} \sum_\lambda C^{[\Lambda]}_{\lambda\lambda} \equiv \frac{1}{\dim(\Lambda)} \sum_\lambda \langle 0|\mathcal{O}^{[J]}_{\Lambda,\lambda} \mathcal{O}^{[J]\dagger}_{\Lambda,\lambda}|0\rangle.$$

Inserting a complete set of meson states between the operators and using the fact that the subduction coefficients form an orthonormal matrix, $\sum_M \mathcal{S}^{J,M}_{\Lambda,\lambda} \mathcal{S}^{J,M*}_{\Lambda',\lambda'} = \delta_{\Lambda,\Lambda'}\delta_{\lambda,\lambda'}$, we obtain terms proportional to $Z^{[J]*}Z^{[J]}$; these terms do not depend upon which $\Lambda$ we have subduced into. Hence, for example, a $J = 3$ meson created by a $[J = 3]$ operator will have the same $Z$ value in each of the $A_2$, $T_1$, $T_2$ irreps. Since this derivation uses smoothed, semiclassical fields it is valid in the continuum limit and at finite lattice spacing we expect there to be small deviations from equality due to discretization effects.

We take advantage of these properties to identify the spin of the extracted states in the following way. Firstly, we consider the relative magnitudes of the extracted $Z$ values for various states. Figure 3 shows that for the $J^{--}$ mesons, each state has large overlap only onto operators of a single spin. The second stage of the identification requires us to match states in different irreps and compare their $Z$ values with common operators subduced across irreps. As shown in Fig. 4, these values agree well. Any small discrepancy could be attributed to two causes: discretization errors from the use of simple central-difference operators to represent derivatives or the effect of renormalization. These operators act on smoothed gluonic and quark fields and this eliminates fluctuations at the cutoff scale so the latter effects will most likely be very small.

Using this method we have extracted a large number of states with all possible $PC$ combinations and confidently identified the spin of these states; the spectrum is shown in Figs. 1 and 2. As well as extracting many excited states, we have for the first time identified states with spin four: $4^{++}$, $4^{-+}$, and $4^{--}$. We have extracted states with exotic quantum numbers ($0^{+-}$, $1^{-+}$ and $2^{+-}$) and these are highlighted in the figures. The presence of these exotics likely points to the influence of explicit gluonic degrees of freedom.

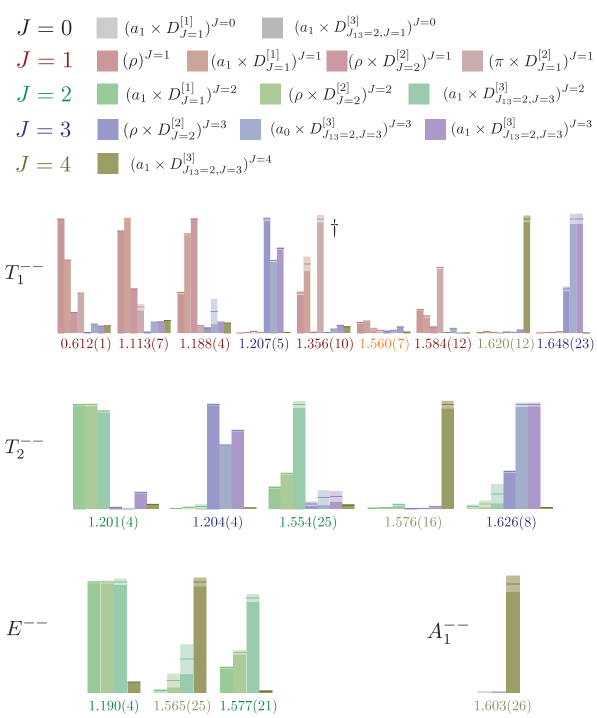


FIG. 3 (color online). Overlaps, $Z$, of a selection of operators onto states labeled by $m/m_\Omega$ in each lattice irrep, $\Lambda^{--}$. $Z$'s are normalized so that the largest value across all states is equal to 1. Lighter area at the head of each bar represents the one sigma statistical uncertainly.

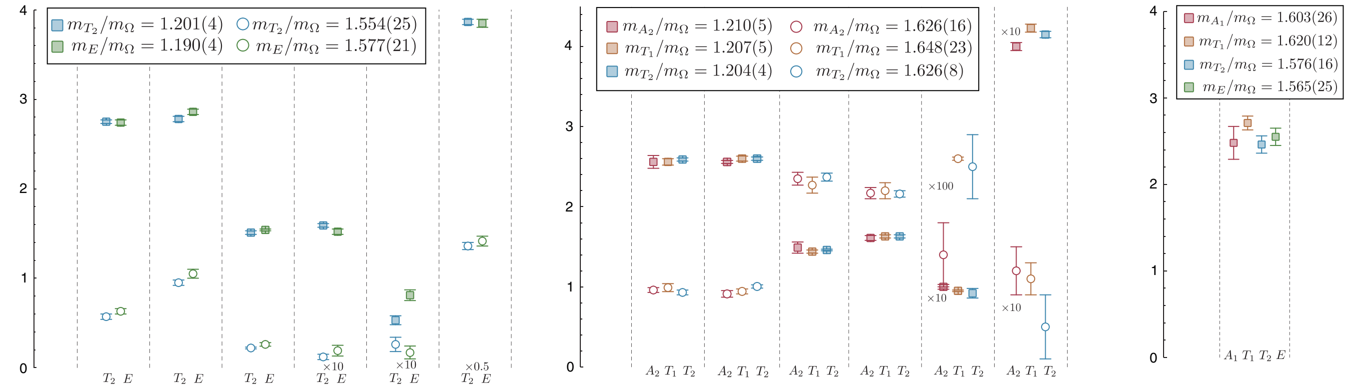


FIG. 4 (color online). A selection of $Z$ values across irreps $\Lambda^{--}$ for states suspected of being $J = 2, 3, 4$. Left to right operators are $(a_1 \times D^{[1]}_{J=1})^{J=2}$, $(\rho \times D^{[2]}_{J=2})^{J=2}$, $(\rho_2 \times D^{[2]}_{J=2})^{J=2}$, $(a_0 \times D^{[3]}_{J_{13}=2,J=2})^{J=2}$, $(b_0 \times D^{[3]}_{J_{13}=1,J=2})^{J=2}$, $(a_1 \times D^{[3]}_{J_{13}=0,J=1})^{J=2}$, $(\rho \times D^{[2]}_{J=2})^{J=3}$, $(\rho_2 \times D^{[2]}_{J=2})^{J=3}$, $(a_0 \times D^{[3]}_{J_{13}=2,J=3})^{J=3}$, $(a_1 \times D^{[3]}_{J_{13}=2,J=3})^{J=3}$, $(a_1 \times D^{[3]}_{J_{13}=2,J=2})^{J=3}$, $(b_1 \times D^{[3]}_{J_{13}=1,J=2})^{J=3}$ and $(a_1 \times D^{[3]}_{J_{13}=2,J=3})^{J=4}$.

Figure 3 shows that the third excited vector state ($m/m_\Omega \sim 1.35$, marked with a †) has a qualitatively different pattern of $Z$ values compared to the lighter vectors. Notably, there is large overlap with the $(\pi \times D^{[2]}_{J=1})^{J=1}$ operator. $D^{[2]}_{J=1}$ corresponds to the commutator of two covariant derivatives which vanishes in the absence of a gluonic field. This commutator is proportional to the field strength tensor and so the significant overlap hints at a gluonic component. This suggests an identification of this state as a crypto-exotic vector hybrid, although the nonzero overlap onto $\bar{\tilde{\psi}}\gamma_i\tilde{\psi}$ suggests some mixing with a conventional vector state.

*Two-meson states.*—We might expect to observe an abundance of two-meson states above $2m_\pi \sim 0.85m_\Omega$, but such states are not apparent in our extracted spectrum. This is most clearly seen in the $A_1^{--}$ channel where the lightest state extracted is a $J = 4$ state above $1.5m_\Omega$, while a pseudoscalar-vector state with the minimum relative momentum allowed in our finite box would be expected close to $1.2m_\Omega$. The operators used in this study featured only a single $\psi$, $\bar{\psi}$ field pair and so do not have overlap onto quark Fock states higher than $q\bar{q}$. QCD dynamics can act to mix $q\bar{q}$ Fock states with two-meson basis states to form mesonic eigenstates. This mixing is expected to be significant when a discrete lattice two-meson state is degenerate with a "single meson" to within that meson's continuum decay width. At this relatively heavy quark mass, we expect low-lying resonances to have small widths due to reduced phase-space for their decay and hence for there to be only small mixing with two-meson states, perhaps explaining our lack of observation of such states. A calculation similar to the one reported herein has been carried out on a lattice of spatial extent ~2.4 fm. The extracted spectrum is found to be identical within statistical fluctuations to that presented here. This is more evidence that we are not seeing two-meson states since their allowed relative momentum, and hence their energy levels, would have changed significantly. These issues can be properly investigated by including in the variational basis operators featuring a product of two fermion bilinears, expected to have good overlap onto two-meson states. This work is underway.

*Summary.*—We have demonstrated a lattice QCD operator construction that enables the identification of continuum spin with some confidence. Using distillation technology to construct the correlators, and a variational analysis to study them, we have extracted an excited-state spectrum featuring well-determined states with exotic quantum numbers and, for the first time, states of spin 4.

It is notable that our extracted spectrum has both features of the $n^{2S+1}L_J$ state assignment of bound-state quark models and also states that do not seem to lie within that classification. We believe that this study is seeing a full spectrum of QCD mesons which includes exotic and nonexotic hybrid mesons [8].

We thank our colleagues within the Hadron Spectrum Collaboration. The CHROMA software suite [9] was used to perform this work on clusters at Jefferson Laboratory using time awarded under the USQCD Initiative. Authored by Jefferson Science Associates, LLC under U.S. DOE Contract No. DE-AC05-06OR23177.

*dudek@jlab.org

[1] M. Peardon *et al.*, Phys. Rev. D **80**, 054506 (2009).
[2] R. G. Edwards, B. Joo, and H.-W. Lin, Phys. Rev. D **78**, 054501 (2008).
[3] H.-W. Lin *et al.*, Phys. Rev. D **79**, 034502 (2009).
[4] R. C. Johnson, Phys. Lett. B **114**, 147 (1982).
[5] M. Luscher and U. Wolff, Nucl. Phys. **B339**, 222 (1990).
[6] B. Blossier, M. Della Morte, G. von Hippel, T. Mendes, and R. Sommer, J. High Energy Phys. 04 (2009) 094.
[7] J. J. Dudek, R. G. Edwards, N. Mathur, and D. G. Richards, Phys. Rev. D **77**, 034501 (2008).
[8] J. J. Dudek and E. Rrapaj, Phys. Rev. D **78**, 094504 (2008).
[9] R. G. Edwards and B. Joo, Nucl. Phys. B, Proc. Suppl. **140**, 832 (2005).