New Insight into the Observation of Spectroscopic Strength Reduction in Atomic Nuclei: Implication for the Physical Meaning of Spectroscopic Factors

N. K. Timofeyuk

Department of Physics, University of Surrey, Guildford, Surrey GU2 7XH, United Kingdom

(Received 14 July 2009; published 8 December 2009)

Experimental studies of one-nucleon knockout from magic nuclei suggest that their nucleon orbits are not fully occupied. This conflicts a commonly accepted view of the shell closure associated with such nuclei. The conflict can be reconciled if the overlap between initial and final nuclear states in a knockout reaction are calculated by a nonstandard method. The method employs an inhomogeneous equation based on correlation-dependent effective nucleon-nucleon interactions and allows the simplest wave functions, in which all nucleons occupy only the lowest nuclear orbits, to be used. The method also reproduces the recently established relation between reduction of spectroscopic strength, observed in knockout reactions on other nuclei, and nucleon binding energies. The implication of the inhomogeneous equation method for the physical meaning of spectroscopic factors is discussed.

DOI: 10.1103/PhysRevLett.103.242501 PACS numbers: 21.10.Jx, 27.20.+n

The concept of magic numbers of neutrons and protons making up a nucleus is fundamental to our understanding of a wide range of phenomena from the properties and binding energies of nuclei themselves to the relative abundance of elements in the universe [1]. The observed magic numbers are usually explained by a model (the shell model) in which independent nucleons fill single-particle energy levels in a mean field according to the Pauli exclusion principle. Such a picture is similar to electronic structure of atoms responsible for organising the chemical elements into the Periodic Table. It has been found, however, that the cross sections of the $(e, e'p)$ reactions on the closed shell nuclei ^{16}O, ^{40,48}Ca, ^{208}Pb are 50%–60% smaller than those expected from the independent particle model [2]. Direct reaction theories of the $(e, e'p)$ reaction predict that its cross section depends on the spectroscopic factor (SF) which is a measure of the occupancy of single proton levels in the target nucleus. The observed reduction of SFs appears to contradict the traditional view of ^{16}O, ^{40,48}Ca, ^{208}Pb as doubly magic nuclei.

Away from the closed shells, nuclei such as ^{7}Li, ^{12}C, ^{30}Si, ^{31}P, ^{51}V, and ^{90}Zr also reveal a similar reduction of SFs as compared to prediction of the $0\hbar\omega$ shell model [3]. The SF reduction is also found for other nuclei in a recent analysis of the (d, p) and (p, d) reactions [4], in which the bound state wave functions of the transferred neutron are fixed by modern Hartree-Fock calculations and have shapes similar to those derived from $(e, e'p)$.

Recently, SF studies with radioactive beams have revealed a new phenomenon. It turned out that reduction of experimental SFs $S_{\rm exp}$, determined as ratios of the measured to theoretical cross sections, from the theoretical values $S_{\rm th}$, obtained in the shell model, depends on the separation energy of the removed nucleon and on the nucleon type. It has been also discovered that the SF reduction factor $R_s = S_{\rm exp}/S_{\rm th}$ is concentrated around a straight line when plotted as a function of the difference between proton (S_p) and neutron (S_n) separation energies, ΔS, taken as $S_p - S_n$ and $S_n - S_p$ for proton and neutron knockout, respectively [5].

It is known that $S_{\rm th}$ agrees better with $S_{\rm exp}$ if the model space, in which $S_{\rm th}$ is calculated, is increased, or in other words, if particle-hole excitations are allowed. Thus, a six-shell treatment of ^{16}O shows that the percentage of the $0\hbar\omega$ component in it is ~48%–60% [6] and that the ^{16}O SF changes from the $0\hbar\omega$ value of 2 to 1.65 when the model space increases to $4\hbar\omega$ [7]. However, it is still higher than the $(e, e'p)$ value of 1.27(13) [3] suggesting that more major shells should be added to the model space, which contradicts the view of ^{16}O as a double magic nucleus. The contributions from missing model spaces can be recovered by using correlated wave functions in *ab initio* approaches. Indeed, the ^{7}Li proton SF calculated in the variational Monte Carlo (VMC) method agrees very well with $S_{\rm exp}$ from $(e, e'p)$ [8]. However, for ^{8,9}Li, ^{8}B and ^{9}C the SF reduction obtained by VMC calculations is not sufficient (see Table I). Also, the *ab initio* calculations are feasible only for light nuclei while the SF reduction is observed for nuclei as heavy as ^{208}Pb.

In this Letter, I show that it is possible to reconcile the double magic nature of ^{16}O with the observed 60% reduction of its spectroscopic strength and at the same time to explain the observed $R_s(\Delta S)$ dependence if an alternative method to calculating SFs is used. This method allows minimal shell model spaces to be used and accounts automatically for excluded orbits. It can be applied to any nucleus and can be introduced into existing shell model codes including those used by the community of nuclear experimentalists studying one-nucleon removal reactions. Below, I present this method, emphasize its importance for explaining the phenomenon of SF reduction and present numerical results for $A < 16$ nuclei.

The theoretical SF for one-nucleon removal, S_{lj}, is defined in a model independent way as the norm of the

TABLE I. $S^{\rm IE} = S^{\rm IE}_{p1/2} + S^{\rm IE}_{p3/2}$ calculated with M3YE and RM3YE in comparison to $S^{\rm DE}$, experimental values $S_{\rm exp}$ [3,4,8–11] and *ab initio* VMC SFs S_{ab} [8,10–12].

A	$A-1$	$S^{\rm DE}$	M3YE	RM3YE	$S_{\rm exp}$	S_{ab}
^{3}H	d	1.5	1.21	1.33		1.30
^{3}He	d	1.5	1.22	1.35		1.32
^{4}He	^{3}He	2.0	1.29	1.42		1.50
^{7}Li	^{6}He	0.69	0.28	0.33	0.42(4)	0.42
^{7}Li	^{6}Li	0.87	0.44	0.46	0.74(11)	0.68
^{8}Li	^{7}He	1.02	0.38	0.44	0.36(7)	0.58
^{8}Li	^{7}Li	1.14	0.65	0.77		0.97
^{8}B	^{7}Be	1.14	0.78	0.91	0.89(7)	0.97
^{9}Li	^{8}Li	1.04	0.60	0.70	0.59(15)	1.14
^{9}Be	^{8}Li	1.13	0.45	0.49		0.73
^{9}C	^{8}B	1.04	0.71	0.82	0.77(6)	1.14
^{10}Be	^{9}Li	1.93	0.81	0.88		1.04
^{10}Be	^{9}Be	2.67	1.48	1.68		1.93
^{12}B	^{11}B	0.99	0.97	0.84	0.40(6)	
^{12}C	^{11}B	2.85	1.55	1.76	1.72(11)	
^{13}C	^{12}C	0.63	0.63	0.51	0.54(8)	
^{14}C	^{13}C	1.87	1.82	1.49	1.07(22)	
^{14}N	^{13}N	0.72	0.60	0.53	0.48(8)	
^{15}N	^{14}N	1.48	1.31	1.06	0.93(15)	
^{16}O	^{15}N	2.13	1.57	1.29	1.27(13)	

radial overlap function $I_{lj}(r)$ with orbital momentum l and angular momentum j, calculated between the wave functions Ψ_{J_B} and Ψ_{J_A} of two neighboring nuclei $B = A-1$ and A with the total spin J_B and J_A:

$$I^{\rm DE}_{lj}(r) = A^{1/2}\langle[[Y_l(\hat{\boldsymbol{r}}) \otimes \chi_{1/2}]_j \otimes \Psi_{J_B}]_{J_A}|\Psi_{J_A}\rangle. \quad (1)$$

All available shell model codes calculate S_{lj} from $I_{lj}(r)$ obtained by direct evaluation (DE) of Eq. (1) using some model wave functions in truncated model spaces. The input to these shell model calculations includes matrix elements of the effective nucleon-nucleon (NN) interaction fitted to a range of nuclear spectra. They carry no information about the radial shapes of $I_{lj}(r)$, crucial for calculating one-nucleon removal cross sections. In most applications, these shapes are found from the separation energy prescription, not related to the shell model NN matrix elements.

An alternative method to calculate $I_{lj}(r)$ is to solve the inhomogeneous equation (IE)

$$\langle\Psi_B|\hat{T}_A - \hat{T}_B - E_A + E_B|\Psi_A\rangle = \langle\Psi_B|V_B - V_A|\Psi_A\rangle, \quad (2)$$

originally introduced by Pinkston and Satchler [13]. Here $\hat{T}_i$ and V_i are the kinetic and potential energy operators while E_i is the total energy of nucleus i. The right-hand side of (2) is treated as known. Equation (2) generates $I_{lj}(r)$ which automatically have the correct asymptotic shape, a feature crucial for transfer reactions but not so for binding energy calculations. Earlier explorations of this method, reviewed in [14], were based on separating the mean field part out of V_i and keeping only the valence nucleon space. They gave little information of utility of the method and were abandoned before 1980s. Later, a different strategy, applied in [15,16] to calculating the source term $\langle\Psi_B|V_B - V_A|\Psi_A\rangle$, resulted in SFs different from traditional shell model values. Neither the Pinkston-Satchler approach nor that of Refs. [15,16] have been considered in the context of the SF reduction phenomenon as both were used at the time when the $R_s(\Delta S)$ dependence was not known. Here, I prove the legitimacy of the method of Ref. [16] and show its relevance to the SF reduction.

According to [16], the solution of Eq. (2) is

$$I^{\rm IE}_{lj}(r) = A^{1/2}\Big\langle\Big[\Big[\frac{G_l(r,r')}{rr'}Y_l(\hat{\boldsymbol{r}}') \otimes \chi_{1/2}\Big]_j \otimes \Psi_{J_B}\Big]_{J_A}||\hat{V}||\Psi_{J_A}\Big\rangle, \quad (3)$$

where integration over r' is implied and $G_l(r,r')$ is the Green function for a bound nucleon in the field of a point charge Z_B corresponding to the momentum $i\kappa$,

$$G_l(r,r') = -\frac{2\mu}{\hbar^2\kappa}e^{-\pi i(l+1+\eta)/2}F_l(i\kappa r_<)W_{-\eta,l+1/2}(2\kappa r_>). \quad (4)$$

Here $\kappa = (2\mu\epsilon/\hbar^2)^{1/2}$, $\epsilon = E_B - E_A$, μ is reduced mass, $\eta = Z_B Z_N e^2\mu/\hbar^2\kappa$, F is the regular Coulomb function and W is the Whittaker function. Also, $\hat{V} = V_A - V_B - Z_B Z_N e^2/r$ and $V_x = \sum_{i<j}^{x} v_{ij}$. In both Eqs. (1) and (3), r (r') is the distance between the center-of-mass of B and the removed nucleon, and Y_l is the spherical function. The advantage of (3) is that it guarantees the correct asymptotic form for $I^{\rm IE}_{lj}(r)$ when the experimental value of ϵ is used, whatever Ψ_{J_B} and Ψ_{J_A} are.

Equation (3) was obtained assuming that Ψ_{J_B} and Ψ_{J_A} are exact solutions of the many-body Schrödinger equation and that $\hat{V}$ contains bare realistic NN interactions. In this case, $I^{\rm DE}_{lj}(r)$ and $I^{\rm IE}_{lj}(r)$, and the corresponding SFs $S^{\rm DE}_{lj}$ and $S^{\rm IE}_{lj}$, should be equal. However, usually Ψ_{J_B} and Ψ_{J_A} are replaced by model wave functions in truncated spaces. This raises the question about what should be used for $\hat{V}$. To answer it, I consider an exact nuclear wave function Ψ constructed from an uncorrelated state Φ, defined in some truncated model space, for example, using the unitary correlation operator method [17]:

$$|\Psi\rangle = C|\Phi\rangle = \exp\Big\{-i\sum_{i<j}^{A} g_{ij}\Big\}|\Phi\rangle. \quad (5)$$

Here C is the unitary correlator designed to shift nucleons away from each other whenever their uncorrelated positions are within the repulsive NN core. Φ is found from an effective Hamiltonian that contain effective interactions $V^{\rm eff}$ consisting of $\hat{V} = C^\dagger V C$ and the terms arising from the kinetic energy operator [17]. If wave functions from Eq. (5) are used in Eq. (3), then

$$\langle \Psi_B|\hat{\mathcal{V}}|\Psi_A\rangle = \langle \Phi_B|C_B^{\dagger}(V_A - V_B)C_A|\Phi_A\rangle = \langle \Phi_B|V_N C_{NB}|\Phi_A\rangle = \langle \Phi_B|\tilde{V}^{\rm eff}|\Phi_A\rangle, \quad (6)$$

where $C_A = C_B C_{NB}$, $C_{NB} = \exp\{-i\sum_{i=1}^{B} g_{iN}\}$ and $V_N = \sum_{i=1}^{B} v_{iA}$, assuming for simplicity that Coulomb interaction is absent. Equation (6) tells us that the effective interaction $\tilde{V}^{\rm eff}$ that approximates $\hat{\mathcal{V}}$ when modeling $I_{lj}^{\rm IE}(r)$ using uncorrelated model functions Φ_B and Φ_A, differs from the effective interaction $V^{\rm eff}$ that generates them. Moreover, Φ_B and Φ_A depend only on matrix elements $\langle \psi_{\alpha_1}(\boldsymbol{r}_1)\psi_{\alpha_2}(\boldsymbol{r}_2)|v^{\rm eff}(r_{12})|\psi_{\alpha_3}(\boldsymbol{r}_1)\psi_{\alpha_4}(\boldsymbol{r}_2)\rangle$ in a chosen truncated space, where $\psi_\alpha(\boldsymbol{r})$ is a single-particle wave function in the state α. Hence, $I_{lj}^{\rm IE}(r)$ and $S_{lj}^{\rm IE}$ depend on them as well. But in addition, they also depend on matrix elements of $\tilde{V}^{\rm eff}$, $\langle G_l(r, r_1)/(r_1 r)\psi_{\alpha_2}(\boldsymbol{r}_2)|\times \tilde{v}^{\rm eff}(r_{12})|\psi_{\alpha_3}(\boldsymbol{r}_1)\psi_{\alpha_4}(\boldsymbol{r}_2)\rangle$ (if center-of-mass motion is neglected), that carry information about coupling to missing model spaces. This conclusion follows from the Green function expansion onto complete set $\{\psi_\alpha(\boldsymbol{r})\}$, which includes states from both truncated and missing spaces. Thus, these matrix elements are not constrained by binding energy calculations and must be constrained by some other means.

A quantity that can serve as a reference to callibrate $\tilde{V}^{\rm eff}$ is the asympotic normalization coefficient (ANC). It determines the magnitude of the $I_{lj}(r)$ tail [18], depends on the same operator $\tilde{V}^{\rm eff}$ and can be determined from peripheral transfer experiments. In [19], the vertex constants, related to the ANCs by a trivial relation [18], were studied for $0p$-shell nuclei in the oscillator $0\hbar\omega$ shell model. It was found that reasonable agreement between measured and calculated vertex constants can be achieved if a version of the M3Y potential, constructed in [20] to fit the oscillator matrix elements derived from the NN scattering phase shifts, is used for $\tilde{V}^{\rm eff}$. Below, I use this interaction (labled as M3YE) to calculate $S_{lj}^{\rm IE}$. I show that they are reduced with respect to $S_{lj}^{\rm DE}$ and, at the same time, are closer to experimental SFs.

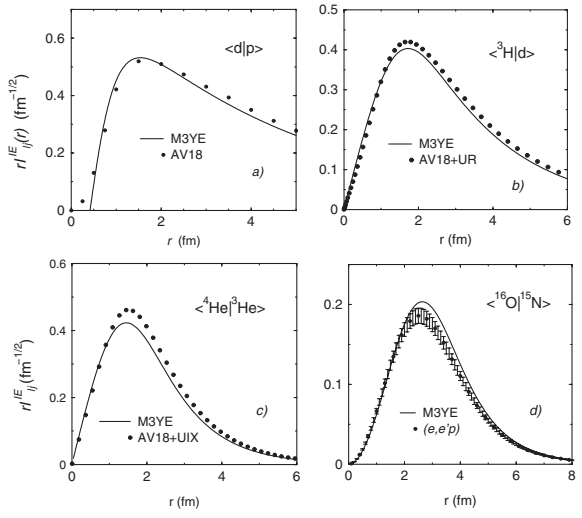

FIG. 1. The overlap functions $rI^{\rm IE}(r)$ calculated for M3YE for (a) $\langle d|p\rangle$, (b) $\langle {}^3{\rm H}|d\rangle$, (c) $\langle {}^4{\rm He}|{}^3{\rm He}\rangle$ and (d) $\langle {}^{16}{\rm O}|{}^{15}{\rm N}\rangle$ in comparison to (a) the realistic deuteron wave function obtained with AV18 potential in [21], (b, c) *ab initio* overlaps obtained with AV18 + UR (or UIX) interaction in [22,23] and (d) the overlap function derived from $(e, e'p)$ reaction [3].

First of all, I test the method as applied to the well understood $A = 2$ system, for which $I_{lj}^{\rm IE}(r)$ is the deuteron wave function and satisfies

$$rI_{0s}^{\rm IE}(r) = \int_0^\infty dr' r' G_0(r, r')\tilde{V}^{\rm eff}(r')\varphi_{0s}(r'), \quad (7)$$

where φ_{0s} is the $0s$ oscillator wave function. $rI_{0s}^{\rm IE}(r)$, calculated with M3YE for $\tilde{V}^{\rm eff}$ and with oscillator radius $r_{\rm osc} = 1.51$ fm, is close to the realistic deuteron wave function generated by the NN potential AV18 [21] [see Fig. 1(a)]. Its norm, $S^{\rm IE} = 0.91$, is close to the s-wave probability of 0.94 established in the deuteron.

For closed shell nuclei, $I_{lj}^{\rm IE}(r)$ depends only on $\tilde{V}^{\rm eff}$ and does not depend on the effective interactions determining their energies. Thus, the SFs for these nuclei, together with their ANCs, can serve in the future as a reference for callibrating the interaction $\tilde{V}^{\rm eff}$. Here, I calculate the overlaps $\langle {}^3{\rm H}|d\rangle$, $\langle {}^4{\rm He}|{}^3{\rm He}\rangle$ and $\langle {}^{16}{\rm O}|{}^{15}{\rm N}\rangle$, involving closed shell nuclei, using M3YE. Only one Slater determinant has been used for Φ_A and Φ_B, which are divided by the $0s$ center-of-mass motion wave function. The $r_{\rm osc}$ is chosen to be 1.53 for ^{3}H and ^{3}He, 1.33 fm for ^{4}He and 1.8 fm for ^{15}N and ^{16}O to reproduce their rms radii. For $A = 3$ and $A = 4$ $I_{lj}^{\rm IE}(r)$ are slightly smaller than the *ab initio* overlaps from [22,23] [see Figs. 1(b)–1(d)] but for $A = 16$ $I_{lj}^{\rm IE}(r)$ is slightly larger than the overlap function derived from the $(e, e'p)$ knockout [3]. In both cases, $S^{\rm IE}$ are reduced with respect to $S^{\rm DE}$ (see Table I).

For open shell nuclei, $S^{\rm IE}$ also depend on occupancies of the single-particle orbits in the chosen model space, or on weights of the $SU(3)$ and $SU(4)$ configurations in the supermultiplet shell model. I generate these weights using phenomenological interaction from [24] which gives improved spectra of $0p$ shell nuclei. I remove the center-of-mass motion explicitly and use $r_{\rm osc}$ chosen as an average of values for nuclei A and B derived in [25] from electron scattering. The resulting SFs $S^{\rm IE}$ for ground states of the $0p$-shell nuclei, obtained with M3YE, are compared in Table I to $S^{\rm DE}$ and to SFs available from knockout and those transfer reactions that use Hartree-Fock wave functions for transfer states. For all of them, $S^{\rm IE} < S^{\rm DE}$, which clearly displays the SFs reduction phenomenon. However, $S^{\rm IE} > S_{\rm exp}$ for $0p_{1/2}$ and $S^{\rm IE} < S_{\rm exp}$ for $0p_{3/2}$. Agreement between $S^{\rm IE}$ and $S_{\rm exp}$ can be improved by tuning the $\tilde{V}^{\rm eff}$ potential. In this letter, for demonstration purpose only, I make the following changes to M3YE. All potentials in even partial waves are multiplied by 1.05. This increases the SFs for d, ^{3}H, ^{3,4}He by 10%. Then the central and spin-

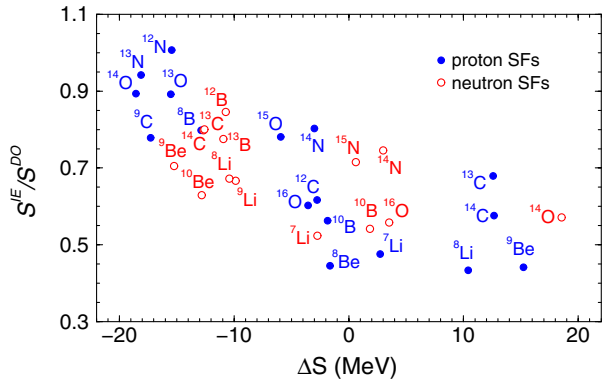

FIG. 2 (color online). The ratio $S^{\mathrm{IE}}/S^{\mathrm{DE}}$, calculated with RM3YE.

orbital odd components are multiplied by 1.7 and 2.5, respectively, which allows S_{exp} for both ^{12}C and ^{16}O to be reproduced. Increasing odd tensor component twice reproduces the SF for ^{13}C. The SFs calculated with such a renormalized potential, called here RM3YE, are shown in Table I. Most SFs agree well with experimental data. Detailed discussion of this comparison will be published elsewhere.

The ratio $R_{\mathrm{DE}}^{\mathrm{IE}} = S^{\mathrm{IE}}/S^{\mathrm{DE}}$, obtained with RM3YE, is shown in Fig. 2 as a function of ΔS. The decrease towards large positive ΔS can partially be explained by the presence of κ-dependent function $G_l(r, r')$ in Eq. (3). Computer calculations show that, for fixed Φ_A, Φ_B and $\tilde{V}^{\mathrm{eff}}$, S^{IE} decreases with increasing κ. Other effects must be also responsible for $R_{\mathrm{DE}}^{\mathrm{IE}}(\Delta S)$ behavior but no rigorous explanation to it is yet available.

The ratio $R_{\mathrm{DE}}^{\mathrm{IE}}(\Delta S)$ is remarkably similar to $R_s(\Delta S)$ from [5]. This suggests that what really is measured in one-nucleon removal experiments is not S^{DE} but S^{IE}, thus implying that these experiments study not occupancies of the shell model orbits but effective interactions $V_N C_{NB}$ for occupancies fixed from other observables, such as binding energies or nuclear spectra. Because of the presence of the Green function in Eq. (3), S^{IE} carries much more information about missing model spaces than S^{DE}. Therefore, it may be difficult to get correct values for SFs by overlapping wave functions directly even if they are obtained in a correlated *ab initio* approach. Indeed, the VMC SFs for light nuclei are systematically larger than S^{IE} calculated in a much simpler model with a reasonably chosen effective interaction, and, except for ^{7}Li, the VMC SFs are in a worse agreement with experiment than those from the present work (see Table I).

Thus, for 50 years SFs have been calculated in a procedure of direct overlapping model wave functions that is sensitive only to effective interactions in truncated model space and does not contain important contributions from excluded model spaces. Calculating SFs from $I_{lj}^{\mathrm{IE}}(r)$ generated by Eq. (3) is a more appropriate procedure that allows small model spaces to be used to explain the large reduction of spectroscopic strength due to coupling to missing model spaces. Moreover, explicitly depending on *NN* matrix elements both in truncated and excluded spaces and having a guaranteed correct asymptotic form, $I_{lj}^{\mathrm{IE}}(r)$ itself becomes an interface between nuclear structure and nuclear reactions theories. Incorporating Eq. (3) into widely used shell model codes and into other microscopic approaches, including *ab initio* ones, would be highly beneficial for modern nuclear physics and for astrophysical applications, in particular.

I am grateful to D. J. Millener for providing me with his supermultiplet shell model code and *NN* interactions and for helpful discussions, M. Viviani and A. Kievsky for sending me overlap functions of three- and four-body systems, L. Lapikás for clarifying accuracy of the ^{7}Li SF, R. C. Johnson for valuable comments and help in preparing this Letter. The support from the UK STFC grant ST/F012012/1 is thankfully acknowledged.

[1] M. Goeppert-Mayer and J. H. D. Jensen, *Nuclear Shell Structure* (John Wiley & Son, N.Y., 1955).
[2] V. R. Pandharipande, I. Sick, and P. K. Huberts, Rev. Mod. Phys. **69**, 981 (1997).
[3] G. J. Kramer, H. P. Blok, and L. Lapikás, Nucl. Phys. **A679**, 267 (2001).
[4] J. Lee *et al.*, Phys. Rev. C **73**, 044608 (2006).
[5] A. Gade *et al.*, Phys. Rev. C **77**, 044306 (2008).
[6] E. K. Warburton, B. A. Brown, and D. J. Millener, Phys. Lett. B **293**, 7 (1992).
[7] B. A. Brown *et al.*, Phys. Rev. C **65**, 061601 (2002).
[8] L. Lapikás, J. Wesseling, and R. B. Wiringa, Phys. Rev. Lett. **82**, 4404 (1999).
[9] J. Enders *et al.*, Phys. Rev. C **67**, 064301 (2003).
[10] R. Kanungo *et al.*, Phys. Lett. B **660**, 26 (2008).
[11] A. H. Wuosmaa *et al.*, Phys. Rev. C **78**, 041302 (2008).
[12] R. Wiringa, http://www.phy.anl.gov/theory/research/overlap.
[13] W. T. Pinkston and G. R. Satchler, Nucl. Phys. **72**, 641 (1965).
[14] G. R. Satchler, *Direct Nuclear Reactions* (Oxford Press, New York, 1983).
[15] A. M. Mukhamedzhanov *et al.*, Sov. J. Nucl. Phys. **52**, 452 (1990).
[16] N. K. Timofeyuk, Nucl. Phys. **A632**, 19 (1998).
[17] H. Feldmeier *et al.*, Nucl. Phys. **A632**, 61 (1998).
[18] L. D. Blokhintsev, I. Borbely, and E. I. Dolinskii, Sov. J. Part. Nucl. **8**, 485 (1977).
[19] A. M. Mukhamedzhanov and N. K. Timofeyuk, Sov. J. Nucl. Phys. **51**, 431 (1990).
[20] G. Bertsch *et al.*, Nucl. Phys. **A284**, 399 (1977).
[21] R. B. Wiringa, V. G. J. Stoks, and R. Schiavilla, Phys. Rev. C **51**, 38 (1995).
[22] M. Viviani, A. Kievsky, and S. Rosati, Phys. Rev. C **71**, 024006 (2005).
[23] A. Kievsky *et al.*, Phys. Lett. B **406**, 292 (1997).
[24] D. J. Millener, Lect. Notes Nucl. Phys. **724**, 31 (2007).
[25] C. W. De Jager, H. De Vries, and C. De Vries, At. Data Nucl. Data Tables **14**, 479 (1974).