Comment on "Kerr Black Holes as Particle Accelerators to Arbitrarily High Energy"

Bañados, Silk, and West (henceforth BSW) [1] recently observed that collisions of point particles falling from rest into rotating (Kerr) black holes (BHs) may have arbitrarily large c.m. energies close to the event horizon if the BH is maximally spinning and one of the particles has orbital angular momentum close to the value $L = L_{\rm scat}$ corresponding to marginally bound geodesics.

Astrophysical limitations on the maximal spin, backreaction effects and sensitivity to the initial conditions impose severe limits on the likelihood of such collisions. An important practical limitation on the achievable c.m. energies occurs because, as pointed out by Thorne [2], the dimensionless spin of astrophysical BHs should not exceed $a/M = 0.998$. For $a/M = 0.998$, Eq. (14) in BSW yields maximum c.m. energies of about $10\,\mu$ for particles of rest mass μ.

Even in the idealized scenario of *being given* an extremal BH, backreaction effects make high-energy scattering very unlikely. Neglecting gravitational radiation, upon absorption of a pair of colliding particles of mass μ, the dimensionless spin must be reduced by $\epsilon \equiv 1 - a/M \sim \mu/M$. After this first collision, Eq. (14) in BSW predicts that the new maximum allowed c.m. energy would be $E_{\rm CM} \lesssim 10^{12}\ (\mu/1\,{\rm MeV})^{3/4}(M/100M_\odot)^{1/4}$ GeV, orders of magnitude below the Planck scale for typical values of the parameters. The collision of (say) a *single* electron pair would reduce the spin of a $100M_\odot$ BH enough to inhibit any further Planck-scale collisions. A hypothetical dark matter particle would need a mass $\mu \gtrsim 10^3$ TeV to allow for more than one Planck-scale event.

The estimates in BSW neglect gravitational radiation, which will significantly affect geodesics with $\delta = 1 - L/L_{\rm scat} \ll 1$ (notice that in the *critical* case $L = L_{\rm scat}$, it takes an infinite amount of proper time for a particle to reach the horizon). For small δ, the particle orbits around the marginally bound circular geodesic of a Schwarzschild BH $N \simeq -(\sqrt{2}\pi)^{-1}\log\delta$ times. For near-extremal Kerr BHs, we get $N \sim -(2\pi\sqrt{2\epsilon})^{-1}\log(8\sqrt{\epsilon}\delta)$. This simple analysis suggests that for $\delta \ll 1$, the radiation should be peaked at frequencies corresponding to marginally bound quasi-circular orbits with orbital frequency $\Omega_{\rm mb}$ and that the total radiated energy $E_{\rm tot} \sim N \sim \log\delta$. In Fig. 1, we confirm these conclusions by a numerical calculation of the energy radiated by particles of mass $\mu \ll M$ plunging from rest into a Schwarzschild BH (see [3] for details and notation). The dominant (quadrupolar) mode of the radiation shows a "bump," as expected, at $\omega = 2\Omega_{\rm mb} = (4M)^{-1}$. Since $E_{\rm tot} \sim -\log\delta$ as $\delta \to 0$, radiative effects cannot be neglected in the analysis.

In conclusion, frame dragging effects in Kerr BHs can *in principle* accelerate particle to high energies, but (1) astrophysical restrictions on the spin severely limit the maximum c.m. energies in the collisions, (2) just as radiative losses constrain the performance of particle accelerators, gravitational radiation and backreaction constrain the maximum c.m. energy for collisions around Kerr BHs, and (3) the exponential sensitivity of whirling orbits to initial conditions requires significant fine-tuning to get sensible cross sections for the highest-energy collisions.

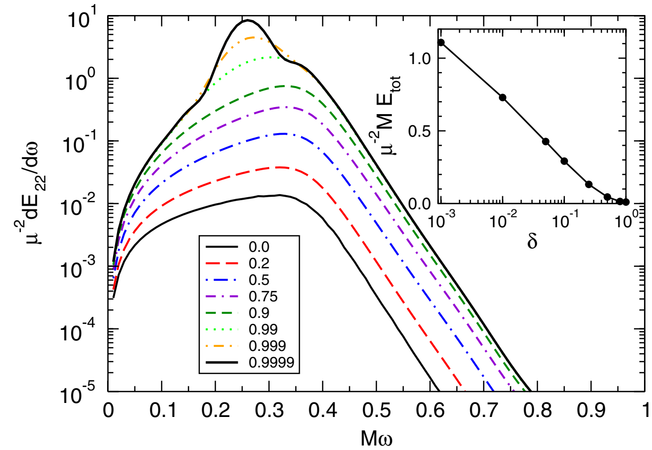

FIG. 1 (color online). Energy spectrum for $l = m = 2$ and different values of $L/L_{\rm scat}$ (as indicated in the legend). Inset: for small δ, a fit of the numerics yields $\mu^{-2}ME_{\rm tot} \sim -0.11 - 0.18\log\delta$.

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