Magnetic Field Delocalization and Flux Inversion in Fractional Vortices in Two-Component Superconductors

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We demonstrate that, in contrast with the single-component Abrikosov vortex, in two-component superconductors vortex solutions with an exponentially screened magnetic field exist only in exceptional cases: in the case of vortices carrying an integer number of flux quanta and in a special parameter limit for half-quantum vortices. For all other parameters, the vortex solutions have a delocalized magnetic field with a slowly decaying tail. Furthermore, we demonstrate a new effect which is generic in two-component systems but has no counterpart in single-component systems: on exactly half of the parameter space of the $U(1)\times U(1)$ Ginzburg-Landau model, the magnetic field of a generic fractional vortex inverts its direction at a certain distance from the vortex core.

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The two-component Ginzburg-Landau (TCGL) model, in which two independent superconducting components interact with each other via a coupling to a vector potential, appears in various physical contexts. It describes the projected quantum liquid states of metallic hydrogen and its isotopes under high pressure [1–4], where superconductivity of electrons coexists with superconductivity of protons or a Bose condensate of deuterons. Similar models describe neutron star interiors, where the two superconducting components represent possible protonic and Σ^- hyperonic Cooper pairs [5]. There are also various physical situations where the $U(1)\times U(1)$ TCGL model arises as an effective description [6]. The crucially important excitations appearing in the physics of rotational and magnetic responses, fluctuations, and phase transitions in these systems are the topological defects (vortex lines and loops). Qualitative analysis of the $U(1)\times U(1)$ symmetric TCGL model indicates that it allows vortex excitations carrying an arbitrary fraction of the standard flux quantum, where the fraction is determined by a continuous parameter, the ratio of superfluid densities [1]. There is also growing interest in various unusual integer-flux vortex solutions which can be viewed, in this model, as bound states of fractional-flux vortices [7,8]. So far, fractional-flux vortices in these theories have been discussed [1] only in the so-called London limit, a mathematical simplification wherein the condensate densities are assumed to be constant outside the vortex core, which is modeled by a sharp cutoff. It is well known that in single-component systems, the London limit gives a qualitatively accurate picture of the behavior of the fields of a vortex in the full Ginzburg-Landau model; in particular, it correctly predicts that the magnetic field varies monotonically and is screened exponentially at large distances.

In this Letter, we demonstrate that the situation in the two-component case is entirely different. We find that vortex solutions in the TCGL model are, in fact, qualitatively different from the solutions obtained in the London limit and exhibit highly unusual behavior for a system that has a Meissner effect. Namely, we find that the magnetic flux of a fractional vortex is generically not exponentially localized in space but has a long tail which decays according to a $1/r^4$ power law. The magnetic field has a tendency to get extremely delocalized for small fractions of flux quanta, where the maximum of the magnetic field at the vortex center becomes barely distinguishable. This effect can be understood using explicit asymptotic formulas we obtain for the magnetic field and condensate densities at long range in terms of the TCGL model parameters. These formulas show, moreover, that under quite generic conditions in the multicomponent superconductor, the magnetic field in a fractional-flux vortex can reverse its direction at a certain distance from the core, in stark contrast to vortex solutions in single-component superconductors.

The system of interest is the $U(1)\times U(1)$ symmetric TCGL model with free energy

$$E=\frac{1}{2}\int dxdy\{|(\partial_k+ieA_k)\psi_1|^2+|(\partial_k+ieA_k)\psi_2|^2 + \eta_1(u_1^2-|\psi_1|^2)^2+\eta_2(u_2^2-|\psi_2|^2)^2+(\epsilon_{ij}\partial_iA_j)^2\}. \tag{1}$$

Here $\psi_{1,2}$ are two complex scalar fields corresponding to two superconducting order parameters. The model (1) is realized in physical systems where the electrodynamics is local and Josephson-like coupling between condensates is forbidden. We have given the condensates equal electric charge, but the results apply equally well for a system of oppositely charged condensates [2–5] since the model (1) is invariant under inversion of the sign of the charge of a condensate accompanied by complex conjugation of that condensate. The results can be straightforwardly generalized to include other terms in the effective potential, or mixed gradient terms, so long as these are consistent with the $U(1) \times U(1)$ symmetry. Vortices in this model are solutions of the Euler-Lagrange equations

$$(\partial_k + ieA_k)^2 \psi_1 + 2\eta_1(u_1^2 - |\psi_1|^2)\psi_1 = 0, \quad (2)$$

$$(\partial_k + ieA_k)^2 \psi_2 + 2\eta_2(u_2^2 - |\psi_2|^2)\psi_2 = 0, \quad (3)$$

$$-\epsilon_{kj}\partial_j B = J_k, \quad (4)$$

where J_k is the supercurrent $J_k = \frac{i}{2}e\{\psi_1(\partial_k - ieA_k)\psi_1^* - \psi_1^*(\partial_k + ieA_k)\psi_1 + \psi_2(\partial_k - ieA_k)\psi_2^* - \psi_2^*(\partial_k + ieA_k)\psi_2\}$. In the first part of this Letter, we seek solutions of this system within the axially symmetric ansatz

$$(A_1, A_2) = \frac{a(r)}{r}(-\sin\theta, \cos\theta); \qquad \psi_i = \sigma_i(r)e^{-in_i\theta}, \quad (5)$$

which amounts to imposing $2\pi n_i$ winding on the phase of condensate ψ_i, where n_i are integers. Here $x + iy = re^{i\theta}$ and $a(r)$, $\sigma_i(r)$ are real profile functions. Note that in certain cases, the axial symmetry of vortex solutions in this model was found to be spontaneously broken [7]. However, in the cases studied below, the solutions are axially symmetric, as verified by the numerical simulations presented in the second part of the Letter. In what follows, we are looking for localized solutions in the sense that $|\mathbf{J}| \to 0$ and $\sigma_i \to u_i$ as $r \to \infty$. It follows that

$$a(r) \to a_\infty = \frac{1}{e}\Phi, \quad \text{where } \Phi = \frac{n_1u_1^2 + n_2u_2^2}{u_1^2 + u_2^2}. \quad (6)$$

By Stokes's theorem, it follows that the total magnetic flux through the xy plane is $\int Bdxdy = \frac{2\pi}{e}\Phi$, which is a fractional multiple Φ of the usual flux quantum if $n_1 \neq n_2$ [1].

Substituting (5) into (2)–(4) yields a coupled system of ordinary differential equations

$$\sigma_1'' + \frac{\sigma_1'}{r} - \frac{(n_1 - ea)^2}{r^2}\sigma_1 + 2\eta_1(u_1^2 - \sigma_1^2)\sigma_1 = 0, \quad (7)$$

$$\sigma_2'' + \frac{\sigma_2'}{r} - \frac{(n_2 - ea)^2}{r^2}\sigma_2 + 2\eta_2(u_2^2 - \sigma_2^2)\sigma_2 = 0, \quad (8)$$

$$a'' - \frac{a'}{r} - e[ae(\sigma_1^2 + \sigma_2^2) - n_1\sigma_1^2 - n_2\sigma_2^2] = 0, \quad (9)$$

subject to the boundary conditions $a \to a_\infty$, $\sigma_1 \to u_1$, and $\sigma_2 \to u_2$ as $r \to \infty$. Solutions with $n_1 = n_2$ carry integer flux and were considered in Ref. [7]. They turn out to have a much richer variety of interaction behavior than Abrikosov vortices. However, just like their single-component counterparts, the modulation of the fields $|\psi_i|$ and $|B|$ is exponentially localized in space. Here we observe that, by contrast, if $n_1 \neq n_2$, neither $n_1 - ea$ nor $n_2 - ea$ approaches zero as $r \to \infty$, and consequently it follows from (7) and (8) that neither σ_1 nor σ_2 can approach its boundary value (u_1, u_2, respectively) exponentially fast. So, in contrast to integer-flux vortices, for fractional-flux vortices the densities $|\psi_i|$ can recover their asymptotic values only according to some power law. Since the third terms in (7) and (8) decay like r^{-2}, it is consistent to assume (the assumption is verified below) that

$$\sigma_i(r) \sim u_i - \alpha_i r^{-2}, \qquad i = 1, 2 \quad (10)$$

at large r for some real coefficients α_1, α_2. Then σ_i'', σ_i'/r are $O(r^{-4})$, and demanding that the leading term (order r^{-2}) vanishes gives the prediction

$$\alpha_i = \frac{(n_i - \Phi)^2}{4\eta_i u_i}, \qquad i = 1, 2. \quad (11)$$

Note that $\alpha_i > 0$, so σ_i approaches its boundary value from below, as one expects. From (9), it is then consistent to assume (again, verified below) that

$$a(r) \sim \frac{\Phi}{e} - \beta r^{-2} \quad (12)$$

at large r for some real coefficient β. Again, a', a'/r are order r^{-4}, and demanding that the leading term in (9) vanishes leads one to predict that

$$\beta = \frac{1}{2e(u_1^2 + u_2^2)}\left\{\frac{(n_1 - \Phi)^3}{\eta_1} + \frac{(n_2 - \Phi)^3}{\eta_2}\right\}. \quad (13)$$

Now $B = r^{-1}a'(r)$, so in the case where $\Phi > 0$ (e.g., if n_1, $n_2 \geq 0$), $a(r)$ interpolates between $a(0) = 0$ and $a_\infty > 0$, so one expects $a'(r) > 0$ uniformly, and hence $B(r) > 0$. In particular, one expects $a(r)$ to approach its boundary value a_∞ from below, so that $\beta > 0$. But in this regard, formula (13) contains a surprise: it is quite possible for β to be negative. In this case, since $B(r) \sim 2\beta r^{-4}$ at large r, we see that the magnetic field has to flip its direction as one travels out from the vortex core: it is positive for small r and negative for large r. Let us introduce polar coordinates on the u_1u_2 and $\eta_1\eta_2$ parameter planes, so $u_1 + iu_2 = ue^{i\zeta}$ and $\eta_1 + i\eta_2 = \eta e^{i\phi}$ where $0 < \zeta, \phi < \frac{\pi}{2}$. Then

$$\beta = \frac{(n_1 - n_2)^3}{2eu^2\eta}\left\{\frac{\sin^6\zeta}{\cos\phi} - \frac{\cos^6\zeta}{\sin\phi}\right\}, \quad (14)$$

so $\beta < 0$ if and only if $\tan\phi < \cot^6\zeta$, which holds on precisely half of the $\zeta\phi$ square. Hence, not only can magnetic flux reversal occur for fractional-flux vortices, it is a generic effect which occurs on half the parameter space of the TCGL model (see also remark [9]).

It is interesting to consider parameter values on the curve $\tan\phi = \cot^6\zeta$, for which $\beta \equiv 0$. At generic points on this curve, $a(r) \sim a_\infty - \beta' r^{-4}$, so for that family of vortices the magnetic field B is power-law localized, but with unusual power, decaying as r^{-6}. However, we find that a very special situation happens when the vortex carries a half of the flux quantum and both condensates have the same coherence length, that is, $u_1 = u_2$, $\eta_1 = \eta_2$ (i.e., $\phi = \zeta = \frac{\pi}{4}$). This regime is relevant for physical situations where such a TCGL model is dictated by symmetry. Substituting power series Ansätze $\sigma_i(r) = \sum_{k=0}^{\infty} \alpha_{i,k} r^{-k}$, $a(r) = \sum_{k=0}^{\infty} \beta_k r^{-k}$ into (7)–(9), we see that it is consistent that $a(r) = a_\infty$ to all orders (i.e., $\beta_k = 0$ for $k \geq 1$): Eqs. (7) and (8) then imply that $\sigma_1(r) = \sigma_2(r)$ to all orders (i.e., $\alpha_{1,k} = \alpha_{2,k}$ for all k), which is consistent with (9) (whose left-hand side is then zero to all orders). One is led to conclude, therefore, that exponential localization of the magnetic field $B(r)$ is recovered for the half-quantum vortex at this symmetric parameter set despite the density fields $|\psi_i(r)|$ still being only r^{-2} localized.

To obtain more detailed knowledge of the behavior of the profile functions of fractional-flux vortices and to confirm accurately the above calculations, we must perform numerical computations. The shooting method for system (7)–(9) described in Ref. [7] turns out to be hopelessly unstable for fractional-flux vortices, so we must resort to a relaxation method. We have discretized the system using the method described in [10] and then used gradient based optimization algorithms to find highly accurate minima of the system energy for a given phase winding. It should be noted that the numerical scheme does not impose rotational symmetry, so if we obtain axially symmetric solutions (as we do), we can be confident that they are stable against all small perturbations.

In this second part of the Letter, we present the numerical results for the parameters $e = 2$, $\eta_1 = \eta_2 = 2/3$, and several values of u_i. These parameters allow us to confirm numerically the analytic calculations from the first part of the Letter. The characteristic unusual features become more pronounced with decreasing e, η_i (i.e., the vortex solution gets more delocalized and has more pronounced field inversion tail). However, our choice of parameters here is motivated by minimizing the effects of the boundary of the numerical grid. We present detailed numerical investigations of the following cases: $[n_1 = 1, n_2 = 1, u_1 = 1, u_2 = \sqrt{0.2}]$ (flux fraction $\Phi = 1$), $[n_1 = 1, n_2 = 0, u_1 = 1, u_2 = \sqrt{0.2}]$ (flux fraction $\Phi = 5/6$), and $[n_1 = 1, n_2 = 0, u_1 = \sqrt{0.2}, u_2 = 1]$ (flux fraction $\Phi = 1/6$).

Equation (10) indicates that the rate at which the density approaches its ground state value at large distances decreases as its corresponding α_i increases. This is indeed confirmed by the plots in Fig. 1. The long distance behavior of all of these agrees with (11): in the integer-flux case, the densities recover their vacuum values exponentially fast, as in the case of the Abrikosov vortex (and $\alpha_1 = \alpha_2 = 0$), while in the fractional-flux cases, the behavior is α_i/r^2. We also find that the component ψ_2 (which does not have phase winding) exhibits very unusual behavior near the origin in the second case: its density has a local maximum in the core. Observe that in our model we do not have terms in the effective potential corresponding to direct interspecies density-density interactions, and this unusual density maximum in the core is caused purely by electromagnetic interaction of the condensates. We explored this behavior for a range of different values of u_2. The results of three characteristic cases with $u_2 \in \{0.2, 0.4, 2\}$ are shown in Fig. 2, which suggests that decreasing u_2 deepens the "W"-shaped density modulation in the condensate without phase winding. The maximum of ψ_2 originates in the fact that the circulation of the supercurrent in the component ψ_2 stems from the vector potential [see Eq. (3)]. At distances $r \ll \lambda$ from the core, we have $\sigma_1 \sim r^{n_1}$, $\sigma_2 \sim r^{n_2}$, and $a \sim r^2$. The behavior of a shows that there is almost no supercurrent circulation in ψ_2 near the origin of the vortex. Consequently, $|\psi_2|$ tries to minimize the energy by recovering the ground state value of density at short r. Since

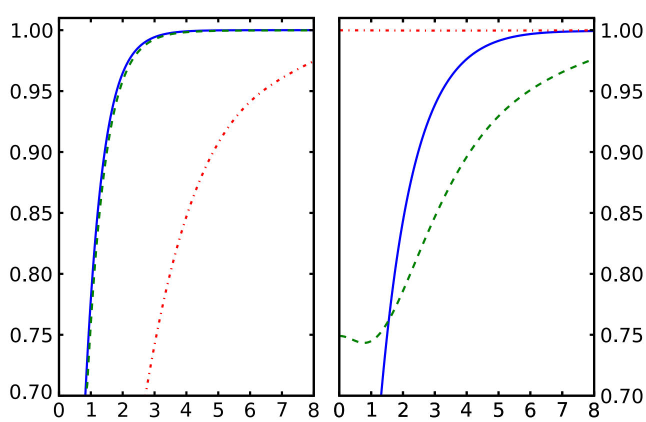

FIG. 1 (color online). Asymptotic behavior of the fields in the two-component vortex: $|\psi_1|$ (left) and $|\psi_2|$ (right) with flux fractions 1 (solid blue line), 5/6 (dashed green line), and 1/6 (dash-dotted red line). In accord with analytic calculations, in the case of 1/6 flux quantum, $|\psi_1|$ is strikingly delocalized; however, in the case of 5/6 flux quantum, the power-law tail is tiny and the difference from the integer-flux case is barely visible. The ψ_2 configuration is coreless but has a dip and local maximum at the origin. The dip is especially pronounced in the case of 5/6 flux quanta and is almost invisible in the case of 1/6 flux quanta [where $|\psi_2(0)| = 1$ and $|\psi_2(3.2)| \approx 0.9996$].

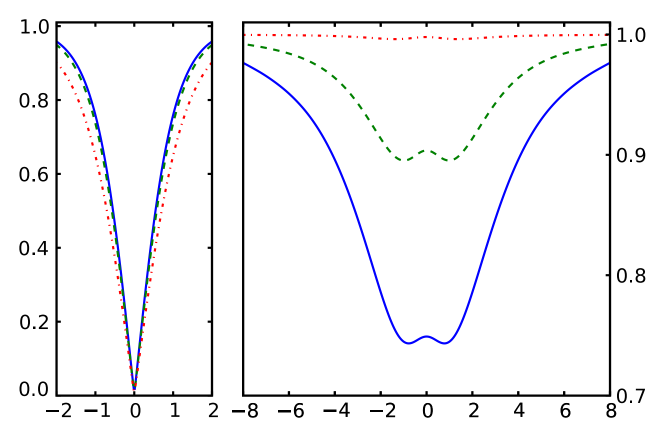

FIG. 2 (color online). The behavior near the vortex core: $|\psi_1|$ (left) and $|\psi_2|$ (right) with flux fractions 5/6 (solid blue line), 5/7 (dashed green line), and 1/3 (dash-dotted red line). The component with the phase winding $|\psi_1|$ always has a singularity. The other component always has a nonsingular W-shaped suppression of density.

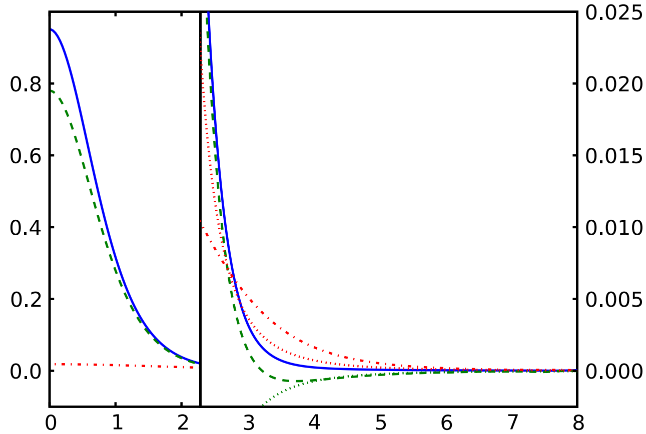

FIG. 3 (color online). The behavior of B_z near the origin of the vortex (left panel) and long-range tail (zoomed in, right panel) with flux fractions 1 (solid blue line), 5/6 (dashed green line), and 1/6 (dash-dotted red line). We see behavior strikingly different from the Abrikosov vortex: in case of 1/6-quantum vortex, the magnetic field is extremely delocalized without a pronounced maximum at the origin but already at $r \approx 3.5$ having larger value than the field of the one-quantum vortex . In the case of 5/6 quantum, the vortex accumulates magnetic flux larger than $(5/6)\Phi_0$ near the origin, almost mimicking in this region the Abrikosov vortex. However, the magnetic field rapidly goes to zero at $r = 3.275 \pm 0.0125$, after which point the magnetic field flips its direction, producing a slowly decaying power-law tail of inverse flux. The delocalized magnetic flux in the outer region subtracts from the strongly localized flux near the origin to produce net flux $(5/6)\Phi_0$. The dotted lines in the right panel depict the curves predicted by Eq. (12).

there are no singularities of superfluid velocity in the component ψ_2, the W-shaped density suppression can be arbitrarily deep; however, it can never produce a zero-density singularity in $|\psi_2|$.

Let us turn our attention to the magnetic field. From the above analytic considerations, we expect the magnetic field to approach zero exponentially if the flux fraction is an integer. Also, exponential and high algebraic power $1/r^6$ localization of magnetic field is found in some cases for half-quantum vortices. But in the general case, the magnetic field should have $1/r^4$ asymptotic behavior. Indeed, this can be seen in Fig. 3, which shows the magnetic field behavior in the same three cases whose density plots appear in Fig. 1.

Figure 3 confirms the two main generic features of vortex solutions in the TCGL model predicted in the first part of the Letter: the delocalization of magnetic flux when the fraction of the flux quantum is 1/6 and the delocalization and reversal of magnetic flux when the fraction of the flux quantum is 5/6. These features get even more pronounced for weaker potentials and larger penetration lengths.

In conclusion, we showed that, quite counterintuitively, considering the solutions of the complete two-component Ginzburg-Landau problem reveals new and unusual physics. Namely, we find that for generic fractional-flux vortex solutions (except for the special parameter set of half-quantum vortices) the magnetic field is delocalized, possessing a slowly decaying $1/r^4$ tail, and that on exactly half of the model's parameter space, the vortices exhibit magnetic flux inversion: near the origin of the vortex there is a peak in magnetic field carrying flux in the positive direction of the z axis, while at a certain distance from the core this field has a rapid reversal of direction, producing a tail of magnetic field in the negative direction along the z axis. These phenomena should have a number of physical consequences. Field delocalization and inversion can serve as an experimental signature of fractional vortices in superconductors with multiple components or in artificial superconducting structures with several magnetically coupled superconducting components. The model describes the projected quantum fluid of metallic hydrogen [2–4], a subject of renewed experimental pursuit. This magnetic field delocalization effect should affect magnetic-response-based techniques proposed to be the main tool to detect the transition to the quantum fluid of metallic hydrogen and suggested similar transitions in hydrogen-rich alloys and deuterium [4].

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[1] E. Babaev, Phys. Rev. Lett. **89**, 067001 (2002).

[2] E. Babaev and N. W. Ashcroft, Nature Phys. **3**, 530 (2007).

[3] E. Babaev, N. W. Ashcroft, and A. Sudbo, Nature (London) **431**, 666 (2004); J. Smiseth *et al.*, Phys. Rev. B **71**, 214509 (2005); E. Smorgrav *et al.*, Phys. Rev. Lett. **94**, 096401 (2005); E. Smorgrav *et al.*, Phys. Rev. Lett. **95**, 135301 (2005).

[4] E. Babaev, A. Sudbo, and N. W. Ashcroft, Phys. Rev. Lett. **95**, 105301 (2005).

[5] P. B. Jones, Mon. Not. R. Astron. Soc. **371**, 1327 (2006).

[6] See, e.g., M. A. Metlitski and S. Sachdev, Phys. Rev. B **77**, 054411 (2008); S. Sachdev, Nature Phys. **4**, 173 (2008).

[7] E. Babaev and M. Speight, Phys. Rev. B **72**, 180502(R) (2005).

[8] V. Moshchalkov *et al.*, Phys. Rev. Lett. **102**, 117001 (2009); L. F. Chibotaru, V. H. Dao, and A. Ceulemans, Europhys. Lett. **78**, 47 001 (2007); E. Babaev, Phys. Rev. Lett. **94**, 137001 (2005).

[9] From the given asymptotical analysis, it follows that these effects occur in $U(1) \times U(1)$ symmetric models when the condensates have differing phase winding and recover their ground state values at different rates. For this reason, the effects should also be present if the effective potential includes density-density interaction terms such as $|\psi_1|^2|\psi_2|^2$ because (i) these preserve $U(1) \times U(1)$ symmetry and (ii) in their presence the two condensates generically have distinct recovery rates.

[10] J. Jäykkä, J. Hietarinta, and P. Salo, Phys. Rev. B **77**, 094509 (2008).