Detecting the Majorana Fermion Surface State of ^{3}He-B through Spin Relaxation

Suk Bum Chung and Shou-Cheng Zhang

Department of Physics, McCullough Building, Stanford University, Stanford, California 94305, USA

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The Majorana fermion, which can be useful for topological quantum computation, has eluded detection. ^{3}He-B, recently shown to be a time-reversal invariant topological superfluid, has a gapless Majorana fermion surface state. We show here that an electron spin relaxation experiment can detect this surface state—its Majorana nature through the Zeeman field direction dependence of the relaxation time $1/T_1 \propto \sin^2\theta$, where θ is the angle between the field and the surface normal. We propose an experimental setup where an electron inside a nanobubble is injected below the ^{3}He liquid surface.

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The Majorana fermion has secured a central place in a wide range of theoretical physics due to recent developments [1]. The chief characteristic of the Majorana fermion is that it has only *half* the degree of freedom as the usual complex fermions. From this characteristic, we can have neutrinoless double β decay if neutrinos are Majorana fermions. In recent years, there has been great interest in condensed matter systems where Majorana fermions can arise. Systematic understanding of such systems has been obtained through investigating their topological properties, which were shown to be analogous to those of topological insulators (TIs) [2–8]. Similar to TIs, topological superconductors (SCs) or superfluids have a full pairing gap inside the bulk, but have protected gapless state at the edge or on the surface. One example is the weak-pairing phase of two-dimensional (2D) spinless chiral SCs with $p + ip$ symmetry [9–12]. This system breaks time-reversal symmetry, and can be understood as the SC analogue of the quantum Hall (QH) state. The main difference is that the chiral edge state of the chiral SC consists of Majorana fermions rather than complex fermions as in the $\nu = 1$ QH state, and thus contains only half the degrees of freedom. In addition, it was shown that a Majorana zero mode is trapped in each vortex core [11], leading to the non-Abelian statistics of vortices [13]. More recently the time-reversal invariant (TRI) SC has been proposed [14,15] and classified [16,17]. Such topological SCs or superfluid states in two and three dimensions are analogous to the TRI quantum spin Hall (QSH) or the TI state discovered recently [14–16]. So far, the only definite candidate for the 3D TRI topological SC state is the ^{3}He-B phase [14–16], the topological invariant of which was first pointed out in Refs. [12,18]. In fact, the Bogoliubov–de Gennes (BdG) Hamiltonian for the ^{3}He-B phase is identical to the simplest model Hamiltonian of the 3D TI [6,19], giving rise to a single surface state described by the Hamiltonian

$$\mathcal{H}_{\text{surf}} = v_F \boldsymbol{\sigma} \cdot (\hat{\mathbf{z}} \times \mathbf{p}), \tag{1}$$

where $\mathbf{p}$ is the in-plane momentum, $\hat{\mathbf{z}}$ is the surface normal, and $\boldsymbol{\sigma}$ is the dimensionless spin operator. Despite having the same Hamiltonian, the surface state of the ^{3}He-B phase consists of a single Majorana cone which has only half the degrees of freedom as the surface state of the TI which consists of a single Dirac cone.

There have been recent experimental efforts to detect the surface states of ^{3}He-B [20,21]. Despite results consistent with the existence of the gapless Andreev bound state at the surface, these experiments were done on a "rough" surface and did not directly detect the Majorana cone or the surface state degree of freedom being half that of the usual complex fermions. We need a probe for a free surface to detect the Majorana nature of the surface mode, i.e., an analogue of neutrinoless double β decay. There are restrictions on external perturbations which can couple to the Majorana surface state of ^{3}He-B; indeed as they are due to the halving of the degrees of freedom, these restrictions are probably the most distinctive features of the surface state. The material properties of ^{3}He-B, mainly its very low energy scale, impose further constraints on possible experimental methods. Nonetheless, we find that the Majorana nature of the surface mode gives rise to some striking and qualitatively distinct experimental signatures.

Surface state of Majorana fermion.—First, we show the basic similarity and difference between the surface modes of ^{3}He-B and the simplest 3D TRI TI [6,19]. The ^{3}He-B surface mode is derived from the BdG Hamiltonian,

$$\hat{\mathcal{H}}_{\text{BdG}} = \begin{bmatrix} \epsilon_{\mathbf{p}} - E_F & 0 & -\frac{\Delta}{p_F}\hat{p}_- & \frac{\Delta}{p_F}\hat{p}_x \\ 0 & \epsilon_{\mathbf{p}} - E_F & \frac{\Delta}{p_F}\hat{p}_x & \frac{\Delta}{p_F}\hat{p}_+ \\ -\frac{\Delta}{p_F}\hat{p}_+ & \frac{\Delta}{p_F}\hat{p}_x & -\epsilon_{\mathbf{p}} + E_F & 0 \\ \frac{\Delta}{p_F}\hat{p}_x & \frac{\Delta}{p_F}\hat{p}_- & 0 & -\epsilon_{\mathbf{p}} + E_F \end{bmatrix}, \tag{2}$$

where we have used the basis $\Psi_{\text{BdG}}(\mathbf{r}) \equiv [\hat{\psi}_{\rightarrow}(\mathbf{r}), \hat{\psi}_{\leftarrow}(\mathbf{r}), \hat{\psi}^{\dagger}_{\rightarrow}(\mathbf{r}), \hat{\psi}^{\dagger}_{\leftarrow}(\mathbf{r})]^T$ with the spin quantization axis along the x axis (up to rotation by the Leggett angle [22,23] around the surface normal $\hat{\mathbf{z}}$). $\epsilon_{\mathbf{p}} = p^2/2m$ is the free fermion Hamiltonian, E_F is the ^{3}He atom Fermi energy, and $\hat{p}_{\pm} = \hat{p}_y \pm i\hat{p}_z$. As noticed in Refs. [14,15],

$\hat{\mathcal{H}}_{\mathrm{BdG}}$ is formally identical to the simplest model of TRI TI with the surface state consisting of a single Dirac cone [6,19]. In both cases, the momentum dependence of the off-diagonal term leads to gapless modes bound to the surface [24,25]. The coupling of the spin and orbital degrees of freedom for the surface state of the 3D TRI system can be understood simply by setting $p_x = 0$ in Eq. (2) and reducing the system to a 2D TRI system, described by the QSH model of Ref. [2]. This enables us to see that when the parallel momentum is aligned along y direction, the quasiparticle spin is polarized in $\pm x$ direction and the $\rightarrow$ [$\leftarrow$]-spin surface quasiparticle will have the dispersion of $E = -(\Delta/k_F)k_y$ [$E = (\Delta/k_F)k_y$]. Because of invariance with respect to simultaneous spin and orbital rotation around $\hat{\mathbf{z}}$, this coupling of orbital and spin degrees of freedom holds for all directions in the xy plane.

Although the BdG Hamiltonian for ^{3}He-B phase is formally similar to the model Hamiltonian for the simplest TI [6,19], the fermionic operators that form the bases of the two Hamiltonians are quite different. In ^{3}He-B we have particle and hole excitations rather than conduction and a valence band as in the TI. Since the spin-triplet pairing in ^{3}He-B implies equal spin pairing, we cannot distinguish the particle and hole excitation through the spin degree of freedom, and thus the annihilation operator of the negative energy state is equivalent to the creation operator of the positive energy state.

This Majorana nature of the ^{3}He-B surface mode imposes strong restriction on its interaction with an external perturbation. To see how this restriction comes about, we need to examine the full mode expansion of fermion creation and annihilation operators near the surface. We impose the boundary conditions that the surface modes vanish at the surface $z = 0$ and decay exponentially in the ^{3}He-B liquid side (where $z < 0$) of the surface, albeit much slower than k_F. Since the wave vector parallel to surface $\mathbf{k}_\parallel$ remains a good quantum number, to satisfy these conditions the surface modes need to be proportional to $e^{i\mathbf{k}_\parallel\cdot\mathbf{r}_\parallel}\sin(k_\perp z)e^{\kappa z}$, where $k_F^2 = k_\parallel^2 + k_\perp^2$ and $\kappa > 0$. Inserting this into the BdG equation Eq. (2) gives $\kappa = \Delta/\hbar v_F$ and reduces Eq. (2) to an effective surface Hamiltonian of Eq. (1). Therefore, for our surface mode expansion we use the result from the TI but also take into account the artificial doubling mentioned above:

$$\begin{bmatrix} \hat{\psi}_\rightarrow(\mathbf{r}) \\ \hat{\psi}_\leftarrow(\mathbf{r}) \\ \hat{\psi}^\dagger_\rightarrow(\mathbf{r}) \\ \hat{\psi}^\dagger_\leftarrow(\mathbf{r}) \end{bmatrix} = \sum_{\mathbf{k}} (\hat{\gamma}_{\mathbf{k}} e^{i\mathbf{k}_\parallel\cdot\mathbf{r}_\parallel} + \hat{\gamma}^\dagger_{\mathbf{k}} e^{-i\mathbf{k}_\parallel\cdot\mathbf{r}_\parallel}) \begin{bmatrix} \cos\frac{\phi_{\mathbf{k}}+\pi/2}{2} \\ \sin\frac{\phi_{\mathbf{k}}+\pi/2}{2} \\ \cos\frac{\phi_{\mathbf{k}}+\pi/2}{2} \\ \sin\frac{\phi_{\mathbf{k}}+\pi/2}{2} \end{bmatrix} \times u_{\mathbf{k}} e^{\Delta z/\hbar v_F}\sin(k_\perp z) + (\text{gapped modes}), \quad (3)$$

where $\phi_{\mathbf{k}} = \arctan(k_y/k_x)$ and $u_{\mathbf{k}}$ is a normalization constant of the mode $\mathbf{k}$ [26]. Note that once we ignore the gapped modes (eigenenergy greater than Δ), we obtain the Majorana condition $\hat{\psi}_\rightarrow(\mathbf{r}) = \hat{\psi}^\dagger_\rightarrow(\mathbf{r})$ and $\hat{\psi}_\leftarrow(\mathbf{r}) = \hat{\psi}^\dagger_\leftarrow(\mathbf{r})$. What this means is that the local creation and annihilation operators for a fermion with its spin polarized parallel to the surface is indistinguishable once we ignore modes with eigenenergy greater than Δ, thus reducing the degrees of freedom by half. Instead of the usual fermion anticommutation relation, these Majorana operators would form Clifford algebra, $\sum_\sigma\{\hat{\psi}_\sigma(\mathbf{r}), \hat{\psi}_{\sigma'}(\mathbf{r}')\} = 2\delta(\mathbf{r}-\mathbf{r}')$ (where $\sigma, \sigma' = \rightarrow, \leftarrow$). It follows that it is impossible to construct the spin-polarized local density $\rho_\sigma(\mathbf{r}) = \hat{\psi}^\dagger_\sigma(\mathbf{r})\hat{\psi}_\sigma(\mathbf{r})$ out of the gapless modes if the polarization axis is parallel to the surface. This means that with the gapless surface mode, we can neither construct the local density operator $\rho(\mathbf{r}) = \sum_\sigma \hat{\psi}^\dagger_\sigma(\mathbf{r})\hat{\psi}_\sigma(\mathbf{r})$ nor the components of the local spin density operator parallel to the surface, $\hat{I}_x = (\hat{\psi}^\dagger_\rightarrow\hat{\psi}_\rightarrow - \hat{\psi}^\dagger_\leftarrow\hat{\psi}_\leftarrow)/2$ and $\hat{I}_y = (\hat{\psi}^\dagger_\rightarrow\hat{\psi}_\leftarrow + \hat{\psi}^\dagger_\leftarrow\hat{\psi}_\rightarrow)/2$. However, it is possible to construct the component of spin density operator perpendicular to the surface, $\hat{I}_z(\mathbf{r}) = -i\hat{\psi}_\rightarrow(\mathbf{r})\hat{\psi}_\leftarrow(\mathbf{r})$ [10]. So in ^{3}He-B the surface state does not contribute to the local density fluctuation while its local spin density is effectively *Ising* for $T \ll \Delta$, which means that the local external perturbation can excite the surface state only if it couples to I_z; this is a direct consequence of the halving of the degrees of freedom.

Therefore, to detect the surface state and its Majorana nature, it is best to measure dynamic susceptibility arising out of these gapless modes. From the discussion above, we see that the imaginary part of the dynamic spin susceptibility of the surface state has only one nonzero component: χ^{zz}, which we can calculate from Eq. (3). Anisotropy this drastic cannot be obtained from spin-orbit coupling of the complex fermions such as we see in the TI surface state. So we conclude that the resonant spin spectroscopy is the best probe for the Majorana surface mode. The extreme anisotropy of the spin susceptibility should be revealed through striking anisotropy in the spin spectroscopy. Because of the gapless dispersion, there will be no $e^{-\Delta/T}$ suppression of this anisotropy. We now need a spin probe that best fits the material property of ^{3}He-B.

ESR—spin spectroscopy.—We find electron spin relaxation (ESR) to be the best spin spectroscopy on the ^{3}He-B surface state. Our basic idea is to introduce some extra electrons to ^{3}He-B, apply a weak DC magnetic field (which satisfies $H \ll T/\mu_B$; note $\Delta/\mu_B \approx 26.2$ G [27]), excite the electron spins through resonance, and then let these electron spins relax through interaction with the surface state. This relaxation process would probe the imaginary part of the dynamic spin susceptibility of the ^{3}He-B atoms analogous to the way the nuclear magnetic relaxation (NMR) probes the imaginary part of the dynamic spin susceptibility of electrons. Such probe should reveal the drastic anisotropy of the dynamic spin susceptibility of the surface state due to its Majorana nature. More explicitly, we start from the spin relaxation rate formula:

$$\frac{1}{T_1} = \frac{T}{\hbar}\sum_{\mathbf{q}} \int dz_e \int dz \int dz_e' \int dz' P(q, z_e)P(q, z_e') \times A_+(\mathbf{q}, z - z_e)A_-(-\mathbf{q}, z' - z_e')\frac{\mathrm{Im}\chi^{zz}(q, \omega_L; z, z')}{\omega_L}, \tag{4}$$

where $P(q, z_e)$ is the static form factor of the electron (obtained from Fourier transforming the xy coordinates of the probability density of a single electron), A_+ is the component of the interaction that flips the electron spin with respect to the direction of the Zeeman field, z (z') and z_e (z_e') are the z coordinates of the ^{3}He atoms and the electron respectively, and $\omega_L = g\mu_B/\hbar$ is the Larmor frequency of the electron. This formula would look like the standard NMR relaxation formula [28] if we drop out the z dependence, the electron form factor P, and restore the isotropy of the dynamic spin susceptibility. Equation (4) implies the dependence of $1/T_1$ on the direction of the Zeeman field, because A_+ couple I_z to the component of the electron spin perpendicular to the Zeeman field.

To illustrate this dependence on the Zeeman field direction, we consider a simple contact interaction model for the coupling between the electron and ^{3}He atom spins. If we set the magnetic field direction as $\hat{\mathbf{z}}' = \hat{\mathbf{z}}\cos\theta + \hat{\mathbf{x}}\sin\theta$, we can write down the contact interaction as $H_{\mathrm{contact}} = -A_{\mathrm{contact}}I_zS_z = -A_{\mathrm{contact}}I_z[S_{z'}\cos\theta - \frac{1}{2}(S_+ + S_-)\sin\theta]$, giving us $A_+ = A_{\mathrm{contact}}\sin\theta$. Inserting this into Eq. (4), we obtain $1/T_1 \propto \sin^2\theta$. In other words, the electron spin does *not* relax at all for perpendicular field. By contrast, the same model gives us $1/T_1$ independent of θ for the surface state of the simplest TI, $\mathbf{q}$ summation canceling out the spin susceptibility anisotropy.

Realistic calculation can still give us this drastic anisotropy of spin relaxation. In ^{3}He-B, the main channel of spin-spin coupling is the dipole-dipole interaction, mainly because an electron strongly avoids contact with ^{3}He atoms. With the dipole-dipole interaction, we do have coupling between I_z and $S_{x,y}$:

$$H_D = -\frac{\mu_0}{4\pi}\frac{r^2\boldsymbol{\mu}_e\cdot\boldsymbol{\mu}_{\mathrm{He}} - 3(\boldsymbol{\mu}_e\cdot\mathbf{r})(\boldsymbol{\mu}_{\mathrm{He}}\cdot\mathbf{r})}{r^5} = -\frac{\mu_0 g\mu_B\gamma\hbar}{4\pi(r_\parallel^2 + z^2)^{5/2}}I_z[(r_\parallel^2 - 2z^2)S_z - 3z(xS_x + yS_y)], \tag{5}$$

where γ is the gyromagnetic ratio of a ^{3}He atom and g is the Landé g factor of an electron. However, for the electron below the liquid surface, the $S_{x,y}$ terms of Eq. (5) may have little effect; because $z > 0$ for helium atoms "below" the electron and $z < 0$ for helium atoms "above" the electrons, the coupling to $S_{x,y}$ from the helium atoms above cancels out the coupling to $S_{x,y}$ from the helium atoms below. Since the spin interaction is effectively Ising (that is, $H_D \propto -I_zS_z$), we have $1/T_1 \propto \sin^2\theta$, as we argued in the previous paragraph. By multiplying $\sin\theta$ to the 2D Fourier transform on the coefficient of the I_zS_z term of Eq. (5), we obtain $A_+(\mathbf{q}, z) = -\frac{\mu_0 g\mu_B\gamma\hbar}{2}qe^{-q|z|}\sin\theta$. As the next step, we need to devise an experimental setup to relax the electron spin by the ^{3}He-B surface state.

Electron bubble.—A crucial constraint on the relaxation rate is how well the electron is localized. Whereas in the NMR, we can assume that a nucleus is a pointlike object, we cannot make the same assumption for electrons in ESR and hence the introduction of the static form factor $P(q)$ in Eq. (4). Because of the Heisenberg uncertainty principle, the more delocalized the electron is in the real space, the more rapidly $P(q)$ falls off with q. This suppresses the spin relaxation for processes that result in a large momentum change for ^{3}He atoms and hence suppresses $1/T_1$. For this reason, $1/T_1$ is very small for an electron sitting on top of the ^{3}He liquid surface. Even when electrons above the surface form a Wigner crystal, the zero-point displacement is greater than 10% of the lattice constant for the lattice constant $\lesssim 1$ μm [29]. In order to enhance the electron localization significantly, we need to place the electron under the ^{3}He liquid surface.

Once it is injected below the ^{3}He liquid, an electron settles into a well-localized metastable state below the surface. It cannot be easily ejected from the liquid due to an electrostatic energy barrier at the surface arising from the induced polarization of ^{3}He atoms. By tuning the electric field perpendicular to the surface, we can adjust the equilibrium distance $|b|$ between the electron and the liquid surface to be as close as 10 nm [30,31]. Below the liquid surface, an electron opens up a nanosized cavity and becomes trapped inside of it to avoid the energy cost due to the negative electron affinity of helium atoms. The size of this "bubble" is determined by competition between the zero-point kinetic energy of the confined electron $E_{\mathrm{ZP}} = h^2/(8mR^2)$ and the surface energy of the cavity $E_S = 4\pi R^2\alpha$, where R is the cavity radius and

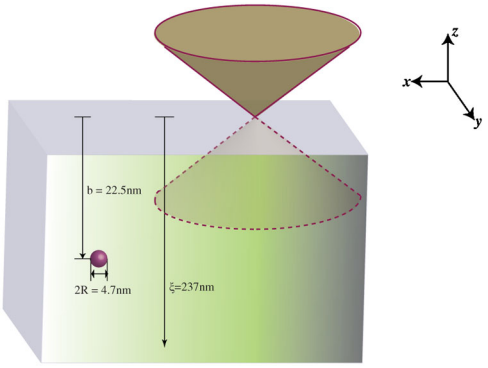

FIG. 1 (color online). Illustration of the surface state of the ^{3}He-B phase consisting of a single Majorana cone, where the $E < 0$ part of the quasiparticle spectrum (with the dashed boundary) is redundant. Also shown are the dimensions of the bubble electron when we apply a perpendicular electric field of 150 V/cm. Note how small the size and depth of the bubble are compared to the depth ξ of the surface state, for which we take the weak coupling approximation $\hbar v_F/\Delta$ as in Eq. (3).

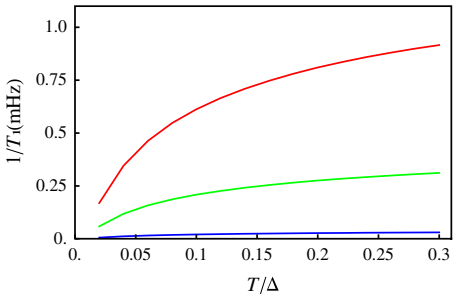

FIG. 2 (color online). The electron spin relaxation rate (in 10^{-3} Hz) due to the surface state through dipole-dipole interaction for the magnetic field applied parallel to the surface. From the top to the bottom curve, the applied perpendicular electric field is 150 V/cm, 10 V/cm, and 1.5 V/cm respectively, which gives us the bubble depth $|b| = 22.5$ nm, 87.4 nm, and 225.2 nm, respectively (for consistency with Eq. (3) the depth of the surface state is set $\xi = \hbar v_F/\Delta 237$ nm).

$\alpha = 0.156$ erg/cm^2 is the surface tension of the helium liquid [32]. This gives us the electron localization $R = [h^2/(32\pi m\alpha)]^{1/4} = 2.35$ nm.

Our ESR rate calculation shows signatures of both the Majorana nature and gapless dispersion. For electron bubbles placed at 22.5 nm, 87.4 nm, and 225.2 nm below the surface, we find that the relaxation rate is 5×10^2 times faster for the parallel field ($\theta = \pi/2$) than for the perpendicular field ($\theta = 0$), implying that we effectively have $1/T_1 \propto \sin^2\theta$ relation. As shown in Fig. 2, for the bubble depth of 22.5 nm, the relaxation rate $1/T_1$ is approximately 10^3 sec [26]. The absence of the $\exp[-\Delta/T]$ suppression in $1/T_1$ versus T behavior characteristic of the bulk quasiparticle is the consequence of the gapless dispersion on the surface. However the relaxation rate anisotropy will be reduced if we include contribution from bulk condensate, which has isotropic nonzero spin susceptibility [22,23].

In conclusion we have proposed a realistic experimental setup to observe the Majorana fermion surface states of the topological superfluid ^{3}He-B phase. Because of the Majorana nature of the surface state, the spin density operator is purely Ising-like, polarized perpendicular to the surface. Through an ESR experiment, we can show both gapless dispersion and extreme anisotropy of the dynamic spin susceptibility. Our experimental setups for the ESR measurement uses electron nanobubbles placed below the liquid helium surface, giving rise to the $1/T_1 \propto \sin^2\theta$ dependence on the magnetic field direction. Such a direct experimental observation of the Majorana fermion would enhance our fundamental understanding this exotic particle and the nature of the topological superfluid, and pave the way for topological quantum computing.

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Note added.—Recently, we learned that Nagato *et al.* also obtained the spin susceptibility anisotropy of the surface state [33].

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