Dissipative Enhancement of the Supercurrent in $Tl_2Ba_2CaCu_2O_8$ Intrinsic Josephson Junctions

P. A. Warburton,[1,*] S. Saleem,[1] J. C. Fenton,[1] M. Korsah,[2] and C. R. M. Grovenor[2]

[1] London Centre for Nanotechnology, UCL, 17-19 Gordon Street, London, WC1H 0AH, United Kingdom

[2] Department of Materials, University of Oxford, Parks Road, Oxford, OX1 3PH, United Kingdom

(Received 11 May 2009; published 17 November 2009)

We have measured dissipation-induced localization of the reaction coordinate for a metastable-state decay process in a model system with moderate damping. Specifically, the supercurrent in an array of $Tl_2Ba_2CaCu_2O_8$ intrinsic Josephson junctions is larger when all the junctions are in the zero-voltage state than when one or more junctions are in the voltage state since the dissipation is larger in the former case.

DOI: 10.1103/PhysRevLett.103.217002 PACS numbers: 85.25.Cp, 74.72.Jt

The decay of metastable states in the presence of dissipation is an important process in many physical, chemical, and electronic systems [1]. Examples of dynamical systems in which dissipation affects the decay rate include protein folding [2], transcription of DNA to RNA [3], the pricing of financial assets [4], and the harvesting of vibrational energy [5]. In the weak damping limit the dynamics are characterized by diffusion of the probability density of the action, whereas in the strong damping limit the diffusive variable is the reaction coordinate. In the crossover regime of moderate damping there is a rich variety of dynamical phenomena [1]. In this Letter we show that in arrays of moderately damped intrinsic Josephson junctions (IJJs), dissipation leads to localization of the reaction coordinate (here the phase difference). This manifests itself at the macroscopic scale as a dissipation-induced enhancement of the array supercurrent.

The Josephson junction is an ideal system for studying decay rates due to thermal activation and quantum tunneling since dissipation can be controlled independently of other parameters [6]. The relevant rates are those between (a) the metastable zero-voltage (or supercurrent) state, in which the phase difference is constant and (b) the voltage (or phase-slip) state, in which the phase continuously advances in time. The escape rate from (a) to (b) depends weakly upon dissipation. The retrapping rate from (b) to (a), however, depends exponentially upon dissipation.

Measurements of decay rates in Josephson junctions have recently been extended to series arrays of N IJJs [7]. The escape rate from the supercurrent branch (where all N junctions are in the zero-voltage state) in the thermal regime [8] is well described using the resistively shunted junction (RSJ) model, confirming that the current-phase relationship is sinusoidal. Decay rate measurements have also been made on IJJs in the macroscopic quantum tunneling (MQT) regime [9]. Jin *et al.* [10] found that the MQT rate was increased by N^2 over that predicted by the RSJ model for those IJJ arrays in which a phenomenon which they refer to as "uniform switching" could be observed. The individual junctions switch from the zero-voltage state to the voltage state in an anomalous order—specifically, the switch from the zero-voltage supercurrent branch occurs at a higher current than switches from the quasiparticle branches. This anomalous switching order (ASO) has been observed in a number of different IJJ types [11–13]. ASO contrasts with the naive expectation for independent junctions in series that the switching current, I_n, from branch n should monotonically increase with n.

To elucidate the mechanism of ASO we have measured switching-current distributions in the thermal regime not only for the supercurrent branch but also for the first quasiparticle branch of a $Tl_2Ba_2CaCu_2O_8$ (Tl-2212) IJJ array. (In previous measurements of switching from the first quasiparticle branch [14,15], the difficulty in biasing on this branch in the case where I_0 is significantly larger than I_1 prevented a detailed analysis of ASO.) A Tl-2212 film is patterned using lithography, Ar milling and focused ion-beam milling to create an IJJ array. The transverse dimensions of the IJJ array are typically submicron. The number of junctions in the array is of order 100. Details of sample preparation are described elsewhere [16]. Four-terminal current-source measurements were made in a liquid helium storage Dewar. Typical current-voltage (I-V) characteristics are shown in Fig. 1. On branch n there are n junctions in the voltage state and $N - n$ junctions in the supercurrent state. The switching current, $I_0 \sim 23\ \mu$A, from the $n = 0$ branch is larger than that from the $n = 1$ branch, $I_1 \sim 20\ \mu$A. Switching-current distributions for switching from both the $n = 0$ and the $n = 1$ branches were measured, with results qualitatively similar for a number of Tl-2212 IJJ arrays. Here we report in detail measurements on one IJJ array with area 0.24 μm^2. The current I is ramped at 1.4 ± 0.1 mA/s to some value $I_{max} > \max(I_0, I_1)$. I is then reduced either to zero (to measure switching from the $n = 0$ branch) or to a nonzero value (to measure switching from the $n = 1$ branch). This current ramp cycle is repeated 5000 times at each temperature in order to create the probability distributions for I_0 and I_1. The resolution, ΔI_{ADC}, of the analogue-to-digital converter for the current measurement is 3.7 nA. The mean and standard deviation of the distributions were found to be independent of the chosen value of I_{max}. This shows that the Ohmic self-heating in the IJJ array does not depend

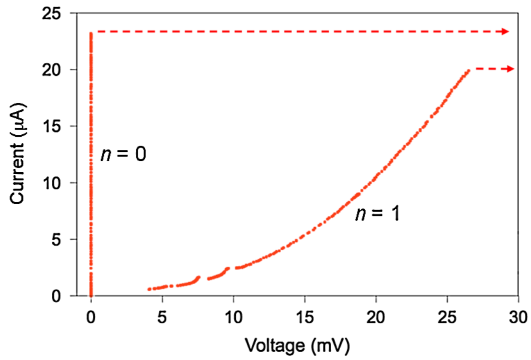

FIG. 1 (color online). Current-voltage characteristics of the Tl-2212 IJJ array at 4.2 K. Only the first two branches are shown. The dashed arrows indicate discontinuous switching.

upon the history of the current sweep; it is entirely determined by the instantaneous value of IV.

The measured distributions are shown in Fig. 2. By way of example, consider the distributions at $T_{\mathrm{bath}} = 26.1$ K. The mean value of I_0 (14.0 μA) is larger than that of I_1 (12.1 μA), and the standard deviation of I_0 (0.15 μA) is smaller than that of I_1 (0.53 μA). This suggests that the first junction ("junction A") and the second junction ("junction B") to switch into the voltage state have different dynamical behavior. Suppression of the critical current of junction B by Josephson "emission" of radiation from junction A when it is in the voltage state (i.e., on the $n = 1$ branch) is negligible at the relevant frequencies (of order 10 THz) at $I \approx I_1$ [17].

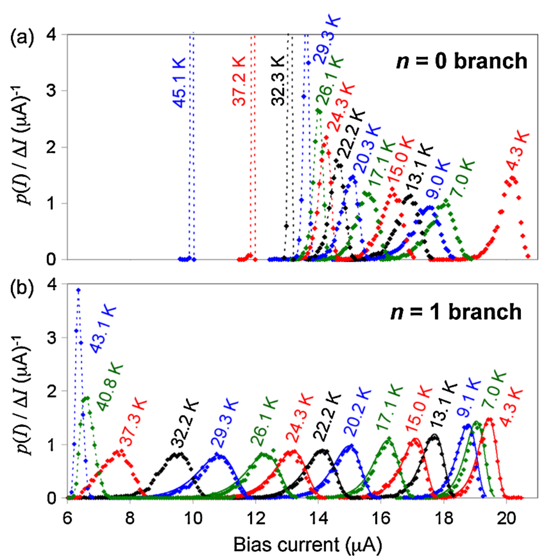

FIG. 2 (color online). Switching-current distributions for the (a) $n = 0$ and (b) $n = 1$ branches of the IJJ array. The points show the measured values at the indicated bath temperature. $p(I)$ is the probability of switching within a current range I to $I + \Delta I$, where ΔI is 74 nA ($\Delta I = 20\Delta I_{\mathrm{ADC}}$). Solid lines in (b) for 4.3 K $< T_{\mathrm{bath}} <$ 29.3 K are fits to Eq. (1) in the limit of negligible damping for a critical current of 27.0 μA, with T_{eff} as the fitting parameter. Dashed lines are guides to the eye.

The switching current of an underdamped Josephson junction in the thermal regime is determined by the unfluctuated critical current I_C and the effective temperature of the activation process T_{eff}. A number of mechanisms resulting from junction A being in the voltage state can lead to a reduction of I_C and/or an increase of T_{eff} for junction B and hence a suppression of I_1. These include Ohmic heating (which is unavoidable due to the dissipation of order 100 nW in junction A), quasiparticle injection [18], and charging effects [19]. We discount the last of these since the charging energy, $E_C = e^2/2C$ (where C is the IJJ capacitance calculated using an estimate of 6 μF cm^{-2} for the specific capacitance) is estimated to be 4 orders of magnitude lower than the Josephson energy and 2 orders of magnitude lower than the thermal energy.

To establish an upper bound on T_{eff} on the $n = 1$ branch we have fitted the data to the standard model of thermal activation as applied to a single junction [20]. Here the decay rate from the zero-voltage state is given by

$$\Gamma_{\mathrm{esc}}(I) = \frac{\omega_a(I)}{2\pi} \exp\left(\frac{-\Delta U(I)}{kT_{\mathrm{eff}}}\right), \quad (1)$$

where $\Delta U(I) \approx (4/3)\sqrt{2}E_J(1 - I/I_{cB})^{3/2}$ is the activation energy, $\omega_a(I) = \omega_{pB}[1 - (I/I_{cB})^2]^{1/4}$ is the attempt frequency (taken here to be dissipation independent), $E_J = \hbar I_{cB}/2e$ is the Josephson energy, and $\omega_{pB} = (2eI_{cB}/\hbar C)^{1/2}$ is the plasma frequency for junction B. Figure 3(a) shows the T_{bath} dependence of T_{eff} extracted by fitting to the data set for each bath temperature. I_{cB} is taken to be constant up to $T_{\mathrm{bath}} = 40$ K, consistent with the Ambegaokar-Baratoff model for $T < \frac{1}{2}T_c$. Hence I_{cB} is a single global fitting parameter over all temperatures in this range, here equal to 27.0 μA. For 11 K $< T_{\mathrm{bath}} <$ 29 K, the extracted T_{eff} is within 1 K of T_{bath}. For $T_{\mathrm{bath}} <$ 11 K there is an increasing amount of self-heating as T_{bath} is lowered, although (as shown in Fig. 2) the fits to the distributions are still good provided T_{eff} is used. This

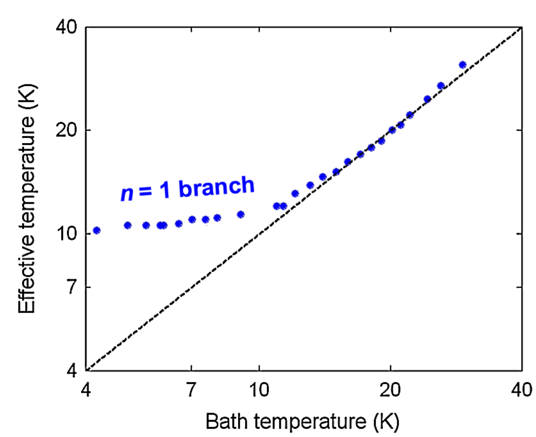

FIG. 3 (color online). A log-log plot of the bath-temperature-dependence of the effective temperature extracted from the fits to the switching-current distributions for the $n = 1$ branch shown in Fig. 2(b). The dashed line shows $T_{\mathrm{eff}} = T_{\mathrm{bath}}$.

confirms that I_{CB} is independent of temperature in the range 4 K $< T_{bath} <$ 29 K. This observation is inconsistent with a model of ASO based on critical current suppression by quasiparticle injection [18] since, over this temperature range, the mean switching current of junction B (and hence the typical quasiparticle injection current) decreases by a factor of ~2 [see Fig. 4(a)]. It also shows that switching on the $n = 1$ branch is unaffected by the presence of Josephson fluxons [12,21] which would have the effect of significantly broadening the distributions.

T_{eff} is an upper bound on the actual sample temperature since the former may be increased by extraneous noise sources. We can therefore say that the self-heating on the $n = 1$ branch is certainly less than 1 K in the range 11 K $< T_{bath} <$ 29 K. Since the voltage on the $n = 0$ branch is close to zero [22] we can also conclude that, in this range of T_{bath}, self-heating on *both* branches is less than 1 K. The switching distributions are therefore negligibly affected by self-heating in the range 11 K $< T_{bath} <$ 29 K. In Fig. 4 we plot the T_{bath} dependence of the mean and the standard deviation σ of the distributions of I_0 and I_1. For $T_{bath} >$ 20 K the mean value of I_0 exceeds that of I_1—i.e. the switching order is anomalous in a temperature range in which self-heating is negligible (20 K $< T_{bath} <$ 29 K). In this temperature range the standard deviation of I_0 is also significantly lower than that of I_1.

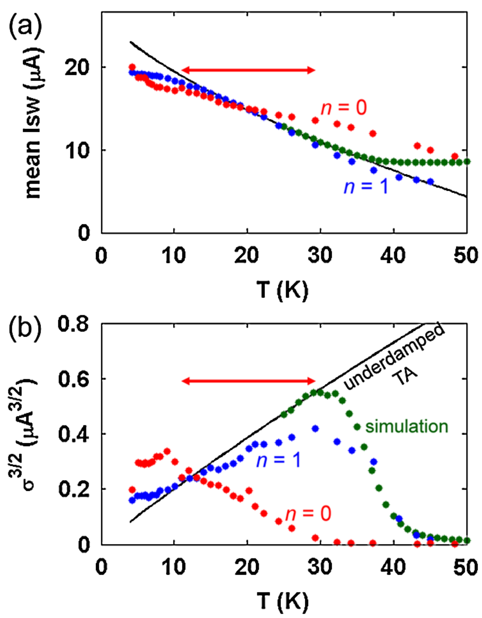

FIG. 4 (color online). The bath-temperature-dependence of the (a) mean and (b) standard deviation σ raised to the power of 3/2, of the switching-current distributions shown in Fig. 2. The red and blue points are the experimental data for $n = 0$ and $n = 1$, respectively. The green points are the results of simulations of repeated escape and retrapping [24] with fitting parameters $I_{cB} = 27.0\ \mu$A [from the fits to the distributions for $T_{bath} <$ 29.3 K as shown in Fig. 2(b)] and $Q_B = 8.6$, both taken to be independent of temperature. The black line shows the result of Eq. (1) with $I_{cB} = 27.0\ \mu$A and $T_{eff} = T_{bath}$. The temperature range where self-heating is certainly less than 1 K on both branches is shown by the arrow.

We now consider the possibility that ASO is caused by different dissipation on the two branches. Note that, for both $n = 0$ and $n = 1$, σ has a maximum at some bath temperature T_n^*, with $T_0^* = 9$ K and $T_1^* = 29$ K. For $T_{bath} > T_n^*$ it is not possible to achieve a good fit to Eq. (1) with any $T_{eff} \geq T_{bath}$. This behavior has been observed in a number of junction types including IJJs [23]. It results from rapid retrapping of the junction into the zero-voltage state after thermally activated escape. We have carried out simulations of this repeated escape and retrapping process [24]. To fit to the data for the $n = 1$ branch we use the temperature-independent value of I_{cB} extracted from fits to the data for $T < T_1^*$ shown in Fig. 2. The McCumber parameter, $\beta_{cB} = Q_B^2$, is likely to be temperature-dependent due to the dissipative effect of thermally excited quasiparticles. In the absence of any prior knowledge of the form of its temperature-dependence, however, we take it to be independent of temperature, the value $Q_B = 8.6$ being selected so as to give a maximum in the distribution width at the same temperature as the experiment. In spite of this simplification we obtain a reasonable qualitative fit to our data for the $n = 1$ branch.

To summarize our results for the $n = 1$ branch, we find $I_{cB} = 27.0\ \mu$A and $Q_B = 8.6$. From this we obtain order-of-magnitude estimates of the plasma frequency $f_{pB} =$ 400 GHz and the real part of the impedance seen by junction B, $\mathrm{Re}(Z_B(f_{pB})) = Q_B/\omega_{pB}C = 300\ \Omega$. These values are of the same order of magnitude as, respectively, the c-axis plasma frequency measured by infrared spectroscopy [25] and the slope resistance of the $n = 1$ branch in the bias range where switching occurs. It is not possible to be similarly quantitative about the critical current of junction A since, for $T_{bath} < T_0^*$, the switching distributions for I_0 are much broader than predicted by Eq. (1) due to the presence of Josephson fluxons [12,21]. Under the assumption that the IJJs are structurally identical, we therefore take $I_{cA} = I_{cB} = 27.0\ \mu$A. We have previously shown [24] that the temperature T^* at which σ is maximum occurs when the escape rate, $\Gamma_{esc}(T^*, I)$ is equal to both the retrapping rate $\Gamma_r(T^*, I)$ and the normalized current sweep-rate $I^{-1}\ dI/dt$ at some value of current I. Numerically solving this for $I_{cA} = 27.0\ \mu$A and $T_0^* = 9$ K we obtain $Q_A = 3.0$. The real part of the impedance seen by junction A, $\mathrm{Re}(Z_A(f_{pA}))$, is thus of order 100 Ω.

We conclude that, when increasing the current, the first IJJ to switch (junction A) does so at a higher current than the second IJJ to switch (junction B) because the value of the McCumber parameter when all IJJs are in the zero-voltage state ($\beta_{cA} = Q_A^2 = 9.0$) is lower than when one IJJ is in the voltage state ($\beta_{cB} = Q_B^2 = 74$). Dissipation enhances the supercurrent, as has previously been observed in low-T_C junctions [26]. The states of junctions A and B as current is increased in the range $I_1 < I < I_0$ are different, entirely because of the different dissipation. On the $n = 0$ branch the viscosity is sufficiently high that escape from the zero-voltage to the phase-slip state in junction A is rapidly followed by retrapping to the zero-voltage state. On the $n = 1$ branch, by contrast, the viscosity of junction B is low enough [27] that retrapping is unlikely and the steady phase-slip state develops. At $I = I_0$ on the $n = 0$ branch the energy provided by the bias current is sufficient to overcome the viscous retrapping and junction A enters the steady phase-slip state. Junction B initially remains in the zero-voltage state. The escape rate to the voltage state for junction B at $I = I_0$ can be estimated from Eq. (1) to be of order 10^6 to 10^7 s^{-1} in the T_{bath} range where ASO occurs. It is therefore not possible to bias stably on the $n = 1$ branch at $I = I_0$, nor presumably on the $n \geq 2$ branches if the critical current is equal for all N IJJs. This gives the impression that all N IJJs switch simultaneously (as implied by the "uniform switching" description used in [10]), whereas they actually switch independently on a time scale too rapid to be resolved by the electronics.

It remains an open question as to why dissipation is lower on the first quasiparticle branch than on the supercurrent branch. On the supercurrent branch $\mathrm{Re}(Z_A(f_{pA}))$ is of order 100 Ω, consistent with the typical impedance of the bias lines attached to the array [28]. This suggests that when all junctions are in the zero-voltage state the Q is determined not by the junction array itself but by its environment, as has been found for high resistance low-T_C junctions [29]. When junction A enters the running state its impedance at the plasma frequency changes, resulting in a change in the dissipative environment seen by junction B. Alternatively nonequilibrium quasiparticle injection effects, while not sufficiently strong to suppress the critical current significantly, may cause a significant change in Q on the $n = 1$ branch [30]. An experimental study of the temperature dependences of Q_A and Q_B would shed light on this.

In conclusion, we have shown that dissipation can enhance the supercurrent in an IJJ array in the thermal regime above that which would be expected for an underdamped junction of equal critical current. This results in the anomalous switching order observed in IJJ experiments. At currents higher than the switching current on the supercurrent branch the IJJs switch *independently* on a time scale of order 1 μs or less. This independent switching appears not to be consistent with collective-switching models [31,32] proposed for the enhancement of the MQT escape rate by a factor N^2 as measured by Jin *et al.* [10]. We emphasize that our measurements were made in the thermal regime. Our result suggests that, provided that the Ohmic self-heating on the $n \geq 1$ branches can be minimized, further experiments to study ASO in the MQT regime would be important for identifying the physics of the enhanced MQT rate. Such work should have general application in elucidating the role of dissipation on macroscopic quantum coherence.

This work is supported by EPSRC under Grants No. EP/D029783/1, No. EP/G061939/1, and No. EP/F048009/1. The authors acknowledge discussions with August Yurgens and Xiaoyue Jin.

*p.warburton@ee.ucl.ac.uk

[1] P. Hanggi *et al.*, Rev. Mod. Phys. **62**, 251 (1990).
[2] X. H. Zhang *et al.*, Science **324**, 1330 (2009).
[3] M. Depken *et al.*, Biophys. J. **96**, 2189 (2009).
[4] V. I. Yukalov *et al.*, J. Econ. Behav. Organ. **70**, 206 (2009).
[5] F. Cottone *et al.*, Phys. Rev. Lett. **102**, 080601 (2009).
[6] E. Turlot *et al.*, Phys. Rev. Lett. **62**, 1788 (1989).
[7] R. Kleiner *et al.*, Phys. Rev. Lett. **68**, 2394 (1992).
[8] V. M. Krasnov *et al.*, Phys. Rev. B **72**, 012512 (2005).
[9] K. Inomata *et al.*, Phys. Rev. Lett. **95**, 107005 (2005).
[10] X. Y. Jin *et al.*, Phys. Rev. Lett. **96**, 177003 (2006).
[11] R. Kleiner *et al.*, Phys. Rev. B **50**, 3942 (1994).
[12] N. Mros *et al.*, Phys. Rev. B **57**, R8135 (1998).
[13] P. A. Warburton *et al.*, Phys. Rev. B **67**, 184513 (2003).
[14] H. Kashiwaya *et al.*, J. Phys. Soc. Jpn. **77**, 104708 (2008).
[15] K. Ota *et al.*, Phys. Rev. B **79**, 134505 (2009).
[16] P. A. Warburton *et al.*, IEEE Trans. Appl. Supercond. **13**, 821 (2003).
[17] Kautz, J. Appl. Phys. **52**, 3528 (1981) provides an expression for the suppression of I_C in underdamped junctions. Putting $Q_B = 8.6$ we find that, even for an ac Josephson current amplitude in junction A as large as its critical current I_{CA}, the suppression of I_{CB} is less than 1% for all voltages larger than 0.26 $I_{CB}R_B$ where R_B is the resistance of the junction B.
[18] D. A. Ryndyk, Phys. Rev. Lett. **80**, 3376 (1998).
[19] T. A. Fulton *et al.*, Phys. Rev. Lett. **63**, 1307 (1989).
[20] T. A. Fulton and L. N. Dunkleberger, Phys. Rev. B **9**, 4760 (1974).
[21] P. A. Warburton *et al.*, J. Appl. Phys. **95**, 4941 (2004).
[22] There may be a small voltage due to incoherent phase-slip events but this is less than the voltage noise (1.0 μV rms) of our measurement. The power dissipated on the $n = 0$ branch is therefore at least 4 orders of magnitude lower than that on the $n = 1$ branch.
[23] V. M. Krasnov *et al.*, Phys. Rev. Lett. **95**, 157002 (2005).
[24] J. C. Fenton and P. A. Warburton, Phys. Rev. B **78**, 054526 (2008).
[25] D. Dulic *et al.*, Phys. Rev. B **60**, R15 051 (1999).
[26] C. D. Tesche *et al.*, IEEE Trans. Magn. **25**, 1424 (1989).
[27] Ben-Jacob *et al.* [Phys. Rev. A **26**, 2805 (1982)] show that the retrapping rate scales approximately as $\exp(-Q^2)$.
[28] M. Tinkham, *Introduction to Superconductivity* (McGraw-Hill, New York, 1996), 2nd ed.
[29] R. L. Kautz and J. M. Martinis, Phys. Rev. B **42**, 9903 (1990).
[30] K. Utsunomiya *et al.*, in *Proceedings of the 4th International Symposium on Mesoscopic Superconductivity and Spintronics*, edited by H. Takayanagi, J. Nitta, and H. Nakano (World Scientific Publ. Co. Pte. Ltd., Atsugi, Japan, 2006), pp. 59–64.
[31] M. V. Fistul, Phys. Rev. B **75**, 014502 (2007).
[32] S. Savel'ev *et al.*, Phys. Rev. Lett. **98**, 077002 (2007).