Gently Modulating Optomechanical Systems

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We introduce a framework of optomechanical systems that are driven with a mildly amplitude-modulated light field, but that are not subject to classical feedback or squeezed input light. We find that in such a system one can achieve large degrees of squeezing of a mechanical micromirror—signifying quantum properties of optomechanical systems—without the need of any feedback and control, and within parameters reasonable in experimental settings. Entanglement dynamics is shown of states following classical quasiperiodic orbits in their first moments. We discuss the complex time dependence of the modes of a cavity-light field and a mechanical mode in phase space. Such settings give rise to certifiable quantum properties within experimental conditions feasible with present technology.

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Periodically driven quantum systems exhibit a rich behavior and display nonequilibrium properties that are absent in their static counterparts. By appropriately exploiting time-periodic driving, strongly correlated Bose-Hubbard-type models can be dynamically driven to quantum phase transitions [1], systems can be dynamically decoupled from their environments to avoid decoherence in quantum information science [2], and quite intriguing dynamics of Rydberg atoms strongly driven by microwaves [3] can arise. It has also been muted that such time-dependent settings may give rise to entanglement dynamics in oscillating molecules [4]. A framework of such periodically driven systems is provided by the theory of linear differential equations with periodic coefficients or inhomogeneities, including Floquet's theorem [5].

In this Letter, we aim at transferring such ideas to describe a new and in fact quite simple regime of optomechanical systems, of micromirrors as part of a Fabry-Perot cavity [6–9], and also to one of the settings [10–14] that are the most promising candidates in the race of exploring certifiable quantum effects involving macroscopic mechanical modes. This is an instance of a regime of driving with mildly amplitude-modulated light. We find that in this regime, high degrees of squeezing below the vacuum noise level can be reached, signifying genuine quantum dynamics. More specifically, in contrast to earlier descriptions of optomechanical systems with a periodic time dependence in some aspect of the description, we will not rely on classical feedback based on processing of measurement-outcomes—a promising idea in its own right in a continuous-measurement perspective [15,16]—or resort to driving with squeezed light. Instead, we will consider the plain setting of a time-periodic amplitude modulation of an input light. The picture developed here, based in the theory of differential equations, gives rise to a framework of describing such situations. We find that large degrees of squeezing can be reached (complementing other very recent nonperiodic approaches based on cavity-assisted squeezing using an additional squeezed light beam [17]). It is the practical appeal of this work that such quantum signatures can be reached without the necessity of any feedback, no driving with additional fields, and no squeezed light input (the scheme by far outperforms direct driving with a single squeezed light mode): in a nutshell, one has to simply gently shake the system in time with the right frequency to have the mechanical and optical modes rotate appropriately around each other, reminiscent of parametric amplification, and to so directly certify quantum features of such a system.

Time-dependent picture of system.—Before we discuss the actual time dependence of the driven system, setting the stage, we start our description with the familiar Hamiltonian of a system of a Fabry-Perot cavity of length L and finesse F being formed on one end by a moving micromirror,

$$H = \hbar\omega_c a^\dagger a + \frac{1}{2}\hbar\omega_m(p^2 + q^2) - \hbar G_0 a^\dagger a q + i\hbar \sum_{n=-\infty}^{\infty} (E_n e^{-i(\omega_0+n\Omega)t} a^\dagger - E_n^* e^{i(\omega_0+n\Omega)t} a). \quad (1)$$

Here, ω_m is the frequency of the mechanical mode with quadratures q and p satisfying the usual commutation relations of canonical coordinates, while the bosonic operators a and $a^\dagger$ are associated to the cavity mode with frequency ω_c and decay rate $\kappa = \pi c/(2FL)$. $G_0 = \sqrt{\hbar/(m\omega_m)}\omega_c/L$ is the coupling coefficient of the radiation pressure, where m is the effective mass of the mode of the mirror being used. Importantly, we allow for any periodically modulated driving at this point, which can be expressed in such a Fourier series, where $\Omega = 2\pi/\tau$ and $\tau > 0$ is the modulation period. The main frequency of the driving field is ω_0 while the modulation coefficients $\{E_n\}$ are related to the power of the associated sidebands $\{P_n\}$ by $|E_n|^2 = 2\kappa P_n/(\hbar\omega_0)$. The resulting dynamics under this Hamiltonian together with an unavoidable coupling of the mechanical mode to a thermal reservoir and cavity losses gives rise to the quantum Langevin equation in the reference frame rotating with frequency ω_0, $\dot{q} = \omega_m p$, and

$$\begin{aligned}\dot{p} &= -\omega_m q - \gamma_m p + G_0 a^\dagger a + \xi,\\ \dot{a} &= -(\kappa + i\Delta_0)a + iG_0 aq\\ &\quad + \sum_{n=-\infty}^{\infty} E_n e^{-in\Omega t} + \sqrt{2\kappa}a^{\text{in}},\end{aligned} \tag{2}$$

where $\Delta_0 = \omega_c - \omega_0$ is the cavity detuning. γ_m is here an effective damping rate related to the oscillator quality factor Q by $\gamma_m = \omega_m/Q$. The mechanical (ξ) and the optical (a^{in}) noise operators have zero mean values and are characterized by their auto correlation functions which, in the Markovian approximation, are $\langle \xi(t)\xi(t') + \xi(t')\xi(t)\rangle/2 = \gamma_m(2\bar{n}+1)\delta(t-t')$ and $\langle a^{\text{in}}(t)a^{\text{in}\dagger}(t')\rangle = \delta(t-t')$, where $\bar{n} = [\exp(\frac{\hbar\omega_m}{k_B T}) - 1]^{-1}$ is the mean thermal phonon number. Here, we have implicitly assumed that such an effective damping model holds [18], which is a reasonable assumption in a wide range of parameters including the current experimental regime.

Semiclassical phase space orbits.—Our strategy of a solution will be as follows: we will first investigate the classical phase space orbits of the first moments of quadratures. We then consider the quantum fluctuations around the asymptotic quasiperiodic orbits, by implementing the usual linearization of the Heisenberg equations of motion [11,12] (excluding the very weak driving regime). Exploiting results from the theory of linear differential equations with periodic coefficients, we can then proceed to describe the dynamics of fluctuations and find an analytical solution for the second moments.

If we average the Langevin equations (2), assuming $\langle a^\dagger a\rangle \simeq |\langle a\rangle|^2$, $\langle aq\rangle \simeq \langle a\rangle\langle q\rangle$ (true in the semiclassical driving regime we are interested in), we have a nonlinear differential equation for the first moments. Far away from instabilities and multistabilities, a power series ansatz in the coupling G_0 $\langle O\rangle(t) = \sum_{j=0}^{\infty} O_j(t)G_0^j$ is justified, where $O = a,\ p,\ q$. If we substitute this expression into the averaged Langevin equation (2), we get a set of recursive differential equation for the variables $O_j(.)$. The only two nonlinear terms in Eq. (2) are both proportional to G_0, therefore, for each j, the differential equation for the set of unknown variables $O_j(.)$ is a *linear* inhomogeneous system with constant coefficients and τ-periodic driving. Then, after an exponentially decaying initial transient (of the order of $1/\gamma_m$), the asymptotic solutions O_j will have the same periodicity of the modulation [5], justifying the Fourier expansion

$$\langle O\rangle(t) = \sum_{j=0}^{\infty}\sum_{n=-\infty}^{\infty} O_{n,j}e^{in\Omega t}G_0^j. \tag{3}$$

Substituting this in Eq. (2), we find the following recursive formulas for the time independent coefficients $O_{n,j}$, $q_{n,0} = p_{n,0} = 0$, $a_{n,0} = E_{-n}/(\kappa + i(\Delta_0 + n\Omega))$, corresponding to the zero coupling $G_0 = 0$, while for $j \geq 1$, we have

$$\begin{aligned}q_{n,j} &= \omega_m \sum_{k=0}^{j-1}\sum_{m=-\infty}^{\infty} \frac{a^*_{m,k}a_{n+m,j-k-1}}{\omega_m^2 - n\Omega^2 + i\gamma_m n\Omega},\\ p_{n,j} &= \frac{in\Omega}{\omega_m}q_{n,j}, \quad a_{n,j} = i\sum_{k=0}^{j-1}\sum_{m=-\infty}^{\infty}\frac{a_{m,k}q_{n-m,j-k-1}}{\kappa + i(\Delta_0 + n\Omega)},\end{aligned} \tag{4}$$

Within the typical parameter space, considering only the first terms in the double expansion (3), corresponding to the first sidebands, leads to a good analytical approximation of the classical periodic orbits, see Fig. 1. On physical grounds, this is expected to be a good approximation, since $G_0 \ll \omega_m$, and because high sidebands fall outside the cavity bandwidth, $n\Omega > 2\kappa$. What is more, the decay behavior of E_n related to the smoothness of the drive inherits a good approximation in terms of few sidebands.

Quantum fluctuations around the classical orbits.—We will now turn to the actual quantum dynamics taking first moments into account separately when writing any operator as $O(t) = \langle O\rangle(t) + \delta O(t)$. The frame will hence be provided by the motion of the first moments. In this reference frame, as long as $|\langle a\rangle| \gg 1$, the usual linearization approximation to (2) can be implemented. In what follows, we will also use the quadratures $\delta x = (\delta a + \delta a^\dagger)/\sqrt{2}$ and $\delta y = -i(\delta a - \delta a^\dagger)/\sqrt{2}$, and the analogous input noise quadratures x^{in} and y^{in}. For the vector of all quadratures we will write $u = (\delta q, \delta p, \delta x, \delta y)^T$, with $n = (0, \xi, \sqrt{2\kappa}x^{\text{in}}, \sqrt{2\kappa}y^{\text{in}})^T$ being the noise vector [11,18]. Then the time-dependent inhomogeneous equations of motion arise as $\dot{u}(t) = A(t)u(t) + n(t)$, with

$$A(t) = \begin{bmatrix} 0 & \omega_m & 0 & 0\\ -\omega_m & -\gamma_m & G_x(t) & G_y(t)\\ -G_y(t) & 0 & -\kappa & \Delta(t)\\ G_x(t) & 0 & -\Delta(t) & -\kappa\end{bmatrix}, \tag{5}$$

where the real $A(t)$ contains the time-modulated coupling constants and the detuning as $G(t) = G_x(t) + iG_y(t)$,

$$G(t) = \sqrt{2}\langle a(t)\rangle G_0, \qquad \Delta(t) = \Delta_0 - G_0\langle q(t)\rangle. \tag{6}$$

From now on we will consider quasiperiodic orbits only, so the long-time dynamics following the initial one, when the first moments follow a motion that is τ periodic. Then, A is

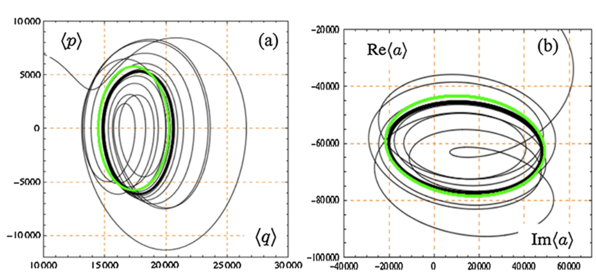

FIG. 1 (color online). Phase space trajectories of the first moments of the mirror (a) and light (b) modes. Numerical simulations for $t \in [0, 50\tau]$ (black) and analytical approximations of the asymptotic orbits (green or light gray).

τ periodic, and hence $A(t) = A(t+\tau) = \sum_{n=-\infty}^{\infty} A_n e^{i\Omega n t}$. In turn, if all eigenvalues of $A(.)$ having negative real parts for all $t \in [0, \tau]$ is a sufficient condition for stability. From the Markovian assumption, we have $\langle n_i(t)n_j(t') + n_j(t')n_i(t)\rangle/2 = \delta(t-t')D_{i,j}$, where $D = \mathrm{diag}(0, \gamma_m(2\bar{n}+1), \kappa, \kappa)$. The formal solution of Eq. (5) is [5]

$$u(t) = U(t, t_0)u(t_0) + \int_{t_0}^{t} U(t, s)n(s)ds, \tag{7}$$

where $U(t, t_0)$ is the principal matrix solution of the homogeneous system satisfying $\dot{U}(t, t_0) = A(t)U(t, t_0)$ and $U(t_0, t_0) = \mathbb{1}$. From Eqs. (5) and (7), we have as an equation of motion of the covariance matrix (CM)

$$\dot{V}(t) = A(t)V(t) + V(t)A^T(t) + D. \tag{8}$$

Here, the CM $V(.)$ is the 4×4 matrix with components $V_{i,j} = \langle u_i u_j + u_j u_i\rangle/2$, collecting the second moments of the quadratures. This is again an inhomogeneous differential equation for the second moments which can readily be solved using quadrature methods, providing numerical solutions that will be used to test and justify analytical approximate results in important regimes. Moreover, now the coefficients and not the inhomogeneity are τ periodic, $A(t) = A(t+\tau)$. Again, we can invoke results from the theory of linear differential equations to Eq. (8) [5]: we find that in the long time limit, the CM is periodic and can be written as $V(t) = \sum_n V_n e^{in\Omega t}$. An analytical solution for $V(.)$, is most convenient in the Fourier domain, $\tilde{f}(\omega) = \int_{-\infty}^{+\infty} e^{-i\omega t} f(t)dt$, giving rise to

$$-i\omega\tilde{u}(\omega) + \sum_{n=-\infty}^{\infty} A_n \tilde{u}(\omega - n\Omega) = -\tilde{n}(\omega). \tag{9}$$

If $A_{n\neq 0} = 0$, corresponding to no modulation, we are in the usual regime where the spectra are centered around $\pm\omega_m$ for the mechanical oscillator and around $\pm\Delta$ for the optical mode. The modulation introduces sidebands shifted by $\pm n\Omega$. If the modulation is weak, only the first two sidebands at $\pm\Omega$ significantly contribute. For strong modulation also further sidebands play a role: Disregarding higher sidebands means truncating the summation to $\pm N$ [valid if $u(\omega \pm N\Omega) \simeq 0$]. Then Eq. (9) can be written as $\bar{A}(\omega)\bar{u}(\omega) = \bar{n}(\omega)$, where $\bar{u}^T(\omega) = (\tilde{u}^T(\omega - N\Omega), \ldots, \tilde{u}^T(\omega), \ldots, \tilde{u}^T(\omega + N\Omega))$ and $\bar{n}^T(\omega) = (\tilde{n}^T(\omega - N\Omega), \ldots, \tilde{n}^T(\omega), \ldots, \tilde{n}^T(\omega + N\Omega))$ are $4 \times (2N+1)$ vectors, while, in terms of 4×4 blocks,

$$\bar{A}(\omega) = \begin{bmatrix} B_{-N} & A_{-1} & A_{-2} & \cdots & A_{-2N} \\ A_1 & B_{-(N-1)} & A_{-1} & & \vdots \\ A_2 & A_1 & B_{-(N-2)} & & \\ \vdots & & & & \\ \vdots & & & & \vdots \\ A_{2N} & \cdots & & A_1 & B_N \end{bmatrix} \tag{10}$$

with $B_k = A_0 - i(\omega + k\Omega)$.

We have that $\phi_{i,j}(\omega, \omega') = \langle\bar{n}_i(\omega)\bar{n}_j^*(\omega') + \bar{n}_j^*(\omega')\bar{n}_i(\omega)\rangle/2 = \sum_{n=-2N}^{2N} \delta(\omega - \omega' - n\Omega)D_n$, where $D_0 = \mathrm{diag}(D, D, \ldots, D)$, then D_1 is the matrix that has D on all first right off diagonal blocks, D_2 on the second off diagonals, with D_n analogously defined, and $D_{-n} = D_n^T$. We now define the two frequency correlation function as $\bar{V}_{i,j}(\omega, \omega') = \langle\bar{u}_i(\omega)\bar{u}_j^*(\omega') + \bar{u}_j^*(\omega')\bar{u}_i(\omega)\rangle/2$. We have $\bar{V}(\omega, \omega') = \bar{A}^{-1}(\omega)\phi(\omega, \omega')[\bar{A}^{-1}(\omega')]^\dagger$. We are interested only on the central 4×4 block of $\bar{V}$ which we call $\tilde{V}(\omega, \omega') = [\bar{V}(\omega, \omega')]_4$. We find $\tilde{V}(\omega, \omega') = \sum_{n=-2N}^{2N} \tilde{V}_n(\omega)\delta(\omega - \omega' - n\Omega)$, where $\tilde{V}_n(\omega) = [\bar{A}^{-1}(\omega)D_n[\bar{A}^{-1}(\omega - n\Omega)]^\dagger]_4$. This means that the driving modulation correlates different frequencies, but only if they are separated by a multiple of the modulation frequency Ω. By inverse Fourier transforms we recover the time-periodic expression for the CM, where the components V_n are given by the integral of their noise spectra, i.e.,

$$V_n = \frac{1}{2\pi}\int_{-\infty}^{+\infty} \tilde{V}_n(\omega)d\omega. \tag{11}$$

Squeezing and entanglement modulation.—We will now see that the mild amplitude-modulated driving in the cooling regime is exactly the tool that we need in order to arrive at strong degrees of squeezing, in the absence of feedback or squeezed light. We will apply the previous general theory to setting of an optomechanical system that is experimentally feasible with present technology. In fact, all values that we assume have been achieved already and reported on in publications with the exception of assuming a relatively good mechanical Q factor. The reasonable set of experimental parameters [9] that we assume is $L = 25$ mm, $F = 1.4\times 10^4$, $\omega_m = 2\pi$ MHz, $Q = 10^6$, $m = 150$ ng, $T = 0.1$ K. We then consider the—in the meantime well known—self-cooling regime [7] in which a cavity eigenmode is driven with a red detuned laser $\Delta_0 \simeq \omega_m$ (with wavelength $\lambda = 1064$ nm), but we also add a small sinusoidal modulation to the input amplitude with a frequency $\Omega = 2\omega_m$, so twice the mechanical frequency. To be more precise we choose the power of the carrier component equal to $P_0 = 10$ mW, and the power of the two modulation sidebands equal to $P_{\pm 1} = 2$ mW.

We approximate the asymptotic classical mean values in Eq. (3) by truncating the series only to the first terms with indexes $j = 0, \ldots, 3$ and $n = -1, 0, 1$. Figure 1 shows that, after less than 50 modulation periods, the first moments reach quasiperiodic orbits which are well approximated by our analytical results.

In order to calculate the variances of the quantum fluctuations around the classical orbits, we truncate the sum in Eq. (9) to $N = 2$ and we apply all the previous theory to find the covariance matrix V. In Fig. 2 we compare two regimes: with or without ($P_{\pm 1} = 0$) modulation (computed analytically and numerically). We see that the modulation of the driving field causes the emergence of significant true quantum squeezing below the Heisenberg limit of the mechanical oscillator state and also the interesting phenomenon of light-mirror entanglement oscillations. This dynamics reminds of the effect of parametric amplification [13,16], as if the spring constant of the mechanical motion

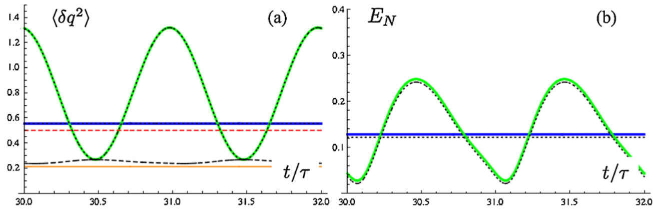

FIG. 2 (color online). (a) Variance of the mirror position and (b) light-mirror entanglement E_N as functions of time. In both (a) and (b) the nonmodulated driving regime (blue or dark gray), the modulated driving regime (green or gray), and the numerical solutions (black dash-dotted) are plotted. (a) also shows the standard quantum limit (red or gray dashed) at $1/2$, the minimum eigenvalue of the mirror covariance matrix (black dashed) and its analytical estimation (12) in the RWA (orange or light gray).

was varied in time with just twice the frequency of the mechanical motion, leading to the squeezing of the mechanical mode. For related ideas of reservoir engineering, making use of bichromatic microwave coupling to a charge qubit of nanomechanical oscillators, see Refs. [20]. Here, it is a more complicated joint dynamics of the cavity field and the mechanical mode—where the dynamics of the first and the second moments can be separated—which for large times yet yields a similar effect. Indeed, this squeezing can directly be measured when considering the output power spectrum, following Ref. [21], and no additional laser light is needed for the readout, giving hence rise to a relatively simple certification of the squeezing. Entanglement here refers to genuine quantum correlations between the mirror and the field mode, as being quantified by the logarithmic negativity defined as $E_N(\rho) = \log\|\rho^\Gamma\|_1$, essentially the trace norm of the partial transpose [22,23]. The minimum eigenvalue of the mirror covariance matrix—the logarithm thereof typically referred to as single mode squeezing parameter—is almost constant and this means that the state is always squeezed but that the squeezing direction continuously rotates in phase space with the same period of the modulation. Calling this rotating squeezed quadrature δx_R, a rough estimate of its variance can be calculated in the rotating-wave approximation (RWA, compare, e.g., Ref. [24]),

$$\langle \delta x_R^2 \rangle = \frac{1}{2} + \bar{n} - \frac{2\kappa(G_0 - G_{-1})(G_0\bar{n} + G_{-1}(\bar{n}+1))}{(\gamma_m + 2\kappa)(G_0^2 - G_{-1}^2 + 2\gamma_m\kappa)}, \quad (12)$$

with $\{G_n\}$ being defined as $G(t) = \sum_{n=-\infty}^{\infty} G_n e^{in\Omega t}$.

Conclusions and outlook.—In this Letter we have introduced a framework of describing periodically amplitude-modulated optomechanical systems. Interestingly, such a surprisingly simple setting feasible with present technology [9] leads to a setting showing high degrees of mechanical squeezing, with no feedback or additional fields needed. We hope that such ideas contribute to experimental studies finally certifying first quantum mechanical effects in macroscopic mechanical systems, constituting quite an intriguing perspective.

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