

## Quark Structure of the Nucleon and Angular Asymmetry of Proton-Neutron Hard Elastic Scattering

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We investigate an asymmetry in the angular distribution of hard elastic proton-neutron scattering with respect to the  $90^\circ$  center of mass scattering angle and demonstrate that its magnitude is related to the helicity-isospin symmetry of the quark wave function of the nucleon. Our estimate of the asymmetry within the quark-interchange model of hard scattering demonstrates that the quark wave function of a nucleon based on the exact SU(6) symmetry predicts an angular asymmetry opposite to that of experimental observations. We found that the quark wave function based on the diquark picture of the nucleon produces a correct asymmetry. Comparison with the data allowed us to show that the vector diquarks contribute around 10% in the nucleon wave function and they are in negative phase relative to the scalar diquarks. These observations are essential in constraining QCD models of a nucleon.

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For several decades elastic nucleon-nucleon scattering at high momentum transfer ( $-t, -u \geq M_N^2 \text{ GeV}^2$ ) has been one of the important testing grounds for QCD dynamics of the strong interaction between hadrons. Two major observables considered were the energy dependence of the elastic cross section and the polarization properties of the reaction.

Predictions for energy dependence are based on the underlying dynamics of the hard scattering of quark components of the nucleons. One such prediction is based on the quark-counting rule [1,2] according to which the differential cross section of two-body elastic scattering ( $ab \rightarrow cd$ ) at high momentum transfer behaves like  $\frac{d\sigma}{dt} \sim s^{-(n_a+n_b+n_c+n_d)}$ , where  $n_i$  represents the number of constituents in particle  $i$  ( $i = a, b, c, d$ ).

For elastic  $NN$  scattering, the quark-counting rule predicts  $s_{NN}^{-10}$  scaling which agrees reasonably well with experimental measurements (see, e.g., Refs. [3–6]). In addition to energy dependence, the comparison [7] of the cross sections of hard exclusive scattering of hadrons containing quarks with the same flavor with the scattering of hadrons that share no common flavor of quarks demonstrated that the quark interchange [8,9] represents the dominant mechanism of hard elastic scattering for up to intersecting storage rings (ISR) energies (see discussion in [10]).

For polarization observables, the major prediction of the QCD dynamics of hard elastic scattering is the conservation of helicities of interacting hadrons. The latter prediction is based on the fact that the gluon exchange in the massless quark limit conserves the helicity of interacting quarks.

The quark-counting rule and helicity conservation, however, do not describe completely the features of hard scattering data. The energy dependence of  $pp$  elastic cross

section scaled by  $s_{NN}^{10}$  exhibits an oscillatory behavior which indicates the existence of other possibly nonperturbative mechanisms for the scattering [11,12]. These expectations are reinforced also by the observed large asymmetry,  $A_{nn}$  at some hard scattering kinematics [13] which indicates an anomalously large contribution from double helicity flip processes. These observed discrepancies, however, do not represent the dominant features of the data and overall one can conclude that the bulk of the hard elastic  $NN$  scattering amplitude is defined by the exchange mechanism of valence quarks that interact through the hard gluon exchange (see, e.g., Refs. [8–10,14]). Quark-interchange mechanism also reasonably well describes the  $90^\circ$  c.m. hard breakup of two nucleons from the deuteron [15,16].

However, the energy dependence of a hard scattering cross section, except for the verification of the dominance of the minimal-Fock component of the quark wave function of the nucleon, provides rather limited information about the symmetry properties of the valence quark component of the nucleon wave function.

In this Letter we demonstrate that an observable such as the asymmetry of a hard elastic proton-neutron scattering with respect to  $90^\circ$  c.m. scattering may provide a new insight into the helicity-flavor symmetry of the quark wave function of the nucleon. Namely we consider

$$A_{90^\circ}(\theta) = \frac{\sigma(\theta) - \sigma(\pi - \theta)}{\sigma(\theta) + \sigma(\pi - \theta)}, \quad (1)$$

where  $\sigma(\theta)$ —is the differential cross section of the elastic  $pn$  scattering. We will discuss this asymmetry in the hard kinematic regime in which the energy dependence of the cross section is  $\sim s^{-10}$ . Our working assumption is the dominance of the quark-interchange mechanism (QIM) in the  $NN$  elastic scattering at these kinematics.

Within the QIM the characteristic scattering diagram can be represented as in Fig. 1. Here one assumes a factorization of the amplitude into the soft and hard parts. The soft part contains the initial and final state wave functions of the nucleons, while the hard part is characterized by a QIM scattering with five hard gluon exchanges generating an energy dependence in accordance with the quark-counting rule. In order to attempt to calculate the absolute cross section of the reaction one needs to sum hundreds of diagrams similar to the one of Fig. 1. However for the purpose of estimating the asymmetry in Eq. (1) the important observation is that the hard scattering kernel is flavor blind and conserves the helicity. As a result one expects that angular asymmetry will be generated mainly through the underlying spin-flavor symmetry of the quark wave functions of the interacting nucleons.

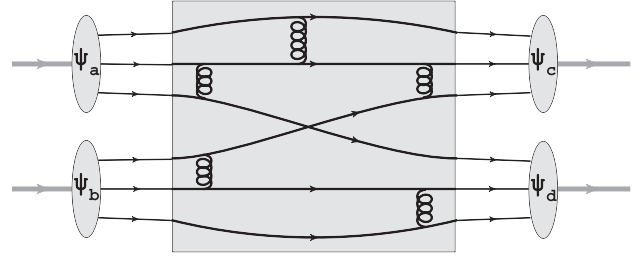


FIG. 1. Typical diagram for quark-interchange mechanism of  $NN \rightarrow NN$  scattering.

The amplitude of the hard elastic  $a + b \rightarrow c + d$  scattering of Fig. 1, within quark-interchange approximation, can be presented as follows:

$$\langle cd|T|ab\rangle = \sum_{\alpha,\beta,\gamma} \langle \psi_c^\dagger | \alpha'_2, \beta'_1, \gamma'_1 \rangle \langle \psi_d^\dagger | \alpha'_1, \beta'_2, \gamma'_2 \rangle \langle \alpha'_2, \beta'_2, \gamma'_2, \alpha'_1 \beta'_1 \gamma'_1 | H | \alpha_1, \beta_1, \gamma_1, \alpha_2 \beta_2 \gamma_2 \rangle \langle \alpha_1, \beta_1, \gamma_1 | \psi_a \rangle \langle \alpha_2, \beta_2, \gamma_2 | \psi_b \rangle, \quad (2)$$

where  $(\alpha_i, \alpha'_i)$ ,  $(\beta_i, \beta'_i)$  and  $(\gamma_i, \gamma'_i)$  describe the spin-flavor quark states before and after the hard scattering,  $H$ , and

$$C_{\alpha,\beta,\gamma}^j \equiv \langle \alpha, \beta, \gamma | \psi_j \rangle \quad (3)$$

describes the probability amplitude of finding the  $\alpha, \beta, \gamma$  helicity-flavor combination of three valence quarks in the nucleon  $j$  [14]. Note that in Eq. (2) the factorization of nucleon wave functions from the hard scattering term is justified by the energies, characteristic of the  $C_{\alpha,\beta,\gamma}^j$  fac-

tors, being of the order of the nucleon mass, while the  $H$  kernel is characterized by the transferred momenta  $-t$ ,  $u \gg m_N^2$ .

To be able to calculate the  $C_{\alpha,\beta,\gamma}^j$  factors one represents the nucleon wave function through the helicity-flavor basis of the valence quarks. We use a rather general form separating the wave function into two parts characterized by two (e.g., second and third) quarks being in spin zero—isinglet and spin one—isotriplet states as follows:

$$\psi^{i^3, h_N} = \frac{1}{\sqrt{2}} \left\{ \Phi_{0,0}(k_1, k_2, k_3) (\chi_{0,0}^{(23)} \chi_{1/2, h_N}^{(1)}) (\tau_{0,0}^{(23)} \tau_{1/2, i^3}^{(1)}) + \Phi_{1,1}(k_1, k_2, k_3) \right. \\ \left. \times \sum_{i_{23}^3=-1}^1 \sum_{h_{23}^3=-1}^1 \left\langle 1, h_{23}; \frac{1}{2}, h_N - h_{23} \left| \frac{1}{2}, h_N \right\rangle \left\langle 1, i_{23}^3; \frac{1}{2}, i^3 - i_{23}^3 \left| \frac{1}{2}, i^3 \right\rangle (\chi_{1, h_{23}}^{(23)} \chi_{1/2, h_N - h_{23}}^{(1)}) (\tau_{1, i_{23}^3}^{(23)} \tau_{1/2, i^3 - i_{23}^3}^{(1)}) \right\}, \quad (4)$$

where  $j_N^3$  and  $h_N$  are the isospin component and the helicity of the nucleon. Here  $k_i$ 's are the light cone momenta of quarks which should be understood as  $(x_i, k_{i\perp})$  where  $x_i$  is a light cone momentum fraction of the nucleon carried by the  $i$  quark. We define  $\chi_{j,h}$  and  $\tau_{I,i^3}$  as helicity and isospin wave functions, where  $j$  is the spin,  $h$  is the helicity,  $I$  is the isospin and  $i^3$  its third component. The Clebsch-Gordan coefficients are defined as  $\langle j_1, m_1; j_2, m_2 | j, m \rangle$ . Here,  $\Phi_{I,J}$  represents the momentum dependent part of the wave function for  $(I = 0, J = 0)$  and  $(I = 1, J = 1)$  two-quark spectator states, respectively. Since the asymmetry in Eq. (1) does not depend on the absolute normalization of the cross section, a more relevant quantity for us will be the relative strength of these two momentum dependent wave functions. For our discussion we introduce a parameter,  $\rho$ :

$$\rho = \frac{\langle \Phi_{1,1} \rangle}{\langle \Phi_{0,0} \rangle} \quad (5)$$

which characterizes an average relative magnitude of the wave function components corresponding to  $(I = 0, J = 0)$  and  $(I = 1, J = 1)$  quantum numbers of two-quark “spectator” states. Note that the two extreme values of  $\rho$  define two well-known approximations:  $\rho = 1$  corresponds to the exact SU(6) symmetric picture of the nucleon wave function and  $\rho = 0$  will correspond to the contribution of only good-scalar diquark configuration in the nucleon wave function (see, e.g., Refs. [17–20] where this component is referred as a scalar or good diquark configuration ( $[qq]$ ) as opposed to a vector or bad diquark configuration denoted by  $(qq)$ ). In further discussions we will keep  $\rho$  as a free parameter. Note that in our approach diquarks represent a  $qq$  component of the nucleon wave function and quarks from the diquark participate in the hard scattering through the quark interchange. We do not introduce additional collective vector and scalar diquark couplings similar to Refs. [20,21].

To calculate the scattering amplitude of Eq. (2) we assume the conservation of the helicities of quarks participating in the hard scattering. This allows us to approximate the hard scattering part of the amplitude,  $H$ , in the following form:

$$H \approx \delta_{\alpha_1 \alpha'_1} \delta_{\alpha_2 \alpha'_2} \delta_{\beta_1, \beta'_1} \delta_{\gamma_1, \gamma'_1} \delta_{\beta_2, \beta'_2} \delta_{\gamma_2, \gamma'_2} \frac{F(\theta)}{s^4}. \quad (6)$$

Inserting this expression into Eq. (2) for the QIM amplitude one obtains [14]:

$$\langle cd|T|ab \rangle = \text{Tr}(M^{ac} M^{bd}) \quad (7)$$

with:

$$M_{\alpha, \alpha'}^{i, j} = C_{\alpha, \beta \gamma}^i C_{\alpha', \beta \gamma}^j + C_{\beta \alpha, \beta}^i C_{\beta \alpha', \beta}^j + C_{\beta \gamma \alpha}^i C_{\beta \gamma \alpha'}^j, \quad (8)$$

where we sum over all the possible values of  $\beta$  and  $\gamma$ . Furthermore, we separate the energy dependence from the scattering amplitude as follows:

$$\langle cd|T|ab \rangle = \frac{\langle h_c, h_d|T(\theta)|h_a, h_b \rangle}{s^4} \quad (9)$$

and define five independent angular parts of the helicity amplitudes as:

$$\begin{aligned} \phi_1 &= \langle ++|T(\theta)|++ \rangle; \\ \phi_2 &= \langle --|T(\theta)|++ \rangle; \\ \phi_3 &= \langle +-|T(\theta)|+- \rangle; \\ \phi_4 &= -\langle -+|T(\theta)|+- \rangle; \\ \phi_5 &= \langle -+|T(\theta)|++ \rangle. \end{aligned} \quad (10)$$

Here the “-” sign in the definition of  $\phi_4$  follows from the Jacob-Wick helicity convention [22] according to which a  $(-1)$  phase is introduced if two quarks that scatter to  $\pi - \theta_{\text{c.m.}}$  angle have opposite helicity (see also Ref. [14]).

Using Eqs. (7) and (8) for the nonvanishing helicity amplitudes of Eq. (10) one obtains: for  $pp \rightarrow pp$ :

$$\begin{aligned} \phi_1 &= (3+y)F(\theta) + (3+y)F(\pi-\theta) \\ \phi_3 &= (2-y)F(\theta) + (1+2y)F(\pi-\theta) \\ \phi_4 &= -(1+2y)F(\theta) - (2-y)F(\pi-\theta) \end{aligned} \quad (11)$$

and for  $pn \rightarrow pn$ :

$$\begin{aligned} \phi_1 &= (2-y)F(\theta) + (1+2y)F(\pi-\theta) \\ \phi_3 &= (2+y)F(\theta) + (1+4y)F(\pi-\theta) \\ \phi_4 &= 2yF(\theta) + 2yF(\pi-\theta) \end{aligned} \quad (12)$$

with  $\phi_2 = \phi_5 = 0$  due to helicity conservation. Here

$$y = x(x+1) \quad \text{with} \quad x = \frac{2\rho}{3(1+\rho^2)} \quad (13)$$

and  $F(\theta)$  is the angular function. Note that the  $\rho = 1$  case reproduces the SU(6) result of Refs. [10,14]. The results of Eqs. (11) and (12) could be obtained also through the

formalism of the  $H$  spin introduced in Ref. [10]. In this case the helicity amplitudes are expressed through the average number of quarks to be found in a given helicity-spin state. These numbers are directly defined through the wave function of Eq. (4).

Introducing the symmetric and antisymmetric parts of the angular function  $F$  as follows:

$$s(\theta) = \frac{F(\theta) + F(\pi-\theta)}{2}; \quad a(\theta) = \frac{F(\theta) - F(\pi-\theta)}{2} \quad (14)$$

and using Eq. (12) for the asymmetry as it is defined in Eq. (1) one obtains

$$A_{90^\circ}(\theta) = \frac{6a(\theta)s(\theta)(1-2y-3y^2)}{a(\theta)^2(1-3y)^2 + 3s(\theta)^2(3+6y+7y^2)}. \quad (15)$$

One can make a rather general observation from Eq. (15), that for the SU(6) model, ( $\rho = 1$ ,  $y = \frac{4}{9}$ ) and for any positive function,  $a(\theta)$  at  $\theta \leq \frac{\pi}{2}$ , the angular asymmetry has a negative sign opposite to the experimental asymmetry (Fig. 2). Note that one expects a positive  $a(\theta)$  at  $\theta \leq \frac{\pi}{2}$  from general grounds based on the expectation that in the hard scattering regime the number of  $t$ -channel quark scatterings dominates the number of  $u$ -channel quark scatterings in the forward direction.

As it follows from Eq. (15), positive asymmetry can be achieved only for  $1-2y-3y^2 > 0$ , which according to Eq. (13) imposes the following restrictions on  $\rho$ :  $\rho < 0.49$  or  $\rho > 2.036$ . The first condition indicates the preference of scalar diquarklike configurations in the nucleon wave function, while the second one will indicate the strong dominance of the vector-diquark component which contradicts the observations [17–19].

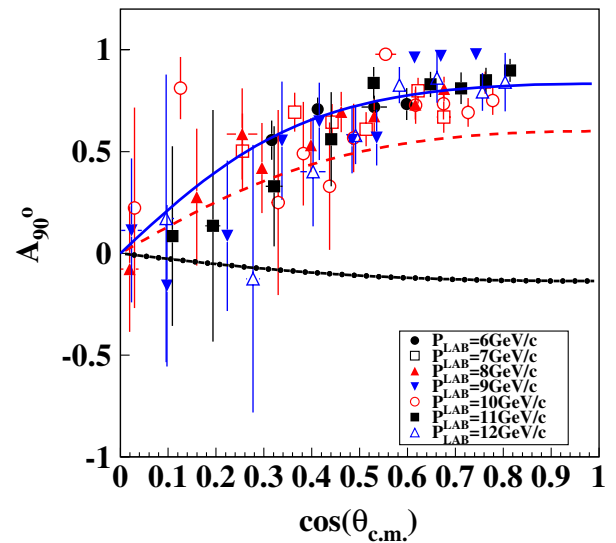


FIG. 2 (color online). Asymmetry of  $pn$  elastic cross section. SU(6), with  $\rho = 1$  (dotted solid line), diquark-model with  $\rho = 0$  (dashed line), fit with  $\rho = -0.3$  (solid line).

In Fig. 2 the asymmetry of  $pn$  scattering calculated with  $SU(6)$  ( $\rho = 1$ ) and pure scalar-diquark ( $\rho = 0$ ) models are compared with the data. In these estimates we use  $F(\theta) = C \sin^{-2}(\theta)[1 - \cos(\theta)]^{-2}$  dependence of the angular function [23] which is consistent with the picture of hard collinear QIM scattering of valence quarks with five gluon exchanges and reasonably well reproduces the main characteristics of the angular dependencies of both  $pp$  and  $pn$  elastic scatterings. Note that using a form of the angular function based on nucleon form-factor arguments [10,14],  $F \approx [1 - \cos(\theta)]^{-2}$  will result in the same angular asymmetry.

The comparisons show that the nucleon wave function (4) with a good-scalar diquark component ( $\rho = 0$ ) produces the right sign for the angular asymmetry. On the other hand, even large errors of the data do not preclude to conclude that the exact  $SU(6)$  symmetry ( $\rho = 1$ ) of the quark wave function of the nucleon is in qualitative disagreement with the experimental asymmetry.

Using the above defined angular function  $F(\theta)$  we fitted  $A_{90^\circ}$  in Eq. (15) to the data at  $-t, -u \geq 2 \text{ GeV}^2$  varying  $\rho$  as a free parameter. We used the maximal likelihood method of fitting excluding those data points from the data set whose errors are too large for meaningful identification of the asymmetry. The best fit is found for

$$\rho \approx -0.3 \pm 0.2. \quad (16)$$

The nonzero magnitude of  $\rho$  indicates the small but finite relative strength of a bad or vector diquark configuration in the nuclear wave function as compared to the scalar diquark component. It is intriguing that the obtained magnitude of  $\rho$  is consistent with the 10% probability of “bad” diquark configuration discussed within the context of baryonic spectroscopy [19].

Another interesting property of Eq. (16) is the negative sign of the parameter  $\rho$ . Within the qualitative quantum-mechanical picture, the negative sign of  $\rho$  may indicate, for example, the existence of a repulsion in the quark-diquark (vector-diquark) channel as opposed to the attraction in the quark-diquark (scalar-diquark) channel. It is rather surprising that both the magnitude and sign agree with the result of the phenomenological interaction derived in the one-gluon exchange quark model discussed in Ref. [18].

In conclusion, we demonstrated that the angular asymmetry of hard elastic  $pn$  scattering represents a new observable in probing the symmetry structure of the valence quark wave function of the nucleon. We demonstrated that the exact  $SU(6)$  symmetry does not reproduce the experimental angular asymmetry of hard elastic  $pn$  scattering. Nucleon wave function consistent with the diquark structure gives a right asymmetry. The fit to the data indicates 10% probability for the existence of bad or vector diquarks in the wave function of nucleons. Most importantly, it also shows that the vector and scalar  $qq$  components of the wave function are in the opposite phase. This indicates the

very different dynamics of  $q - [qq]$  and  $q - (qq)$  interactions. One expects our observation to be valid in valence quark sector for up to ISR energies for which QIM is expected to be dominant (see, e.g., [10]).

The opposite sign of the vector ( $qq$ ) and scalar  $[qq]$  components in the nucleon wave function may have important implications for different coherent QCD processes such as form factors and hard exclusive scatterings involving baryons. These quantities in principle can be checked in lattice calculations. The angular asymmetry studies can be extended also to include the scattering of other baryons such as  $\Delta$  isobars (which may have a larger fraction of the vector diquark component) as well as strange baryons which will allow us to study the relative strength of  $(qq)$  and  $[qq]$  configurations involving strange quarks. Another extension of these studies is calculation of elastic  $p\bar{p}$  scattering. All these processes could be studied experimentally in new facilities such as J-PARC and GSI-FAIR.

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