Supersoft Symmetry Energy Encountering Non-Newtonian Gravity in Neutron Stars

De-Hua Wen,[1,2] Bao-An Li,[2,*] and Lie-Wen Chen[3]

[1]Department of Physics, South China University of Technology, Guangzhou 510641, P.R. China
[2]Department of Physics and Astronomy, Texas A&M University-Commerce, Commerce, Texas 75429-3011, USA
[3]Department of Physics, Shanghai Jiao Tong University, Shanghai 200240, P.R. China

(Received 13 August 2009; published 16 November 2009)

Considering the non-Newtonian gravity proposed in grand unification theories, we show that the stability and observed global properties of neutron stars cannot rule out the supersoft nuclear symmetry energies at suprasaturation densities. The degree of possible violation of the inverse-square law of gravity in neutron stars is estimated using an equation of state of neutron-rich nuclear matter consistent with the available terrestrial laboratory data.

DOI: 10.1103/PhysRevLett.103.211102 PACS numbers: 26.60.−c, 14.70.Pw, 97.60.Jd

The density dependence of nuclear symmetry energy $E_{\rm sym}(\rho)$ is an important ingredient for understanding many interesting phenomena in astrophysics, cosmology [1–4], and nuclear physics [5–9]. However, theoretical predictions on the $E_{\rm sym}(\rho)$ especially at suprasaturation densities are currently rather diverse [8–13]. Unfortunately, there is no known first-principle guiding the high-density behavior of the $E_{\rm sym}(\rho)$. Presently, while many theories, see, e.g., Refs. [4,13–17], predict that the $E_{\rm sym}(\rho)$ increases continuously at all densities, many other models, see, e.g., Refs. [9,18–32], predict that the $E_{\rm sym}(\rho)$ first increases and then decreases above certain suprasaturation densities. The $E_{\rm sym}(\rho)$ may even become negative at high densities [2,5,8–12,20,24]. This latter kind of symmetry energy functions are generally regarded as being soft. Some (e.g., the UV14 + TNI in [20] and group II in [24]) of them can describe very well all observed properties of neutron stars (NSs). However, the supersoft ones (e.g., the original Gogny-Hartree-Fock (GHF) prediction [27] and group III in [24]) that quickly drop to zero around 3 times the saturation density ρ_0 either can not keep the NSs stable or predict maximum NS masses significantly below $1.4M_\odot$ depending on the EOS used for symmetric nuclear matter. Given the above theoretical situation, experimental indications on the high density $E_{\rm sym}(\rho)$ are thus utmost important. Very interestingly, circumstantial evidence for a supersoft $E_{\rm sym}(\rho)$ [33] was found very recently from analyzing the FOPI/GSI experimental data on the π^-/π^+ ratio in relativistic heavy-ion collisions [34] within a transport model [35] using the MDI (momentum-dependent-interaction) EOS [27]. While the symmetric part of the MDI EOS is consistent with the existing terrestrial nuclear laboratory data [6,8], the total pressure of NS matter obtained using the supersoft $E_{\rm sym}(\rho)$ (which is actually the original GHF prediction) preferred by the FOPI/GSI data can not keep neutron stars stable. Among possibly many important ramifications in astrophysics and cosmology, this finding posts immediately a serious scientific challenge: how can the NSs be stable with such kind of supersoft symmetry energies? In fact, this question has been raised repeatedly and the answer has been negative long before any experimental indication was available. In the literature, the supersoft symmetry energies were often regarded by some people as either "unpleasant", see, e.g., [23], or "unphysical", see, e.g., [24,36,37]. These assertions, of course, are all based on the assumption that gravity is well understood. However, it is really remarkable that gravity, despite being the first to be discovered, is actually still considered by far the most poorly understood force [38–40]. In fact, in pursuit of unifying gravity with the three other fundamental forces, conventional understanding about gravity has to be modified due to either the geometrical effect of the extra space-time dimensions predicted by string theories and/or the exchange of the weakly interacting bosons newly proposed in the supersymmetric extension of the standard model, see, e.g., Refs. [41,42] for reviews. Consequently, the inverse-square-law (ISL) of gravity is expected to be violated. In stable neutron stars at β equilibrium which is determined by the weak and electromagnetic interactions, the gravity has to be balanced by the strong interaction. Neutron stars are thus a natural testing ground of grand unification theories. In this Letter, we show that the supersoft $E_{\rm sym}(\rho)$ preferred by the FOPI/GSI data can readily keep neutron stars stable if the non-Newtonian gravity is considered.

The deviation from the ISL of gravity can be characterized effectively by adding a Yukawa term to the normal gravitational potential [43,44], i.e.,

$$V(r) = -\frac{G_\infty m_1 m_2}{r}(1 + \alpha e^{-r/\lambda}), \qquad (1)$$

where α is a dimensionless strength parameter, λ is the length scale and G_∞ is the universal gravitational constant. Alternatively, the Yukawa term can also be considered as due to the putative "fifth force" [41–43] coexisting with gravity or a nonuniversal gravitational "constant" [41,45] of $G(r) = G_\infty[1 + \alpha e^{-r/\lambda}(1 + r/\lambda)]$. In the scalar or vector boson exchange picture, $\alpha = \pm g^2/(4\pi G_\infty m_b^2)$ and

$\lambda = 1/\mu$ (in natural units). The g^2, μ and m_b are the boson-baryon coupling constant, the boson and baryon mass, respectively. To reduce gravity from the ISL, the exchange of a vector boson is necessary. It is worth noting that a neutral spin-1 vector U-boson has been a favorite candidate. It is very weakly coupled to baryons [46], can mediate the interactions among dark matter (DM) candidates [47,48] and has been used to explain the 511 keV γ-ray observation from the galactic bulge [49–51].

According to Fujii [52], the Yukawa term is simply part of the matter system in general relativity. Consequently, the Einstein equation remains the same and only the EOS is modified. Within the mean-field approximation, the extra energy density due to the Yukawa term is [44,46]

$$\varepsilon_{\rm UB} = \frac{1}{2V}\int \rho(\vec{x}_1)\frac{g^2}{4\pi}\frac{e^{-\mu r}}{r}\rho(\vec{x}_2)d\vec{x}_1 d\vec{x}_2 = \frac{1}{2}\frac{g^2}{\mu^2}\rho^2, \quad (2)$$

where V is the normalization volume, ρ is the baryon number density and $r = |\vec{x}_1 - \vec{x}_2|$. The corresponding addition to the pressure is then $P_{\rm UB} = \frac{1}{2}\frac{g^2\rho^2}{\mu^2}(1 - \frac{2\rho}{\mu}\frac{\partial\mu}{\partial\rho})$. Assuming a constant boson mass independent of the density, one obtains $P_{\rm UB} = \varepsilon_{\rm UB} = \frac{1}{2}\frac{g^2}{\mu^2}\rho^2$. For the purposes of the present study, it is sufficient to consider neutron stars as simply consisting of neutrons (n), protons (p) and electrons (e). Including the Yukawa term the total pressure inside neutron stars is $P = P_{npe} + P_{\rm UB}$. For the inner and outer crusts we use for P_{npe} the EOS of Carriere *et al.* [53] and that of Baym *et al.* [54], respectively. They are smoothly connected to the EOS in the core [55]. For the latter we use $P_{npe}(\rho, \delta) = \rho^2[dE_0/d\rho + dE_{\rm sym}/d\rho\delta^2] + \frac{1}{2}\delta(1 - \delta)\rho E_{\rm sym}(\rho)$. The value of the isospin asymmetry δ at β equilibrium is determined by the chemical equilibrium condition $\mu_e = \mu_n - \mu_p = 4\delta E_{\rm sym}(\rho)$ and the charge neutrality requirement $\rho_e = \frac{1}{2}(1 - \delta)\rho$. The $E_0(\rho)$ and $E_{\rm sym}(\rho)$ obtained consistently within the modified GHF approximation are [27,55], respectively,

$$\begin{aligned}
E_0(\rho) &= \frac{8\pi}{5mh^3\rho}p_f^5 + \frac{\rho}{4\rho_0}(-216.55) + \frac{B}{\sigma+1}\left(\frac{\rho}{\rho_0}\right)^\sigma \\
&\quad + \frac{1}{3\rho_0\rho}(C_l + C_u)\left(\frac{4\pi}{h^3}\right)^2\Lambda^2\left[p_f^2(6p_f^2 - \Lambda^2) - 8\Lambda p_f^3 \arctan\frac{2p_f}{\Lambda} + \frac{1}{4}(\Lambda^4 + 12\Lambda^2 p_f^2)\ln\frac{4p_f^2 + \Lambda^2}{\Lambda^2}\right], \\
E_{\rm sym}(\rho) &= \frac{8\pi}{9mh^3\rho}p_f^5 + \frac{\rho}{4\rho_0}[-24.59 + 4Bx/(\sigma+1)] - \frac{Bx}{\sigma+1}\left(\frac{\rho}{\rho_0}\right)^\sigma + \frac{C_l}{9\rho_0\rho}\left(\frac{4\pi}{h^3}\right)^2\Lambda^2\left[4p_f^4 - \Lambda^2 p_f^2\ln\frac{4p_f^2+\Lambda^2}{\Lambda^2}\right] \\
&\quad + \frac{C_u}{9\rho_0\rho}\left(\frac{4\pi}{h^3}\right)^2\Lambda^2\left[4p_f^4 - p_f^2(4p_f^2 + \Lambda^2)\ln\frac{4p_f^2+\Lambda^2}{\Lambda^2}\right], \qquad (3)
\end{aligned}$$

where $p_f = \hbar(3\pi^2\frac{\rho}{2})^{1/3}$ is the Fermi momentum for symmetric nuclear matter at density ρ. The values of the parameters are $\sigma = 4/3$, $B = 106.35$ MeV, $C_l = -11.70$ MeV, $C_u = -103.40$ MeV and $\Lambda = p_f^0 \equiv p_f(\rho_0)$ [27]. The resulting symmetric EOS contribution $dE_0/d\rho$ to the pressure is consistent with that extracted from studying kaon production and nuclear collective flow in relativistic heavy-ion collisions using hadronic transport models assuming no hadron to quark-gluon plasma phase transition up to about $5\rho_0$ [6,8]. The parameter x in Eq. (3) was introduced to vary the density dependence of the $E_{\rm sym}(\rho)$ without changing any property of symmetric nuclear matter and the value of $E_{\rm sym}(\rho_0) = 31$ MeV [27]. Shown in the inset of Fig. 1 are two typical $E_{\rm sym}(\rho)$ denoted as MDIx1 and MDIx0 obtained by using $x = 1$ and $x = 0$, respectively. While the MDIx0 $E_{\rm sym}(\rho)$ increases continuously, the MDIx1 $E_{\rm sym}(\rho)$ becomes negative above $3\rho_0$. Only the MDIx1 $E_{\rm sym}(\rho)$ can reproduce the FOPI/GSI pion production data within the transport model analysis [33]. It is seen that the corresponding MDIx1 pressure decreases with increasing density as shown with the lowest curve in Fig. 1. However, the Yukawa term makes the pressure grow continuously with increasing density with a value of g^2/μ^2 higher than about 10 GeV^{-2}.

Shown in Fig. 2 is the mass-radius relation of static neutron stars obtained from solving the Tolman-Oppenheimer-Volkoff (TOV) equation using the MDIx1 $E_{\rm sym}(\rho)$ and various values for the g^2/μ^2. The result

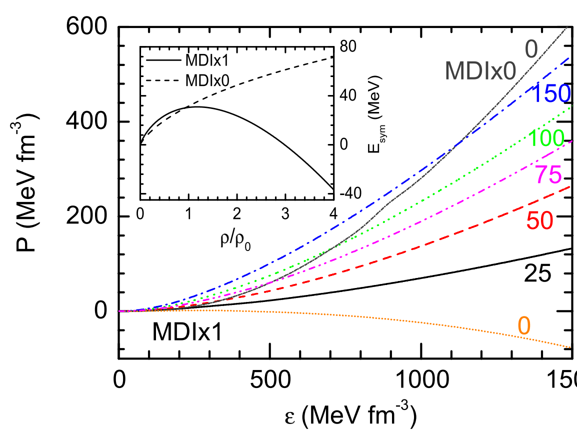

FIG. 1 (color online). The inset shows two typical examples (MDIx0 and MDIx1) of the density dependence of the nuclear symmetry energy. The MDIx1 (MDIx0) EOS with (without) the Yukawa contribution using different values of the g^2/μ^2 in units of GeV^{-2} are shown.

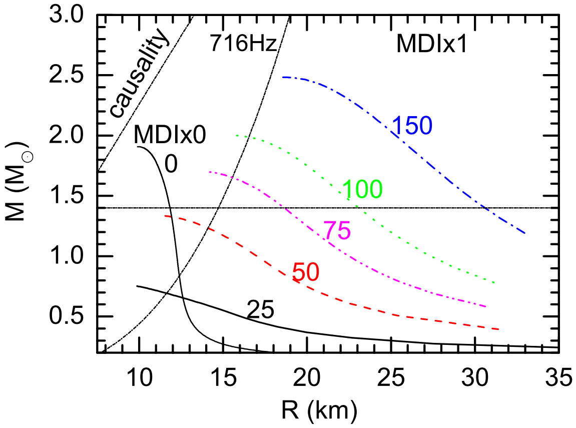

FIG. 2 (color online). The mass-radius relation of static neutron stars with the MDIx1 (MDIx0) EOS with (without) the Yukawa contribution. The static neutron star sequences constrained by the rotational frequency 716 Hz of the J1748-2446ad [58] are taken from Haensel *et al.* [57]. The numbers above the lines are the g^2/μ^2 values in units of GeV^{-2}.

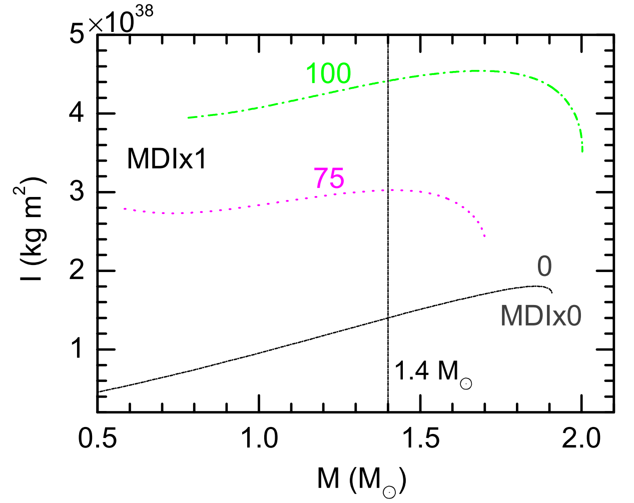

FIG. 3 (color online). The momenta of inertia of neutron stars with the MDIx1 (MDIx0) $E_{\rm sym}(\rho)$ with (without) the Yukawa contribution. The numbers above the lines are the g^2/μ^2 values in units of GeV^{-2}.

obtained using the MDIx0 without including the Yukawa term is included as a reference [56]. The causality [3] and rotational constraint [57] are also shown. The Keplerian (mass-shedding) frequency is approximately [57] $\nu_k \approx 1.08(\frac{M}{M_\odot})^{1/2}(\frac{R}{10\ \mathrm{km}})^{-3/2}$ kHz. So far, the fastest pulsar observed is the J1748-2446ad spinning at 716 Hz [58]. Taking 716 Hz as the Keplerian frequency, the $M-R$ relation is restricted to the left side of the rotational limit. The latter restricts the value of g^2/μ^2 to less than 150 GeV^{-2}. It is seen that to produce a neutron star with a maximum mass above $1.4M_\odot$, the g^2/μ^2 has to be higher than about 50 GeV^{-2}. More specifically, with the MDIx1 $E_{\rm sym}(\rho)$ and the $g^2/\mu^2 = 50$–150 GeV^{-2}, or equivalently $|\alpha|\lambda^2 = (2.6-7.8)\times 10^7\ \mathrm{m}^2$, neutron stars can have a maximum mass between 1.4 and $2.5M_\odot$ and a corresponding radius between 12 and 18 km.

For canonical neutron stars of $1.4M_\odot$, the radius is quite sensitive to the g^2/μ^2 value used. Thus, besides the accurate measurement of neutron star radii, additional measurements related to the mass distribution, such as the moment of inertia, will be very useful in setting astrophysical constraints on the $E_{\rm sym}(\rho)$ and g^2/μ^2. According to Lattimer and Schutz [59], at the slow rotation limit the moment of inertia can be well approximated as $I \approx (0.237 \pm 0.008)MR^2[1 + 4.2\frac{M}{M_\odot}\cdot\frac{\mathrm{km}}{R} + 90(\frac{M}{M_\odot}\cdot\frac{\mathrm{km}}{R})^4]$. Shown in Fig. 3 is the I as a function of M. For $M = 1.4M_\odot$, the MDIx0 without the Yukawa contribution gives an I no more than $1.8\times 10^{38}\ \mathrm{kg\,m}^2$ [60]. However, significantly larger I values are obtained with the MDIx1 $E_{\rm sym}(\rho)$ and the Yukawa contribution. The discovery of the double-pulsar system PSR J0737-3039 *A*&*B* provides a great opportunity to determine accurately the moment of inertia I_A of the star A [61,62]. Our results shown here add to the importance of measuring the moment of inertia accurately.

To constrain the values of α and λ has been a longstanding goal of many terrestrial experiments and astrophysical observations as limits on them may provide useful guidance for developing grand unification theories, see, e.g., Refs. [41–43,63–69]. These studies have estimated various upper limits on the α. In the range of $\alpha = 10^{-10}$–10^{38} and $\lambda = 10^{15}$–10^{-14} m, there is a clear trend of increased strength α at shorter length λ. What we have constrained is the value of g^2/μ^2 or equivalently the $|\alpha|\lambda^2$ from the pressure necessary to support both static neutron stars and the fastest pulsars. While we expect that the range parameter λ has to be much larger (smaller) than the radii of finite nuclei (neutron stars), we can not set separate constraints on the values of α and λ. Compared to other efforts to constrain the α and λ, our study here is unique in that the estimated minimum value of g^2/μ^2 is a lower limit satisfying all known constraints from both terrestrial nuclear experiments and observations of global properties of neutron stars. Moreover, very interestingly, our estimated range of g^2/μ^2 overlaps well with the upper limits estimated from analyzing the neutron-proton and neutron-lead scattering data in the range of $\lambda \approx 10^{-14}$–10^{-8} m [69–72].

In summary, neutron stars are a natural testing ground of grand unification theories of fundamental forces. Considering the possible violation of the ISL of gravity, the stability and observed properties of NSs can not rule out supersoft symmetry energies at suprasaturation densities. Given the uncertainties and model dependence involved in extracting information about the EOS and symmetry energy from heavy-ion reactions, it is very important to test the possible supersoft symmetry energy at suprasaturation densities using several observables simultaneously from independent experiments analyzed using different models. If confirmed, it may point towards a violation of the ISL in neutron stars.

We would like to thank M. I. Krivoruchenko for useful communications and G. C. Yong, C. Xu, and J. Xu for helpful discussions. The work is supported in part by the National Natural Science Foundation of China under Grant No. 10647116, 10710172, 10575119, 10675082, and 10975097, the Young Teachers' Training Program from China Scholarship Council under Grant No. 2007109651, the MOE of China under project NCET-05-0392, Shanghai Rising-Star Program under Grant No. 06QA14024, the SRF for ROCS, SEM of China, and the National Basic Research Program of China (973 Program) under Contract No. 2007CB815004, the US National Science Foundation under Grants No. PHY0652548 and No. PHY0757839, the Research Corporation under Grant No. 7123 and the Texas Coordinating Board of Higher Education Grant No. 003565-0004-2007.

*Corresponding author.
Bao-An_Li@Tamu-Commerce.edu

[1] K. Sumiyoshi and H. Toki, Astrophys. J. **422**, 700 (1994).
[2] I. Bombaci, *Ch. 2 in Isospin Physics in Heavy-Ion Collisions at Intermediate Energies*, edited by Bao-An Li and W. Udo Schröder (Nova Science Publishers, Inc., New York, 2001).
[3] J. M. Lattimer and M. Prakash, Astrophys. J. **550**, 426 (2001); Science **304**, 536 (2004).
[4] A. W. Steiner *et al.*, Phys. Rep. **411**, 325 (2005).
[5] B. A. Li *et al.*, Int. J. Mod. Phys. E **7**, 147 (1998).
[6] P. Danielewicz *et al.*, Science **298**, 1592 (2002).
[7] V. Baran *et al.*, Phys. Rep. **410**, 335 (2005).
[8] B. A. Li *et al.*, Phys. Rep. **464**, 113 (2008).
[9] B. A. Brown, Phys. Rev. Lett. **85**, 5296 (2000).
[10] M. Kutschera, Phys. Lett. B **340**, 1 (1994).
[11] S. Kubis and M. Kutschera, Nucl. Phys. **A720**, 189 (2003).
[12] A. Szmaglinski *et al.*, Acta Phys. Pol. B **37**, 277 (2006).
[13] A. E. L. Dieperink *et al.*, Phys. Rev. C **68**, 064307 (2003).
[14] C.-H. Lee *et al.*, Phys. Rev. C **57**, 3488 (1998).
[15] C. J. Horowitz *et al.*, Phys. Rev. Lett. **86**, 5647 (2001).
[16] L. W. Chen *et al.*, Phys. Rev. C **76**, 054316 (2007).
[17] Z. H. Li *et al.*, Phys. Rev. C **74**, 047304 (2006).
[18] V. R. Pandharipande *et al.*, Phys. Lett. B **39**, 608 (1972).
[19] B. Friedman *et al.*, Nucl. Phys. **A361**, 502 (1981).
[20] R. B. Wiringa *et al.*, Phys. Rev. C **38**, 1010 (1988).
[21] N. Kaiser *et al.*, Nucl. Phys. **A697**, 255 (2002).
[22] P. Krastev *et al.*, Phys. Rev. C **74**, 025808 (2006).
[23] E. Chabanat *et al.*, Nucl. Phys. **A627** 710 (1997); **A635** 231 (1998).
[24] J. R. Stone *et al.*, Phys. Rev. C **68**, 034324 (2003).
[25] L. W. Chen *et al.*, Phys. Rev. C **72**, 064309 (2005).
[26] J. Decharge *et al.*, Phys. Rev. C **21**, 1568 (1980).
[27] C. B. Das *et al.*, Phys. Rev. C **67**, 034611 (2003).
[28] W. D. Myers *et al.*, Acta Phys. Pol. B **26**, 111 (1995).
[29] D. T. Khoa *et al.*, Nucl. Phys. **A602**, 98 (1996).
[30] D. N. Basu *et al.*, Nucl. Phys. **A811**, 140 (2008).
[31] S. Banik *et al.*, J. Phys. G **26**, 1495 (2000).
[32] P. Roy Chowdhury *et al.*, Phys. Rev. C **80**, 011305(R) (2009).
[33] Z. G. Xiao *et al.*, Phys. Rev. Lett. **102**, 062502 (2009).
[34] W. Reisdorf *et al.*, Nucl. Phys. **A781**, 459 (2007).
[35] B. A. Li *et al.*, Nucl. Phys. **A735**, 563 (2004).
[36] N. Glendening, *Compact Stars* (Springer, New York, 2000), ISBN 0387989773.
[37] J. R. Stone, J. Phys. G **31**, R211 (2005).
[38] R. Pease, Nature (London) **411**, 986 (2001).
[39] C. D. Hoyle, Nature (London) **421**, 899 (2003).
[40] N. Arkani-Hamed *et al.*, Phys. Lett. B **429**, 263 (1998).
[41] E. Fischbach and C. L. Talmadge, *The Search for Non-Newtonian Gravity* (Springer-Verlag, Inc., New York, 1999), ISBN 0-387-98490-9.
[42] E. G. Adelberger *et al.*, Annu. Rev. Nucl. Part. Sci. **53**, 77 (2003).
[43] Y. Fujii, Nature (London) **234**, 5 (1971).
[44] J. C. Long *et al.*, Nature (London) **421**, 922 (2003).
[45] J. P. Uzan, Rev. Mod. Phys. **75**, 403 (2003).
[46] M. I. Krivoruchenko *et al.*, Phys. Rev. D **79**, 125023 (2009).
[47] P. Fayet, Phys. Lett. B **675**, 267 (2009).
[48] C. Boehm and P. Fayet, Nucl. Phys. **B683**, 219 (2004).
[49] P. Jean *et al.*, Astron. Astrophys. **407**, L55 (2003).
[50] C. Boehm *et al.*, Phys. Rev. Lett. **92**, 101301 (2004); Phys. Rev. D **69**, 101302 (2004).
[51] S. H. Zhou, Phys. Rev. D **75**, 115004 (2007).
[52] Y. Fujii, in *Large Scale Structures of the Universe*, edited by J. Audouze *et al.* (International Astronomical Union, Garching, Germany, 1988), p. 471.
[53] J. Carriere *et al.*, Astrophys. J. **593**, 463 (2003).
[54] G. Baym *et al.*, Astrophys. J. **170**, 299 (1971).
[55] J. Xu *et al.*, Astrophys. J. **697**, 1549 (2009).
[56] B. A. Li *et al.*, Phys. Lett. B **642**, 436 (2006).
[57] P. Haensel *et al.*, Astron. Astrophys. **502**, 605 (2009).
[58] J. W. T. Hessels *et al.*, Science **311**, 1901 (2006).
[59] J. M. Lattimer *et al.*, Astrophys. J. **629**, 979 (2005).
[60] A. Worley *et al.*, Astrophys. J. **685**, 390 (2008).
[61] A. G. Lyne *et al.*, Science **303**, 1153 (2004).
[62] J. M. Lattimer *et al.*, Phys. Rep. **442**, 109 (2007).
[63] G. W. Gibbons and B. F. Whiting, Nature (London) **291**, 636 (1981).
[64] M. Bordag *et al.*, Phys. Rep. **353**, 1 (2001).
[65] Serge Reynaud *et al.*, Int. J. Mod. Phys. A **20**, 2294 (2005).
[66] R. S. Decca *et al.*, Phys. Rev. Lett. **94**, 240401 (2005).
[67] E. G. Adelberger *et al.*, Phys. Rev. Lett. **98**, 131104 (2007).
[68] D. J. Kapner *et al.*, Phys. Rev. Lett. **98**, 021101 (2007).
[69] Y. Kamyshkov *et al.*, Phys. Rev. D **78**, 114029 (2008).
[70] R. Barbieri *et al.*, Phys. Lett. B **57**, 270 (1975).
[71] Yu. N. Pokotilovski, Phys. At. Nucl. **69**, 924 (2006).
[72] V. V. Nesvizhevsky *et al.*, Phys. Rev. D **77**, 034020 (2008).