Hamiltonian Theory of Disorder at $\nu = 1/3$

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The Hamiltonian theory of the fractional quantum Hall regime provides a simple and tractable approach to calculating gaps, polarizations, and many other physical quantities. In this Letter we include disorder in our treatment and show that a simple model with minimal assumptions produces results consistent with a range of experiments. In particular, the interplay between disorder and interactions can result in experimental signatures which mimic those of spin textures.

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Static disorder is crucial to the phenomenology of the integer quantum Hall effect (IQHE) [1,2]. The strong perpendicular magnetic field quantizes the kinetic energy of the two-dimensional electron gas into Landau levels. While static disorder broadens each Landau Level (LL), only the state at the center of the broadened LL is extended at $T = 0$, while all other states are localized [3]. This produces the plateau in the Hall resistance as the chemical potential μ traverses the localized states and the plateau transition when μ crosses the extended state. Merely broadening the LLs is not enough to decrease the transport gap. Because the transport gap depends on *extended* states, one needs to *shift* the energies of the extended states relative to μ to change the transport gap.

In the fractional quantum Hall effect (FQHE) [4], the qualitative role of disorder is to localize the quasiparticles, while the quantum Hall condensate carries the Hall current. Theoretical gaps $\Delta_{\rm Th}$ computed by numerical methods [5,6] are much larger than the measured gaps $\Delta_{\rm Ex}$ [7]. Empirically, one uses $\Delta_{\rm Ex} = \Delta_{\rm Th} - \Gamma$ [7], where Γ is a measure of the disorder broadening. As stated above, disorder broadening by itself cannot account for the reduction of the transport gap.

In previous work, the effect of dopant disorder on the magnetoroton minimum at $\frac{1}{3}$ has been investigated [8] within the single-mode and self-consistent Born approximations, and it shows a large broadening of the minimum. Also, pointlike disorder at $\nu = \frac{1}{3}$ [9] has been treated numerically in a finite-size system, and the transport gap is found to be suppressed.

It is the purpose of this Letter to use the Hamiltonian theory [10] of composite fermions to develop an approximate approach which can treat long-range dopant disorder for arbitrary fractions in the bulk at nonzero temperatures. We start with Jain's composite fermion (CF) picture [6], in which electrons are bound to $2s$ units of statistical flux to form CFs. The statistical flux cancels part of the external magnetic flux, leading the CFs at the principal fractions $\nu = p/(2sp + 1)$ to see just the right effective field to fill p CF-LLs. Thus, the FQHE of electrons is mapped into the IQHE of CFs [6]. Following the Chern-Simons [11] approaches, Shankar and the present author developed a Hamiltonian theory [10] to describe the dynamics of CFs in the LLL, which we now very briefly describe.

In the LLL, only the electron guiding center coordinate $\mathbf{R}_e$ (with $[R_{ex}, R_{ey}] = -il^2$, where $l = \sqrt{h/eB}$ is the magnetic length), is active, and the interaction Hamiltonian of the projected density $\bar{\rho}(\mathbf{q}) = \sum e^{-i\mathbf{q}\cdot\mathbf{R}_{ej}}$ controls the dynamics. To get to a starting point from which one can apply conventional many-body techniques, we introduce fictitious pseudovortex coordinates $\mathbf{R}_v$ ($[R_{vx}, R_{vy}] = il^2/2\nu$) and combine $\mathbf{R}_e$, $\mathbf{R}_v$ to obtain the coordinates $\mathbf{r}$ and velocity operators $\mathbf{\Pi}$ ($[\Pi_x, \Pi_y] = i(1 - 2\nu)/l^2$) of CFs seeing a reduced field. Now the Hartree-Fock (HF) ground state is just one where the CFs fill the lowest p CF-LLs. One also has to constrain the pseudovector density $\bar{\bar{\chi}}(\mathbf{q}) = \sum e^{-i\mathbf{q}\cdot\mathbf{R}_{vj}}$ to have no fluctuations. While this can be taken into account by a conserving approximation [12,13], we have found that the "preferred density" $\bar{\rho}^p(\mathbf{q}) = \bar{\rho}(\mathbf{q}) - c^2\bar{\bar{\chi}}(\mathbf{q})$, which has the correct charge and dipole moment of the CF, is a convenient shortcut [10]. When used in combination with the CF-HF state this approximation produces semiquantitative agreement with experiment for many quantities [14,15]. In the following, we will use this approximation to compute excitation energies with disorder. Our starting point for the clean system is

$$H = -E_Z S_z + \frac{1}{2}\int \frac{d^2q}{(2\pi)^2}\tilde{v}(q)\bar{\bar{\rho}}^p(\mathbf{q})\bar{\bar{\rho}}^p(-\mathbf{q}), \quad (1)$$

where now we include an implicit sum over spins in the density operator, and E_Z is the Zeeman energy. Applying the HF approximation to this Hamiltonian, one obtains the energies of the CF-LLs ($\sigma = \pm 1$ is the spin index)

$$\epsilon_{n\sigma} = -\frac{E_Z\sigma}{2} + \int \frac{d^2q}{(2\pi)^2}\tilde{v}(q)\sum_m \frac{1 - n_F(m\sigma)}{2}|\bar{\bar{\rho}}^p(\mathbf{q})_{mn}|^2, \quad (2)$$

where $\bar{\bar{\rho}}^p(\mathbf{q})_{mn}$ is the matrix element of the preferred density operator between the CF-LLs m, n. Note that the energy is strongly dependent on the occupations of the CF-LLs via CF-exchange terms.

Now we turn to disorder. Efros [16] pointed out that since a quantum Hall state is incompressible, it cannot screen the disorder from the distant dopants linearly. The 2DEG forms compressible puddles of size s (the distance between the dopants and the 2DEG), with incompressible strips separating them. The electrons feel a self-consistent short-range potential which has the natural scale of $E_c = e^2/\varepsilon l \approx \sqrt{B}$. In contrast, in the compressible $\nu = \frac{1}{2}$ system [11,12], a B-independent disorder width gives good agreement [17] with nuclear relaxation rate data [18].

Inspired by the Efros picture [16], we construct a model which treats disorder in the self-consistent Born approximation (SCBA) and interactions in the HF approximation. Since disorder can mix the different CF-LLs, we allow independent dimensionless coupling coefficients α_{mn} [the average of the square of the matrix elements of the disorder potential (in units of E_c) between CF-LLs m and n]

$$\alpha_{mn} = \int \frac{d^2q}{(2\pi)^2} |\bar{\bar{\rho}}^p(\mathbf{q})_{mn}(q)|^2 \frac{\langle\langle |V_{\rm dis}(q)|^2 \rangle\rangle}{E_c^2 l^2}. \qquad (3)$$

Here $V_{\rm dis}$ is the self-consistent screened disorder potential, and $\langle\langle\rangle\rangle$ denotes ensemble averaging over the disorder. While a microscopic calculation of α_{mn} is beyond the scope of this work, one can make a number of robust statements. (i) α_{mn} are roughly B independent. (ii) α_{mn} generically decreases as m, n increase due to the structure of the matrix elements, with α_{00} being the largest. Physically, since the wave functions of the higher CF-LLs are more extended and have nodes, they "average out" the disorder more. (iii) For the same m, n, different spin species can have somewhat different α, since the majority species also sees an exchange potential. (iv) The typical magnitude of α should correspond to the Γ used in the empirical fits, that is, $E_c\sqrt{\alpha} \simeq 1$ K, implying $\alpha \simeq 10^{-3}$ for typical B. (v) Though there are many free parameters, only $\alpha_{00\uparrow}$ and $\alpha_{00\downarrow}$ are relevant to the physics at $\nu = \frac{1}{3}$, as will be seen below. The rest can all be given the same value.

Combining SCBA and HF we obtain

$$\Sigma_{n\sigma}(\omega) = E_c^2 \sum_m \alpha_{mn} G_{m\sigma}(\omega),$$
$$G_{m\sigma}(\omega) = \frac{1}{\omega - \epsilon_{m\sigma} - \Sigma_{m\sigma}(\omega)}, \qquad (4)$$
$$n_F(m\sigma) = -\int \frac{d\omega}{\pi} \frac{\mathrm{Im}[\Sigma_{m\sigma}(\omega)]}{1 + \exp[-\beta(\omega - \mu)]}.$$

Equations (2) and (4) are iterated to self-consistency at $T \neq 0$, with the global condition of $\frac{1}{3}$ filling being maintained by adjusting μ.

To obtain the transport gaps and other extended excitations, we will assume, in analogy with the IQHE [3], that the CF states are localized except at the "band center" of the disorder-broadened CF-LL. This is plausible, since the motion of a CF now occurs in some (self-consistent) random potential in a set of CF-LLs. There are two natural ways to identify the band center: (i) as the CF-HF energy $\epsilon_{m\sigma}$ or (ii) as the energy at which the density of states of the disorder-broadened CF-LL is the highest. The author has verified that the two choices exhibit no qualitative differences and only very small quantitative ones, and in what follows we will use choice (i). At this level the theory does not treat magnetoexcitons, except in the $q \to \infty$ limit, when they converge to the gaps between the CF-LLs. Note that the structure of the puddles can be complex, involving Wigner crystallites, etc., but since the extended states lie in the incompressible strip, the detailed structure of the puddles is irrelevant for the extended states, which makes the self-consistent Born approximation plausible.

We use the Zhang–Das Sarma potential [19] $v(q) = 2\pi E_c e^{-\lambda q}/(ql)$ with the thickness parameter $\lambda = 0.6l$. This choice makes our CF-HF gap for the clean system reproduce the numerical gap of $0.103E_c$ [5,6] for the pure Coulomb interaction in the LLL. All results we present below are for this choice and $T = 50$ mK.

CF exchange depending on CF-LL occupations is an important contribution to the gaps [Eq. (2)]. The $n = 0$, $\sigma = \uparrow$ CF-LL, which was fully occupied in the clean system, is now partially occupied, as are the rest of the CFs in higher CF-LLs whose densities of states overlap the $n = 0\uparrow$ CF-LL. At self-consistency, the energy of the $n = 0\uparrow$ CF-LL *increases* compared to the clean system, while that of all other CF-LLs *decreases*, leading to a decrease of all the gaps. We define the transport gap as $2 \times \min(\mu - \epsilon_{0,\uparrow}, \epsilon_{0,\downarrow} - \mu, \epsilon_{1,\uparrow} - \mu)$.

Recently, Dethlefsen *et al.* [7] have studied the transport gap at $\frac{1}{3}$ as a function of perpendicular field for two different samples. Using the empirical fit $\Delta_{\rm ex} = \Delta_{\rm th} - \Gamma$ they interpret the measurements for sample A (mobility 4.5×10^6 cm^2/V s) as showing a crossing of the $n = 0\downarrow$ CF-LL with the $n = 1\uparrow$ CF-LL with increasing E_Z, while sample B (mobility 3.5×10^5 cm^2/V s) does not show this. In Fig. 1, we show the experimental results and the results of our approach with a few different sets of parame-

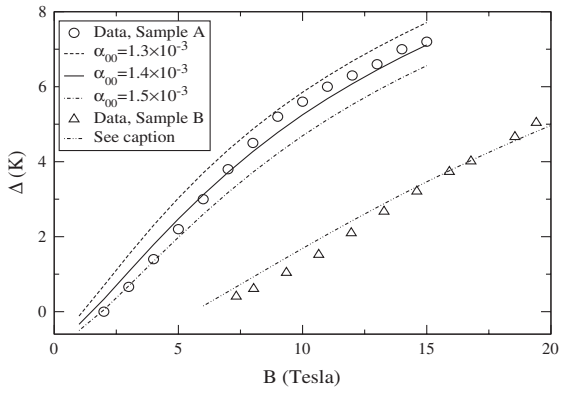

FIG. 1. A comparison of experimental gaps at $\frac{1}{3}$ of Dethlefsen *et al.* [7] as a function of total perpendicular field and the predictions of our approach. For sample A, $\alpha_{mn} = 5 \times 10^{-4}$ and vary α_{00}, while for sample B we use $\alpha_{00\uparrow} = 1.65 \times 10^{-3}$, $\alpha_{00\downarrow} = 2.2 \times 10^{-3}$, $\alpha_{mn} = 5.25 \times 10^{-4}$.

ters, all of which are assumed to be *independent of B*. The agreement is at the few percent level. Theoretically, the system is not fully polarized even at the highest field. The $n = 0 \downarrow$ CF-LL crosses the $n = 1, \uparrow$ CF-LL around 4.5 T in sample *A*. At low fields (below 2 T for sample *A* and below 5 T for sample *B*), the calculated chemical potential lies beneath the band center of the lowest CF-LL, showing the absence of the QHE.

Finally, a caveat about the assumed *B* independence of α_{mn}: as *B* increases, the self-consistent gap increases, the width of the Efros strip [16] also increases as $\sqrt{\Delta}$, and the disorder can be expected to decrease. On the other hand, as *B* increases, the thickness (λ) of the 2DEG in units of l increases (like $B^{1/6}$), which will tend to decrease the gap. However, in the absence of a microscopic theory α_{mn} and λ have been kept constant.

Let us now turn to inelastic light-scattering experiments [20], which can access excitations invisible in transport, such as spin-wave and spin-flip excitations [20] and, potentially, magnetoroton excitations. Recently Groshaus *et al.* [21] have reported measurements in an extremely clean sample (mobility $\approx 7 \times 10^6$ cm^2/V s) in a tilted field. They see two excitations: one which hardly disperses with E_Z, which they identify as the magnetoroton (MR), and another mode which disperses sharply upwards with E_Z, which they consequently identify as a spin-texture (ST) mode with spin 2. The two excitations are observed to have very different *T* dependences: the MR mode strength decreases with increasing *T* while that of the ST mode increases.

In analyzing light-scattering, the disorder is necessary (to couple the photon of $q \approx 0$ to nonzero q excitations) but is usually assumed to be weak, so that the structure of the magnetoexciton dispersions are unchanged. In the weak-disorder case, the extrema of the magnetoexciton dispersions are picked out as the energies with the highest density of states and appear as peaks in the spectra.

However, it is not clear that the quantum Hall system is in the weak-disorder limit. The analysis of Ref. [8] shows that, generically, the disorder-induced width of the magnetoroton minimum should be of the order of its energy ($\approx 0.05 E_c$ for a clean system). Physically, this corresponds to the magnetoexciton energy being strongly modified by its local environment so that it attains a broad range of values. However, peaks seen in light scattering are considerably sharper [20,21].

We will use the gaps between the extended states at the center of the CF-LLs and not the magnetoroton minimum (which, in any case, the theory presented here cannot access) to compare to light-scattering data. Since the incident light has a wavelength ≈ 800 nm $\gg l$ the coherent response to the light comes from regions much bigger than the puddles. Extended states occur at the same energy at every location in the sample. Thus, transitions between extended states produce a sharp feature in light scattering, a view supported by a recent calculation [22]. Figure 2 compares the predicted $n = 0 \uparrow \Rightarrow n = 1, \uparrow$ and $n = 1, \uparrow \Rightarrow n = 0, \downarrow$ CF-LL gaps to the MR and ST modes, respectively.

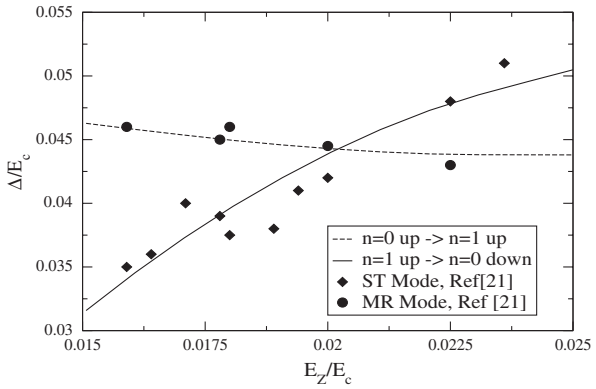

FIG. 2. A comparison of the light-scattering data of Groshaus *et al.* [21] to theoretical results for the $n = 0 \uparrow \Rightarrow n = 1, \uparrow$ and $n = 1, \uparrow \Rightarrow n = 0, \downarrow$ CF-LL gaps. The theoretical results are for $\alpha_{00\uparrow} = 1.2 \times 10^{-3}$, $\alpha_{00\downarrow} = 1.1 \times 10^{-3}$, and other $\alpha_{mn} = 6.5 \times 10^{-4}$.

Our interpretation makes it natural that, as observed [21], the strength of the ST mode should increase as *T* increases, while that of the MR mode should decrease, due to the *T*-dependent factors of $n_F(i)[1 - n_F(f)]$ in the transition amplitude. The slight reduction in energy of the MR mode seen in the data [21] is also reproduced theoretically and is a CF-exchange effect.

In summary, we have presented an approach which allows the phenomenological treatment of disorder in the bulk FQHE at any *T*. The key ingredients are the Hamiltonian theory [10] of composite fermions [6] and the disorder-averaged self-consistent Born approximation for treating the disorder, modeled by the Efros picture [16] of puddles of size *s* (the distance between the 2DEG and the dopant layer) separated by incompressible strips of the typical size of a few magnetic lengths. A maximum of three parameters $\alpha_{00\uparrow}$, $\alpha_{00\downarrow}$, and the other α_{mn} are sufficient to quantitatively fit the data over a broad range of fields.

We find that the observed reduction of the gap at $\frac{1}{3}$ is the result of an interplay between the disorder broadening of the CF-LLs and the strong occupation dependence of the CF-LL energies (CF exchange). The interplay of disorder and interactions produces strong Zeeman dependences of transport and other gaps which can easily be mistaken for Skyrmions [23] or other spin structures. As in previous work by the present author at $\nu = 1$ [24], the observed large slope of the transport gap [25] is consistent with an exchange-enhanced disorder effect. This explanation is not new [26] and is worthy of reexamination in light of recent nuclear magnetic resonance (NMR) results, which indicate that Skyrmions are actually localized [27] at and around $\nu = 1$ at $T = 0$. It is also consistent with numerical work [28] showing that localized spin textures occur in the ground state for realistic disorder strength and with NMR

data around $\nu = \frac{1}{3}$ which show no signatures of Skyrmions [29].

A key assumption here is that the CF extended state is at the disorder-averaged HF energy of that CF-LL. While it is plausible that there is one and only one extended state per CF-LL at $T = 0$, there is no microscopic understanding of where it should lie. Another whole class of quite mysterious data on the compressibility [30] of the FQH states exists to which the methods developed here are applicable. Finally, the theory needs to be developed to the point where the α parameters are calculable from first principles, even if approximately.

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