Dark Matter, Constrained Minimal Supersymmetric Standard Model, and Lattice QCD

Joel Giedt,[1] Anthony W. Thomas,[2,3] and Ross D. Young[4]

[1]*Department of Physics, Applied Physics, and Astronomy, Rensselaer Polytechnic Institute, 110 Eighth Street, Troy, New York 12180-3590, USA*

[2]*Jefferson Lab, 12000 Jefferson Avenue, Newport News, Virginia 23606, USA*

[3]*Special Research Centre for the Subatomic Structure of Matter, School of Chemistry and Physics, University of Adelaide, Adelaide SA 5005 Australia*

[4]*Physics Division, Argonne National Laboratory, Argonne, Illinois 60439, USA*

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Recent lattice measurements have given accurate estimates of the quark condensates in the proton. We use these results to significantly improve the dark matter predictions in benchmark models within the constrained minimal supersymmetric standard model. The predicted spin-independent cross sections are at least an order of magnitude smaller than previously suggested and our results have significant consequences for dark matter searches.

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The astronomical evidence for dark matter [1,2] presents modern physics with some of its greatest challenges. We need to find ways to detect it directly and a number of very sophisticated searches are underway [3–5], with at times confusing results [6,7]. To guide those searches we need theoretical models for what the dark matter might be and what cross section one might expect it to have for scattering from hadronic matter. The constrained minimal supersymmetric standard model (CMSSM—see [8] for a review) has the advantage of being consistent with modern nuclear and particle physics while incorporating the compelling concepts of supersymmetry, coupling unification, and viable cold dark matter [9]. Benchmark CMSSM models have over time been tuned to ensure that they are consistent with constraints on relic abundance [10–13], with the favored dark matter candidate being a neutralino.

Extensive studies of the spin-independent interaction of neutralinos with hadronic matter have established that it is determined by their coupling to the quark sigma commutators ($(m_u + m_d)\langle \bar{u}u + \bar{d}d\rangle/2$ and $m_s\langle \bar{s}s\rangle$); see, e.g., [12,13]. Given the favored values 50 [14] and 300 MeV [15] of these quantities, the cross section has been dominated by the strange-quark content of the nucleon. Realizing the importance of these quantities, Ellis *et al.* recently made a plea for more accurate experimental data [12] (see also [16]). The answer to their plea has recently been provided from an unexpected source, through the lattice study of octet baryon masses as a function of quark mass. Indeed, a sophisticated dynamical 2 + 1 flavor chiral analysis of recent lattice measurements has yielded surprisingly accurate estimates [17], with subsequent consistent results [18]. We note also a recent dynamical 2 flavor lattice measurement of σ_s by Ohki *et al.* [19]. Although it agrees with [17], we will not use it here because of unknown systematic errors of quenching the strange quark.

In this Letter, we use the precise new values of the light and strange-quark sigma commutators [17,18] to update the cross sections predicted within the benchmark CMSSM models that have been studied for several years by Ellis *et al.* [12] and collaborators [10,11]. We show that the sizeable reduction in the strange sigma term from the values previously favored leads to a rather dramatic reduction in the expected neutralino cross sections, with important implications for the interpretation of experiments to search for dark matter.

Sigma terms from lattice QCD.—These are given by the scalar form factors evaluated at $t = 0$, denoted by $\sigma_q = m_q\langle N|\bar{q}q|N\rangle$. These are difficult to directly probe in experiment. The light-quark sigma term, $\sigma_\ell = (m_u + m_d) \times \langle \bar{u}u + \bar{d}d\rangle/2$, has most reliably been accessed by invoking a chiral low-energy relation between $\pi - N$ scattering and the scalar form factor [14,20,21], $\Sigma_{\pi N} \equiv \sigma_\ell = \Sigma_{\pi N}^{\rm CD} - \Delta_R - \Delta_\sigma$. The remainder term, Δ_R, describes a correction to the low-energy theorem and is estimated to be less than 2 MeV [21,22]. The shift in the scalar form factor can be inferred from a dispersion analysis and found to be rather large [23], $\Delta_\sigma \equiv \sigma_\ell(2m_\pi^2) - \sigma_\ell = 15.4 \pm 0.4$ MeV. $\Sigma_{\pi N}^{\rm CD}$ is the Born-subtracted, isoscalar πN scattering amplitude evaluated at the (unphysical) Cheng-Dashen point. An early experimental extraction [24] gave $\Sigma_{\pi N}^{\rm CD} = 64 \pm 8$ MeV, to be compared with a more recent determination $\Sigma_{\pi N}^{\rm CD} = 79 \pm 7$ MeV [25]. These two values lead to light-quark sigma terms of $\sigma_\ell = 45 \pm 8$ MeV and 64 ± 7 MeV, respectively. These are to be compared with the recent lattice QCD determination $\sigma_\ell = 47 \pm 9$ MeV [17]. While this lattice analysis tends to favor the lower value, it is not inconsistent with the higher extraction. We use the lattice determination in the current analysis.

Extracting the strangeness sigma term is significantly more challenging, because the same prescription would lead to both a poorly converged low-energy relation and a large extrapolation of $K - N$ scattering to the Cheng-Dashen point—see Reya [26]. A much more practical approach has been to resolve the patterns of SU(3) breaking among the baryon octet [15,27,28]. In essence, the baryon masses give guidance with respect to the symmetry breaking component $\sigma_0 = m_\ell\langle N|\bar{u}u + \bar{d}d - 2\bar{s}s|N\rangle$, with $m_\ell = (m_u + m_d)/2$. The commonly reported value $\sigma_0 = 36 \pm 7$ MeV is based on the phenomenological analysis of corrections to the linear mass-splitting relations [27], with further uncertainties from yet higher order quantified by [28]. The difference between the estimated σ_0 and the extracted σ_ℓ then gives a best estimate for the strange-quark sigma term, as related by

$$\sigma_s/\sigma_\ell = m_s(\Sigma_{\pi N} - \sigma_0)/(2m_\ell\Sigma_{\pi N}). \qquad (1)$$

In contrast, lattice QCD allows one to study the quark mass dependence of the baryon masses from first principles. New lattice results, based on an SU(3) baryon mass analysis [17] and a novel application of the Feynman-Hellman theorem at the correlator level [18], find $\sigma_s = 31 \pm 15$ MeV and 59 ± 10 MeV, respectively. An important feature of these calculations is the demonstrated internal consistency, where both studies have successfully shown the ability to predict the mass splittings between simulations with different strange-quark mass parameters. We argue that these results show a vast improvement over the earlier phenomenological analyses. This, in turn, implies that it is appropriate to update the earlier estimates of dark matter cross sections. In the present analysis, we assume that the reported uncertainties in the lattice QCD determinations of the strange-quark sigma term are independent and use a naïve average of the two results, $\sigma_s = 50 \pm 8$ MeV. We report the results of our analysis at the 95% confidence level to ensure that the conclusions are minimally sensitive to our input. There could be some degree of correlation between [17,18] stemming from some of the same underlying MILC gauge configurations. We suggest that these correlations are small because of the differences in both the lattice and analysis techniques. We also note that the difference between the two determinations is more than 1 standard deviation. This is not unusual, but in the extreme case that one were concerned about a possible inconsistency we note that applying the PDG technique for such a case [29] would only increase the error on the combined result from 8 to 13 MeV. This would lead to no significant change in our conclusions.

Constrained MSSM.—The CMSSM is inspired by supergravity mediated scenarios of spontaneous supersymmetry breaking, in the context of supersymmetric grand unified theories [8]. Then one has a universal soft scalar mass m_0, a universal gaugino mass $m_{1/2}$, a universal trilinear scalar coupling A_0 (set to zero in the models considered here), and a higgsino mass μ. In practice one trades the ratio of vacuum expectation values $\tan\beta = v_u/v_d$ of the Higgs fields H_u and H_d at the scale m_Z for μ; the sign of μ is also a parameter, and unification of gauge couplings is imposed. The benchmark models "*A*-*M*" of [10,11] were selected to be representative of the regions of parameter space that yield neutralino dark matter with the correct abundance, consistent with WMAP constraints.

Because of the incredible accuracy of the WMAP results for relic cold dark matter abundance, a fine tuning of high scale parameters is typically required in order to have the lightest supersymmetric particle (LSP) fit this constraint. However, the dark matter cross section itself only changes by a few percent when this tuning is done.

Cross sections for benchmark models.—We have computed the spin-independent cross section for neutralino dark matter for benchmark models *A*-*M* of [10,11]. Three of the benchmark models were also studied in [12], where the reader may find the cross section formulas that we likewise use. We have used SOFTSUSY [30] to compute the running parameters between the grand unified scale and the electroweak scale. Minor modifications were necessary in two benchmark models, because of the slight differences between renormalization group codes. These were made to avoid a stau ($\tilde{\tau}$) LSP for models that are on the edge of the $\tilde{\tau}$ exclusion region of CMSSM. In model *J* we changed m_0 from 285 to 290 GeV. In model *L* we changed m_0 from 300 to 315 GeV.

Our results for the spin-independent cross section with the proton are shown in Figs. 1–3. In all cases we see that there is only a mild dependence on $\Sigma_{\pi N}$. As can be seen in the last column of Table I, the cross section is dominated by

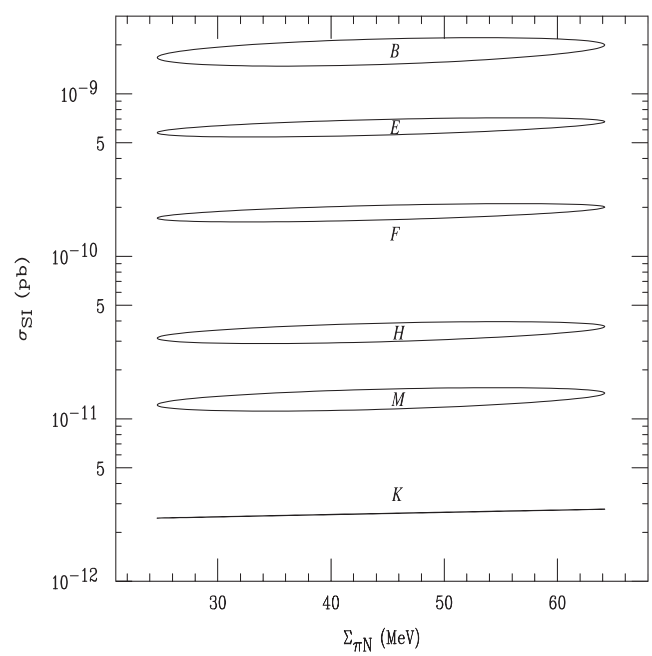

FIG. 1. Proton, benchmark models (labeled by letters *A, B, ...*), 95% C.L. using lattice inputs for sigma commutators. Remaining models are shown in other figures below, since they would overlap with these. The neutron predictions are not shown because they are basically degenerate with the proton cross sections on the logarithmic scale that is shown.

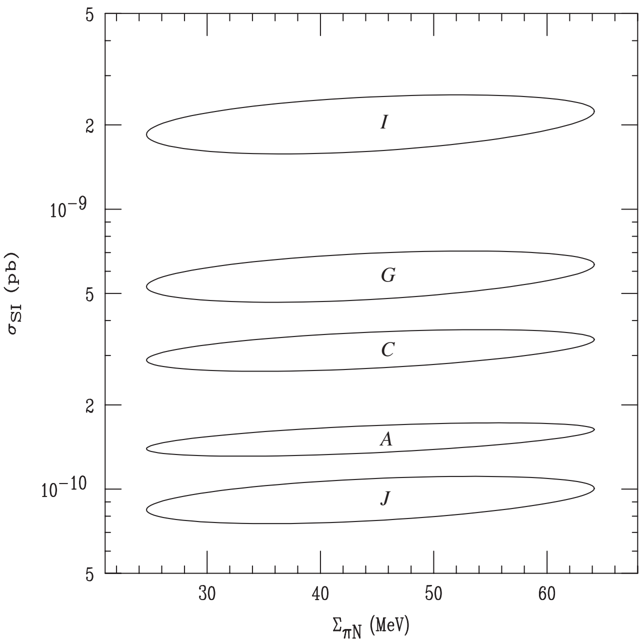

FIG. 2. Cross section estimates (95% CL) for benchmark models A, C, G, I, J.

heavier quarks (e.g., less than 15% from u, d). In the approach of [12], which uses Eq. (1), varying $\Sigma_{\pi N}$ leads to large variation in σ_s. Because of Eq. (2), f_{TG} also varies significantly, which leads to a large variation in the c, b, t quark contribution. By contrast, using the lattice data for σ_s makes it independent of $\Sigma_{\pi N}$, so that σ_s only varies within the theoretical error. That variation is much smaller than what comes out of Eq. (1), and hence we get a cross section that varies by a much smaller amount. In summary, $\Sigma_{\pi N}$ has little impact since it is essentially independent of σ_s in our approach, and it only affects the u, d quark contribution, which is of order 15%. Furthermore, most models have rather small cross sections, $\sigma_{SI} < 10^{-9}$ pb. We do not show model L because it is essentially degenerate with model I. Nor do we show model D because it has such small cross sections ($\sigma_{SI} < 10^{-14}$ pb) that it is unobservable in the foreseeable future. The cross sections for the neutron are essentially degenerate with those of the proton. To contrast with previous estimates, in Fig. 4 we compare σ_{SI} for model C using σ_ℓ and σ_s extracted from lattice QCD with the determination of σ_s inferred from the phenomenological estimate for $\sigma_0 = 36$ MeV, using Eq. (1) above. Not only is the cross section calculated here far more weakly dependent on $\Sigma_{\pi N}$, but it is also typically much smaller.

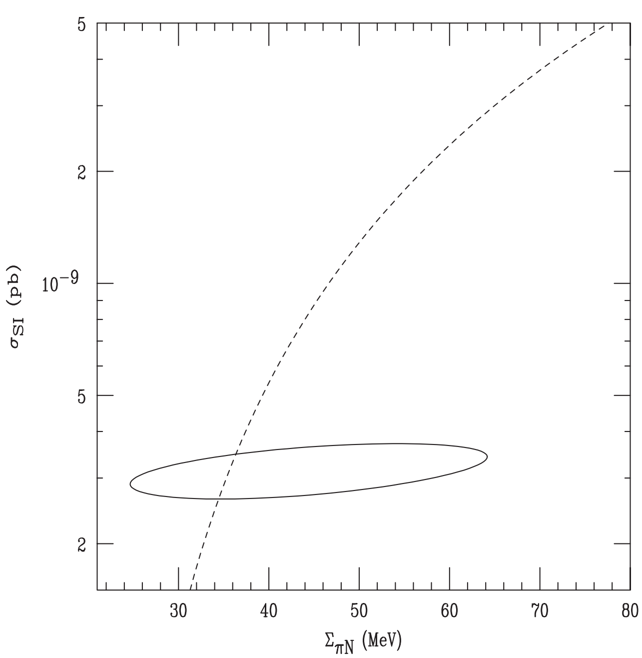

FIG. 3. A comparison of our results (solid ellipse) for model C, versus the traditional approach (dashed line) which relates the strange-quark sigma commutator to the light-quark one through Eq. (1) with $\sigma_0 = 36$ MeV.

Details of quark flavor contributions.—It is interesting to investigate the breakdown of how each quark flavor contributes to the overall cross section. First, we note that for the proton $\sigma_{SI} \propto |f_p|^2$, where

$$\frac{f_p}{m_p} = \sum_{q=u,d,s} \frac{\alpha_{3q}}{m_q} f^p_{Tq} + \frac{2}{27} f^p_{TG} \sum_{q=c,b,t} \frac{\alpha_{3q}}{m_q}, \qquad f^p_{TG} = 1 - \sum_{q=u,d,s} f^p_{Tq}. \tag{2}$$

The coefficients α_{3q} are determined by CMSSM diagrams and mixing coefficients of the LSP neutralino. The dimensionless sigma commutators $f^p_{Tq} = \sigma_q/m_p$ are taken from the lattice results whereas f^p_{TG} addresses the heavy flavor sigma commutators through SVZ relations (see the review [9]). We define the contribution of each quark flavor through $f_p = \sum_q f^p_q$. For instance, the bottom quark contribution $f^p_b = (2/27) m_p f^p_{TG} \alpha_{3b}/m_b$ turns out to be rather large. Its increased importance in our computations can be traced to the fact that the lattice results give a smaller result for f^p_{Ts}, enhancing f^p_{TG}. Results are given in Table I for the point with maximum σ_{SI}, which occurs in model L.

Conclusions.—We have shown that recent lattice results [17,18] have a dramatic effect on predictions for direct

TABLE I. Example breakdown of quark flavor contributions to σ_{SI}. This is for model L at the maximum value of its cross section, within our 95% CL region.

model	q	α_{3q}/m_q	f^p_{Tq} or f^p_{TG}	f^p_q/f_p
L (maxσ_{SI})	u	-1.019×10^{-09}	0.0280	0.0105
$\sigma_{SI} =$	d	-1.302×10^{-08}	$\cdots$	0.1342
2.8×10^{-9} pb	c	-1.031×10^{-09}	0.8751	0.0261
	s	-1.522×10^{-08}	0.0689	0.3633
$\sigma_\ell = 52.5$ MeV,	t	-1.936×10^{-09}	0.8751	0.0462
$\sigma_s = 64.6$ MeV	b	-1.670×10^{-08}	$\cdots$	0.3984

detection of neutralino cold dark matter. Furthermore, the theoretical uncertainty is considerably smaller than in the traditional approach. Unfortunately, the most important effect of the improved estimates is the reduction of the strange-quark sigma commutator, which leads to rather small dark matter cross sections. On the other hand, even within the 95% confidence intervals, our estimates come with small errors; hence any observation of dark matter would be highly selective amongst the benchmark models that we have considered. In future work we hope to report on dark matter cross section predictions in models other than the CMSSM, such as the nonuniversal Higgs boson mass (NUHM) models, which are known [13] to allow for larger cross sections.

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