Reversed Cherenkov-Transition Radiation by a Charge Crossing a Left-Handed Medium Boundary

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We analyze the radiation from a charged particle crossing the boundary between an ordinary medium and a ‘‘left-handed’’ metamaterial. We obtain exact and approximate expressions for the field components and develop algorithms for their computation. The spatial radiation in this system can be separated into three distinct components, corresponding to ordinary transition radiation having a relatively large magnitude, Cherenkov radiation, and reversed Cherenkov-transition radiation (RCTR). The last one is explained by reflection and refraction of reversed Cherenkov radiation at the interface. Conditions for generating of RCTR are obtained. We note properties of this radiation that have potential applications in the detection of charged particles and accelerator beams and for the characterization of metamaterial macroscopic parameters (ε, μ).

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In the 1960s, Veselago introduced the concept of ‘‘left-handed media’’ (LHM), i.e., media having simultaneously negative permittivity and permeability [1,2]. In LHM, the electric field vector, magnetic field vector, and wave vector form a left-handed orthogonal set. The direction of the energy flow and the direction of the phase velocity are opposite in LHM, resulting in very unusual properties of electromagnetic waves propagating in these media.

Note that the ‘‘left-handed’’ properties can be realized only in a limited frequency range [1,2]. Therefore, it would be more correct to refer to a ‘‘left-handed frequency band’’ (LHFB) as opposed to a ‘‘right-handed frequency band’’ (RHFB) where the familiar properties of the medium occur. However, the term LHM is widespread now in the scientific literature, and we will use it as well, with the understanding that a LHM is a medium possessing a LHFB.

Artificial materials possessing left-handed properties in the gigahertz frequency band have been demonstrated recently (see, for example, [3–6]). These metamaterials (MTMs) are composed of discrete conducting elements having their size and spacing much smaller than the wavelengths of interest. Therefore, such media can be described by the macroscopic parameters $\varepsilon(\omega)$ and $\mu(\omega)$.

Radiation from a charge traversing the interface between two media is one of the principal problems of electrodynamics. In the case of ordinary (‘‘right-handed’’) media (RHM), this question was investigated as early as 1946 [7,8]. The case of an interface between RHM and LHM was partially discussed in [9,10]. However, a quantitative investigation of the field was not performed.

Cherenkov radiation (CR) in LHM has been investigated in more detail [11–14]. In particular, it was shown that the moving particle generates both ordinary (forward) and reversed (backward) CR [11].

We analyze the electromagnetic field generated by a small bunch with a charge q passing through the interface (located at $z = 0$) separating two homogeneous isotropic frequency dispersive media described by permittivity and permeability: $\varepsilon_1(\omega)$, $\mu_1(\omega)$ for $z < 0$ and $\varepsilon_2(\omega)$, $\mu_2(\omega)$ for $z > 0$ (Fig. 1). There are no surface charges and currents located at $z = 0$. The bunch moves uniformly along the z axis in accordance with $z = Vt = c\beta t$. The dimensions of the bunch are assumed to be negligible. Therefore, the charge density ρ and the current density $\vec{j} = j\vec{e}_z$ can be written in the form

$$\rho = q\delta(x)\delta(y)\delta(z - Vt), \qquad j = V\rho. \tag{1}$$

Further, we will assume that both media possess nonzero losses, resulting in small positive values of $\mathrm{Im}\varepsilon_{1,2} > 0$ and $\mathrm{Im}\mu_{1,2} > 0$. We will let these terms go to 0 in final results. The medium filling the volume $z < 0$ is right-handed, that is, $\mathrm{Re}\varepsilon_1 > 0$, $\mathrm{Re}\mu_1 > 0$ for all frequencies where $\mathrm{Re}\varepsilon_1 \mathrm{Re}\mu_1 > 0$. The medium filling the region $z > 0$ is supposed to have both RHFB and LHFB for propagating waves: (I) RHFB where $\mathrm{Re}\varepsilon_1 > 0$, $\mathrm{Re}\mu_1 > 0$ and (II) LHFB where $\mathrm{Re}\varepsilon_1 < 0$, $\mathrm{Re}\mu_1 < 0$. The conditions of continuity of tangential components of electric ($\vec{E}$) and magnetic ($\vec{H}$) strengths must be satisfied in the plane $z = 0$.

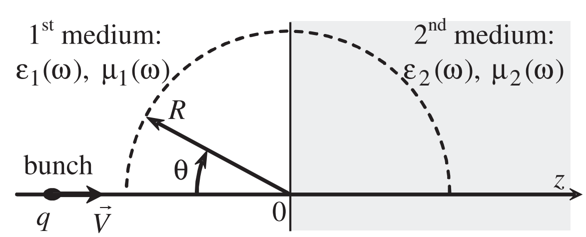

FIG. 1. Geometry of the problem.

It is easy to show that the general solution of the problem can be written in the same form as in the case of two RHMs [7,8]. If the index 1 refers to the area $z < 0$ and the index 2 refers to the area $z > 0$, then we can write

$$\begin{Bmatrix} \vec{A}_{1,2} \\ \Phi_{1,2} \end{Bmatrix} = \int_{-\infty}^{\infty} \begin{Bmatrix} \vec{e}_z A_{\omega 1,2} \\ \Phi_{\omega 1,2} \end{Bmatrix} e^{-i\omega t} d\omega, \quad (2)$$

$$\begin{Bmatrix} A_{\omega 1,2} \\ \Phi_{\omega 1,2} \end{Bmatrix} = \begin{Bmatrix} A^b_{\omega 1,2} \\ \Phi^b_{\omega 1,2} \end{Bmatrix} + \begin{Bmatrix} A^q_{\omega 1,2} \\ \Phi^q_{\omega 1,2} \end{Bmatrix}, \quad (3)$$

$$\begin{Bmatrix} A^q_{\omega 1,2} \\ \Phi^q_{\omega 1,2} \end{Bmatrix} = \frac{iq}{2} \begin{Bmatrix} c^{-1}\mu_{1,2}(\omega) \\ [\beta c \varepsilon_{1,2}(\omega)]^{-1} \end{Bmatrix} e^{i(\omega/\beta c)z} H_0^{(1)}(\rho s_{1,2}), \quad (4)$$

$$\begin{Bmatrix} A^b_{\omega 1,2} \\ \Phi^b_{\omega 1,2} \end{Bmatrix} = \frac{q}{\pi \beta c} \int_0^{\infty} \begin{Bmatrix} A^b_{\omega k_\rho 1,2} \\ \Phi^b_{\omega k_\rho 1,2} \end{Bmatrix} e^{ik_{z1,2}|z|} k_\rho dk_\rho, \quad (5)$$

$$\begin{Bmatrix} A^b_{\omega k_\rho 1,2} \\ \Phi^b_{\omega k_\rho 1,2} \end{Bmatrix} = -\begin{Bmatrix} \mu_{1,2}\mathrm{sgn}(z) k_{z1,2}^{-1} \\ c\omega^{-1}\varepsilon_{1,2}^{-1} \end{Bmatrix} J_0(\rho k_\rho) B_{1,2}, \quad (6)$$

$$B_{1,2} = \frac{k_{z1,2}}{g_3}\left(\frac{\frac{\omega \varepsilon_{2,1}}{c} \mp \beta \varepsilon_{1,2} k_{z2,1}}{g_{1,2}} - \frac{\beta^2 \varepsilon_{1,2}}{\frac{\omega}{c} \pm \beta k_{z2,1}} \right), \quad (7)$$

where $H_0^{(1)}(\xi)$ is the Hankel function, $J_0(\xi)$ is the Bessel function, $\rho = \sqrt{x^2 + y^2}$, $s_{1,2}(\omega) = \sqrt{\frac{\omega^2}{\beta^2 c^2}(n_{1,2}^2 \beta^2 - 1)}$, $k_{z1,2} = \sqrt{\frac{\omega^2 n_{1,2}^2}{c^2} - k_\rho^2}$, $n_{1,2}^2 = \varepsilon_{1,2}\mu_{1,2}$, $g_{1,2} = k_\rho^2 - s_{1,2}^2$, $g_3 = \varepsilon_1 k_{z2} + \varepsilon_2 k_{z1}$, and the functions $s_{1,2}$, $k_{z1,2}$ are determined according to the rules

$$\mathrm{Im}\, s_{1,2} > 0, \qquad \mathrm{Im} k_{z1,2} > 0. \quad (8)$$

Fourier harmonics of the fields are determined by the formulas $\vec{E}_\omega = -\nabla \Phi_\omega + i\omega c^{-1} \vec{e}_z A_\omega$, $\vec{B}_\omega = \mathrm{rot}(\vec{e}_z A_\omega)$. The potentials $A^q_{1,2}$, $\Phi^q_{1,2}$ describe the so-called "forced" field or the charge field in the corresponding unbounded medium [8]. These potentials describe CR if the charge velocity exceeds the Cherenkov threshold. The potentials $A^b_{1,2}$, $\Phi^b_{1,2}$ describe the so-called "free" field arising from the discontinuity in the medium properties in the plane $z = 0$. The conditions (8) mean that the Fourier component of the forced field must exponentially decay with increasing distance ρ from the charge trajectory and that the Fourier component of the free field must exponentially decrease with increasing distance $|z|$ from the boundary.

In the region $z < 0$ (RHM), we have $\mathrm{Re} s_1 > 0$, $\mathrm{Re} k_{z1} > 0$ for all frequencies. In the region $z > 0$ (LHM), one obtains that $\mathrm{Re} s_2 > 0$, $\mathrm{Re} k_{z2} > 0$ for the RHFB but $\mathrm{Re} s_2 < 0$, $\mathrm{Re} k_{z2} < 0$ for the LHFB. The same results can be obtained for lossless media if the so-called Mandelshtam condition is used. According to this condition, the group velocity (and the energy flow density) of the forced field and free field must be directed away from the charge trajectory and away from the boundary, respectively. Since the group velocity is parallel to the phase velocity for RHFB and antiparallel for LHFB, we arrive at the aforementioned conclusions.

Two methods were used to investigate integrals (5)–(7), and both of them extensively use the techniques of complex function theory. Note that we had previously developed such an approach for analysis of CR in passive and active RHM [15–18]. In the first method, we used the steepest descent technique and obtained asymptotic expressions for (5) that are valid for $\omega c^{-1}|n_{1,2}|R \gg 1$, where $R = \sqrt{\rho^2 + z^2}$. The most interesting and novel effects are connected with the saddle point contribution [determining transition radiation (TR)] and especially with the contribution of one of the poles [indicated below as reversed Cherenkov-transition radiation (RCTR)]. Note that the asymptotic expression obtained is valid regardless of the distance between the saddle point and this pole in the complex plane (it is uniform).

In the second method, we produced an effective algorithm for numerical computation of integral (5) using a transformation of the integration path in the complex plane. The results of both methods were in a good agreement in the domain of their asymptotic validity. The results presented below were obtained using the second method.

The typical behavior of the Fourier harmonics of the full field ("forced" + "free") on the semicircle $R = \mathrm{const}$ in the xz plane (dashed line in Fig. 1) is shown in Figs. 2–4. The angle θ increases from 0 (negative part of the z axis) to π (positive part). The half-space $z < 0$ is assumed to be vacuum. We compare two cases: the frequency under consideration is in the RHFB for the second medium (case 1) or in the LHFB (case 2).

The situation where the charge velocity does not exceed the Cherenkov threshold for the medium ($\beta < \beta_{\mathrm{CR}} = |n_2|^{-1}$) is presented in Fig. 2. One can see that, as a rule, TR in vacuum in the case of an electron crossing a vacuum-LHM boundary is much larger than in the case of a vacuum-RHM transition.

The situation where the charge velocity exceeds the Cherenkov threshold is shown in Figs. 3 and 4. In the ordinary case of the vacuum-RHM interface, the Fourier harmonic of the fields is relatively small both in the whole vacuum half-space and in the medium outside of the Cherenkov cone ($\theta < 130°$ in Fig. 3). In the case of vacuum-LHM interface, the Fourier harmonic is of the same order of magnitude all over the whole half-space $z > 0$ (Fig. 4 for $\beta > 0.7$). The important finding here is that a significant electromagnetic field can be observed in vacuum for a certain range of charge velocities. This effect is explained by the reversed CR in the LHM: the energy flux density of CR forms an obtuse angle with the direction of the charge motion (Fig. 5, top). Therefore, CR falls on the boundary and produces both the reflected and transmitted fields. We call these fields the reversed Cherenkov-transition radiation because they possess certain features of both CR and TR. Actually, RCTR is generated during the whole time the charge moves in the medium, similar to

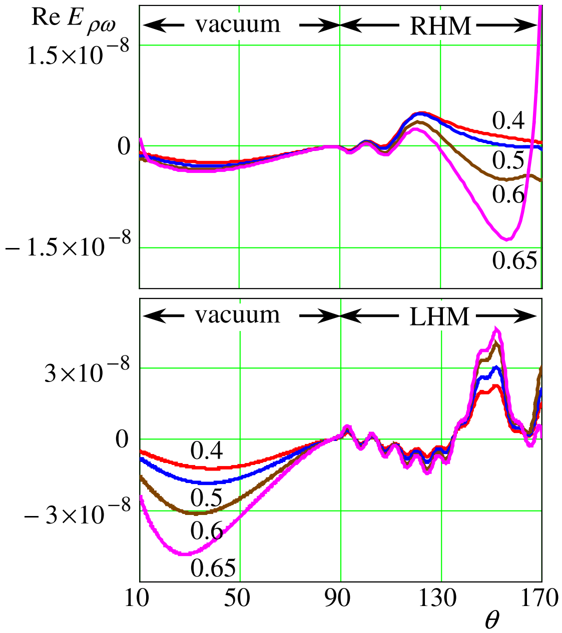

FIG. 2 (color online). Dependence of the electric field Fourier harmonic Re$E_{\rho\omega}$ (V m^{-1} s) on the angle θ for a charged particle crossing the vacuum-RHM (top) and the vacuum-LHM (bottom) boundary for velocities below the Cherenkov threshold $\beta < \beta_{CR} = 0.7$; $\varepsilon = 1.7$, $\mu = 1.2$ for RHM, $\varepsilon = -1.7$, $\mu = -1.2$ for LHM; $q = -1$ nC, $\nu = 10$ GHz, and $R = 15$ cm. The values of β are indicated near the curves.

CR but in contrast to the TR generated in a small part of the charge trajectory close to the boundary (the formation length [8]). On the other hand, the necessary condition for RCTR excitation is the presence of the interface, similar to TR. Another essential difference between CR and RCTR is that the RCTR in vacuum consists of waves propagating both away from the z axis and towards it. The fact is that the law of refraction at the LHM-RHM interface is an unusual one: the power flux density of the transmitted wave has opposite tangential projection with respect to the incident wave [2]. Therefore, in the vacuum half-space we obtain interference of the waves transmitted

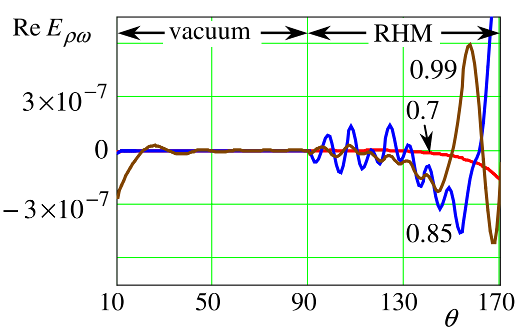

FIG. 3 (color online). Dependence of the electric field Fourier harmonic Re$E_{\rho\omega}$ (V m^{-1} s) on the angle θ for vacuum-RHM transition and for $\beta > \beta_{CR} = 0.7$. Other parameters are the same as in Fig. 2. The values of β are indicated near the curves.

from different parts of the boundary. The reflected wave of RCTR interferes with reversed CR in the area outside of some cone (Fig. 5, bottom). One can see evident interference effects at angles $\theta < 50°$ for $\beta = 0.85$ and at angles $90° < \theta < 130°$ for $\beta = 0.99$ in Fig. 4.

Analytically, the RCTR is the contribution of a pole giving a cylindrical wave. This contribution appears when transformation of the initial integration path to the steepest descent path is accompanied by the intersection of this pole. Our analysis gives the following condition for the presence of RCTR in the vacuum area: $\beta_{CR} < \beta < \beta_{TIR}$,

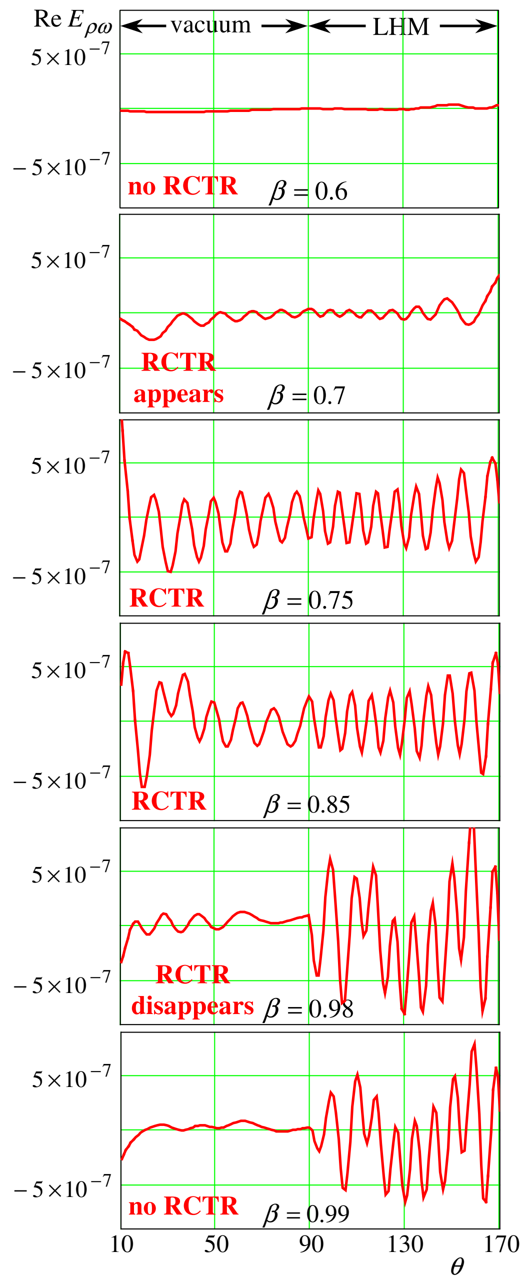

FIG. 4 (color online). Modification of the spatial distribution of the electric field Fourier harmonic Re$E_{\rho\omega}$ (V m^{-1} s) with an increase in β for a vacuum-LHM interface. Parameters are the same as in Fig. 2; $\beta_{CR} = 0.7$, $\beta_{TIR} = 0.98$.

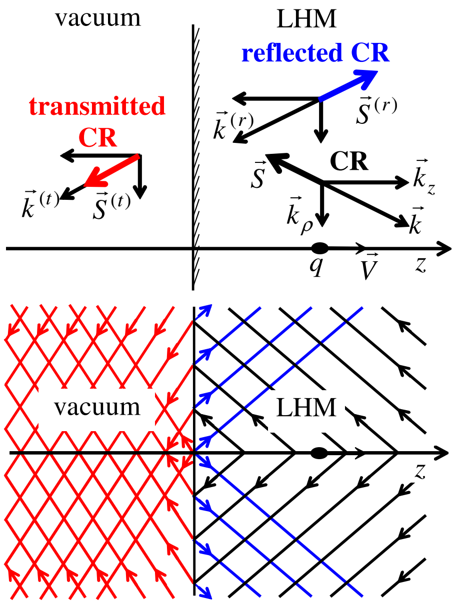

FIG. 5 (color online). Top: wave vectors and power flux densities of CR and RCTR (reflected and transmitted waves). Bottom: the lines are parallel to the power flux density $\vec{S}$ of CR, the reflected wave, and the transmitted wave; one can see areas of interference.

with $\beta_{\mathrm{TIR}} = (n_2^2 - n_1^2)^{-1/2}$. The lower threshold of RCTR is the Cherenkov threshold for the medium. The upper threshold is explained by the total internal reflection. It should be noticed that the upper threshold β_{TIR} is essential for $n_2^2 > 1 + n_1^2$ only (otherwise $\beta_{\mathrm{TIR}} > 1$). The condition for the existence of RCTR in the medium is $\beta > \beta_{\mathrm{CR}}$.

In conclusion, we have shown that three types of spatial radiation can be generated in the case of an interface between vacuum and LHM. The first type of radiation is transition radiation which is generated for any properties of the medium. For LHM, TR has, as a rule, relatively large magnitude. The second type of radiation is Cherenkov radiation generated in the medium for charge velocities exceeding the Cherenkov threshold. The third type of radiation is reversed Cherenkov-transition radiation, generated in the case when the medium is left handed. Generation of RCTR takes place only if the charge velocity lies between certain lower and upper thresholds. Note that we consider the RHM-isotropic LHM interface in this Letter. It should be noted that the RCTR effect can, in principle, occur in the case of an interface between an isotropic and a specific anisotropic medium because reversed CR is possible in anisotropic media as well [19].

The properties of the RCTR offer the prospect of new capabilities for detection of charged particles. They allow the design of a detector for charged particles having two energy thresholds. This system would be sensitive, for example, to particles with energies lying within the energy range defined by the two thresholds. Note that another active area of research is the determination of the frequency dependent bulk permittivity and permeability of metamaterials from scattering measurements [6,20]. The spectrum of RCTR generated by a monoenergetic electron bunch provides independent information on the macroscopic properties (ε, μ) of the LHM structure that can be used for characterization of the LHM.

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