

Chirped Biphotons and their Compression in Optical Fibers

G. Brida,¹ M. V. Chekhova,^{2,1} I. P. Degiovanni,¹ M. Genovese,¹ G. Kh. Kitaeva,² A. Meda,¹ and O. A. Shumilkina²

¹*Istituto Nazionale di Ricerca Metrologica, Strada delle Cacce 91, 10135 Torino, Italy*

²*Department of Physics, M. V. Lomonosov Moscow State University, Leninskie Gory, 119992 Moscow, Russia*

(Received 20 June 2009; published 5 November 2009)

We show that broadband biphoton wave packets produced via spontaneous parametric down-conversion in crystals with linearly aperiodic poling can be easily compressed in time using the effect of group-velocity dispersion in optical fibers. This result could foster important developments in quantum metrology and lithography.

DOI: 10.1103/PhysRevLett.103.193602

PACS numbers: 42.50.Dv, 03.67.Hk, 42.62.Eh

One of the central problems in quantum optics is the generation of nonclassical light with given spectral and spatiotemporal properties. In particular, for the needs of quantum metrology and quantum lithography it is important to obtain two-photon wave packets with small correlation times. Such wave packets should naturally manifest a broad frequency spectrum. Several ideas have been put forward in this direction, all based on two-photon states produced via spontaneous parametric down-conversion (SPDC). Among them, one can mention prisms or diffraction gratings introducing a frequency chirp [1], SPDC in aperiodically poled crystals [2–4], and SPDC in crystals with temperature gradients [5]. However, a broad spectrum of two-photon light does not necessarily imply small correlation times, although the inverse is true [6–8]. This is similar to the fact that a broadband pulse does not have to be short in time, although the spectrum of a short pulse is always broad. The spectrum broadening introduced in Refs. [1,4,5] is in fact inhomogeneous; as a result, the two-photon spectral amplitude in all these cases has a phase depending nonlinearly on the frequency. This phase (a frequency chirp [9]) makes two-photon wave packets not Fourier-transform limited. Therefore, they are not short in time despite their broad frequency spectrum. As it was mentioned in Ref. [3], time compression of such two-photon wave packets requires compensation for their frequency chirp. At the same time, the way to eliminate the chirp was not specified.

In this Letter we show that, under certain conditions, biphoton wave packets can be made nearly Fourier-transform limited and hence compressed by injecting one of the photons of a pair in a standard optical fiber and exploiting the effect of group-velocity dispersion (GVD). No specially engineered fibers (for instance, with negative GVD) are necessary. This suggests an easy way of achieving extremely short correlation times for two-photon light.

Consider generation of two-photon light via spontaneous parametric down-conversion (SPDC) from a cw pump in an aperiodically poled crystal. From the viewpoint of applications and for simplifying the calculation, it is convenient to assume that signal and idler photons are distin-

guishable, due to either frequency nondegenerate or type-II phase matching. Below, we consider phase matching to be type-II, collinear, and frequency degenerate, with idler (extraordinary) and signal (ordinary) radiations centered at frequency ω_0 . The two-photon state can be written as

$$|\psi\rangle = \int d\Omega F(\Omega) |\omega_0 - \Omega\rangle_i |\omega_0 + \Omega\rangle_s, \quad (1)$$

where $|\omega\rangle_{i(s)}$ denotes the idler (signal) photon state with frequency ω . The two-photon spectral amplitude (TPSA) $F(\Omega)$ determines all spectral and temporal properties of two-photon light. In particular, its squared module gives the frequency spectra of signal and idler radiation, $I_{s,i}(\omega) \propto |F(\omega - \omega_0)|^2$. Its Fourier transform can be called time two-photon amplitude (TPA) [10],

$$F(\tau) = \int d\Omega e^{i\Omega\tau} F(\Omega), \quad (2)$$

its squared module giving the second-order Glauber's correlation function [6,7]: $G^{(2)}(\tau) = |F(\tau)|^2$. The TPSA is determined by the distribution of the quadratic nonlinearity $\chi(z)$ along the crystal [3,10,11]:

$$F(\Omega) \propto \int_{-L}^0 dz \chi(z) e^{i(k_i + k_s - k_p)z}. \quad (3)$$

Here, L is the crystal length and k_i, k_s, k_p are wave vectors of the idler, signal, and pump waves, respectively. Let the spatial dependence of the quadratic nonlinearity be $\chi(z) = \chi_0 e^{iK(z)(z+L/2)}$, where the inverse grating vector K has a linear dependence on the coordinate, $K(z) = K_0 - \alpha(z + L/2)$ [3,4,9], and K_0 provides quasiphasematching: $k_i(\omega_0) + k_s(\omega_0) - k_p + K_0 = 0$ at the center of the crystal. It is convenient to expand the wave vectors around the exact quasiphasematching frequency,

$$\begin{aligned} k_i &= k_i(\omega_0) - k'_i \Omega + \frac{1}{2} k''_i \Omega^2, \\ k_s &= k_s(\omega_0) + k'_s \Omega + \frac{1}{2} k''_s \Omega^2. \end{aligned} \quad (4)$$

Here, $k'_{i,s}$ and $k''_{i,s}$ are the first and second derivatives of the dispersion law evaluated at ω_0 , related to the group velocity and group velocity dispersion, respectively.

Denoting $D \equiv k'_s - k'_i$ and $\kappa \equiv \frac{1}{2}(k''_i + k''_s)$, we obtain the TPSA in the form

$$F(\Omega) \propto e^{-iD\Omega(L/2) - i\kappa\Omega^2(L/2)} \int_{-L/2}^{L/2} d\xi \chi_0 e^{i(D\Omega + \kappa\Omega^2)\xi - i\alpha\xi^2}, \quad (5)$$

where $\xi = z + L/2$.

Suppose that the spectrum is not too broad compared to the difference of group velocities of the signal and idler radiation, so that the condition

$$\left| \frac{\kappa\Omega}{D} \right| \ll 1 \quad (6)$$

holds true. Then the wave vector mismatch can be written up to linear terms in frequency detuning Ω , and the TPSA becomes

$$F(\Omega) \propto e^{-iD\Omega(L/2)} \int_{-(L/2)}^{L/2} d\xi \chi_0 e^{iD\Omega\xi - i\alpha\xi^2}, \quad (7)$$

which yields, similarly to Ref. [3],

$$F(\Omega) \propto \exp\left\{-iD\Omega \frac{L}{2} + i \frac{D^2\Omega^2}{4\alpha}\right\} \times \left\{\text{erf}\left[\sqrt{\frac{i}{\alpha}} \frac{L\alpha - D\Omega}{2}\right] + \text{erf}\left[\sqrt{\frac{i}{\alpha}} \frac{L\alpha + D\Omega}{2}\right]\right\}. \quad (8)$$

In the case of large aperiodicity α , the spectral amplitude has a rectangular shape. Indeed, let us introduce the “rectangle function” $\Pi(x, a, b) \equiv 1$ for $a \leq x \leq b$ and $\Pi(x, a, b) \equiv 0$ otherwise. Then, rewriting the integral in (7) in terms of the rectangle function and applying the convolution theorem, we get

$$F(\Omega) \propto e^{-iD\Omega(L/2) + i(D^2\Omega^2/4\alpha)} \int_{-\infty}^{\infty} dx \text{sinc}\left\{Dx \frac{L}{2}\right\} \times e^{i(D^2(x^2 - 2\Omega x)/4\alpha)}, \quad (9)$$

where $\text{sinc}(x) \equiv \sin(x)/x$. The first exponential term in the integral, $\exp\{i \frac{D^2x^2}{4\alpha}\}$, can be omitted if the typical scale of its variation is much larger than the sinc-function width, π/DL . This is the case if the aperiodicity is large enough,

$$|\alpha| \gg \frac{\pi^2}{4L^2}. \quad (10)$$

Note that condition (10) is well satisfied in Refs. [3,4]. Then, the integral in Eq. (9) becomes

$$F(\Omega) \propto e^{-iD\Omega(L/2) + i(D^2\Omega^2/4\alpha)} \Pi\left(\Omega, -\frac{\alpha L}{D}, \frac{\alpha L}{D}\right). \quad (11)$$

We see that the spectrum of SPDC in a crystal with linear $K(z)$ dependence and large α is a rectangular function of width $\Delta\Omega = \frac{2\alpha L}{D}$. The condition (10) means physically that the aperiodicity should induce a substantial spectrum broadening. Now we can explicitly write the condition (6) for the GVD of the nonlinear crystal to be

negligible,

$$\left| \frac{\kappa L \alpha}{D^2} \right| \ll 1. \quad (12)$$

For given α and L , this condition is realized if the difference of signal (ordinary) and idler extraordinary group velocities is large enough.

Increasing the aperiodicity α , one can make the spectrum as broad as desired. At the same time, this does not make the TTPA (2) short in time because, due to the nonlinear frequency-dependent phase factor in (8), the TPSA is not Fourier-transform limited. Here we would like to stress that because the squared module of TTPA is the second-order Glauber's correlation function, its width gives the correlation time of the biphoton, i.e., the biphoton entanglement time [6,7]. It is this time that is important for two-photon effects such as two-photon absorption, two-photon ionization, or up-conversion, and which can be measured for two-photon light using these techniques [12]. At the same time, coherence time of biphoton light is defined as the width of the first-order Glauber's correlation function, which is the Fourier transform of the spectrum $|F(\Omega)|^2$. This is why coherence time does not depend on the phase factor in (8) and it is given by the inverse width of the spectrum [4].

The Fourier transform of TPSA (7) is easily obtained by introducing the rectangle function under the integral, extending the integration to infinite limits and using the convolution theorem. As a result, we get

$$F(\tau) \propto e^{-i\alpha((L/2) - (\tau/D))^2} \Pi(\tau, 0, DL), \quad (13)$$

with the amplitude being the same as in the case of a bulk or periodically poled crystal of length L . This means that the TTPA of an aperiodically poled crystal is as broad as in the absence of the aperiodicity α .

Consider now propagation of the extraordinary photon of the biphoton field through an optical fiber of length l with the inverse group velocity given by $k'_f \equiv \frac{dk}{d\omega}|_{\omega=\omega_0}$ and the GVD given by $\kappa_f \equiv \frac{1}{2} \frac{d^2k}{d\omega^2}|_{\omega=\omega_0}$. Propagation through such a fiber leads to a phase factor $\exp\{i(k'_f\Omega + \kappa_f\Omega^2)l\}$ in the two-photon spectral amplitude [6,7]. The first term in the phase is linear in frequency and hence only shifts the two-photon wave packet in time. The second term, being quadratic in frequency, can compensate for the TPSA chirp. This will happen under the condition

$$\kappa_f l + \frac{D^2}{4\alpha} = 0. \quad (14)$$

For a fiber with positive GVD, this condition can be satisfied for negative α , i.e., for the case where the poling period reduces along the pump propagation through the crystal.

In the case of large aperiodicity α satisfying (10), the resulting TTPA can be calculated analytically as the Fourier-transform of expression (11) with the quadratic phase term removed. Clearly, it has the form of a sinc-

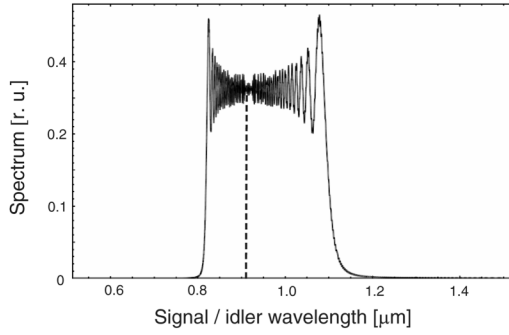


FIG. 1. Calculated spectrum of the signal or idler radiation (solid line). Dashed line: the corresponding spectrum for a crystal with $K = K_0$, reduced in the vertical scale by $2 \cdot 10^2$.

function with the width being almost αL^2 times as narrow as for a periodically poled sample of the same length.

Figure 1 shows the spectrum of SPDC radiation calculated for the case of aperiodically poled KTP crystal with $L = 0.8$ cm, $K_0 = 2441.8$ cm $^{-1}$ and $\alpha = 1200$ cm $^{-2}$, which corresponds (using the first-order quasi-phase-matching) to the poling period varying from 18.47 to 42.40 μ m. The pump at 458 nm is y polarized, as well as the idler radiation, and all three wave vectors, as well as the inverse grating vector, are directed along x. The dispersion dependencies are given by Sellmeier equations from Ref. [13], without any additional assumptions. One can see that the spectrum of the biphoton field is quite broad (from 800 to 1200 nm) and has nearly rectangular shape. This is due to the fact that condition (10) is fulfilled very well. In fact, the aperiodicity leads to the spectral broadening of more than 2 orders of magnitude. For comparison, the same figure shows the spectra of signal and idler radiation for a crystal with the same length but constant poling period $K = K_0$, corresponding to $\alpha = 0$. Condition (12) is reasonably satisfied, since $|\frac{\kappa L \alpha}{D^2}| \approx 0.16$.

Figure 2 shows the second-order correlation function calculated as the squared module of the Fourier transform of expression (8). For comparison, second-order correlation function of a crystal with constant poling period is plotted in the same graph. Clearly, both distributions have

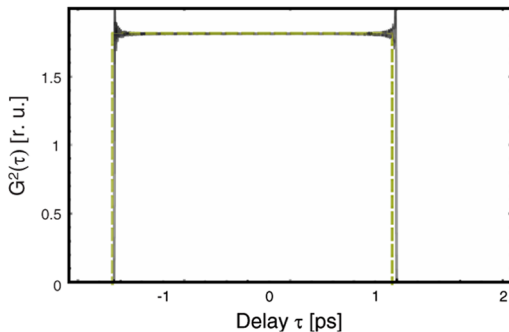


FIG. 2 (color online). Calculated second-order correlation function of the biphoton in the case of aperiodic poling (black solid line) and periodic poling (green dashed line).

the same width, which means that the biphoton with the broadened spectrum has the same correlation time as the narrow band biphoton; i.e., it is not Fourier transform limited. Evolution of the correlation time in an optical fiber is demonstrated in Fig. 3 for the cases of $\alpha > 0$ and $\alpha < 0$, when only the idler photon is transmitted through the fiber. We see that at $\alpha > 0$, propagation through the fiber only broadens the biphoton wave packet, while in the case $\alpha < 0$, which is achieved by simply exchanging the input and output faces of the crystal, the biphoton is compressed. For the calculation, we used the GVD of bulk fused silica [14], because the waveguide contribution into GVD far from zero dispersion point is negligibly small [15]. The value of κ_f we used in the calculation was $1.359 \times 10^{-28} \frac{s^2}{cm}$. The largest compression of the biphoton wave packet is achieved at the fiber length $l = 16.927$ cm. The second-order correlation function in this case has a typical shape of squared sinc-function with the FWHM equal to 12 fs. At other lengths of the fiber, the two-photon wave packet is broader. As the length of the fiber increases, the shape of $G^{(2)}(\tau)$ becomes similar to the shape of the spectrum, an effect that was studied in detail in Refs. [6,7]. Figure 4 shows the shapes of $G^{(2)}(\tau)$ after the biphoton propagation through fibers of different length; the corresponding points are shown in Fig. 3.

The model we have been using so far is based on the linear dependence of the wave vector mismatch on the frequency detuning from exact phase matching. This is valid for type-II or frequency nondegenerate type-I SPDC [16], under the condition (12). Similarly, the dispersion law of the fiber in our consideration was described by a quadratic dependence; i.e., the third-order GVD of the fiber was neglected. In order to see the effect of higher-order GVD terms, we have performed exact numerical calculation for the same case as considered above. The results (Fig. 5) show that the effect of compression is slightly reduced but there still remains a significant narrowing of the TTPA, useful for applications. Even without any optimization, the correlation time is reduced by more than an order of magnitude. An exhaustive study of this

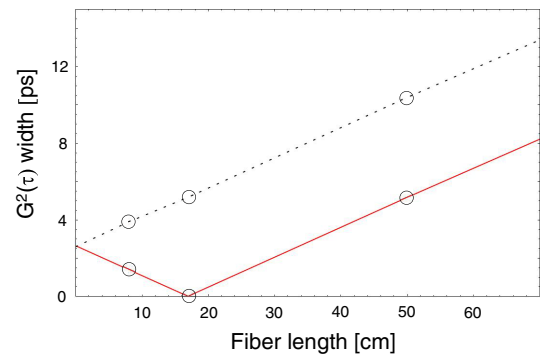


FIG. 3 (color online). TTPA width at the output of the fiber versus the fiber length for $\alpha < 0$ (red solid line) and $\alpha > 0$ (grey dashed line). Circles denote the points for which the shapes of the second-order correlation function are shown in Fig. 4.

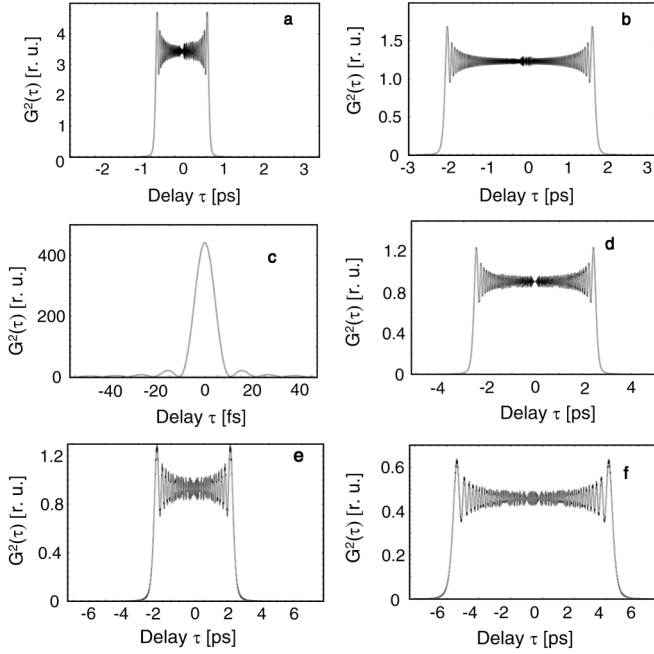


FIG. 4. Second-order Glauber's correlation function of the biphoton with $\alpha < 0$ (a),(c),(e) and $\alpha > 0$ (b),(d),(f), after its propagation through a fiber of length 8 cm (a),(b), 16.927 cm (c),(d), 50 cm (e),(f).

effect and a search for optimized parameters of both the crystal and the fiber will be presented in the nearest future [17].

An interesting feature of Fig. 5 is that at the output of the crystal, TTPA widths for the cases $\alpha > 0$ and $\alpha < 0$ are different. This can be explained as follows. Although at the center of the crystal signal and idler photons are generated with the same frequencies, biphotons generated at the back face are nondegenerate. Because of GVD, there is a delay accumulated between the photons of a pair in the course of its propagation through the crystal. At $\alpha > 0$, this delay is

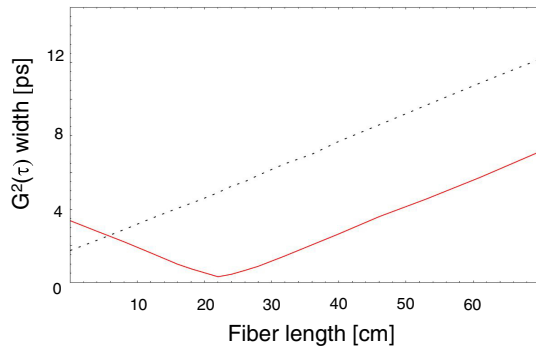


FIG. 5 (color online). TTPA width at the output of the fiber versus the fiber length for $\alpha < 0$ (red line) and $\alpha > 0$ (grey dashed line) calculated without neglecting higher-order GVD terms.

compensated by the one appearing due to birefringence and at $\alpha < 0$, both delays add up.

In conclusion, a biphoton whose spectrum is broadened due to a linear aperiodicity of the crystal poling can be compressed in time using normal GVD of an optical fiber. To describe the compression, it is sufficient to take into account first-order terms in the crystal dispersion dependence and second-order terms in the fiber dispersion dependence. Exact calculation shows that higher-order terms reduce the compression but the effect is still present. It is worth mentioning an interesting result: the two-photon correlation time as well as its evolution due to the propagation of the biphoton through an optical fiber strongly depend on the position of the crystal.

This work has been supported in part by the joint grant RFBR-Piedmont 07-02-91581-ASP, RFBR grant 09-02-92003-NNS, MIUR (PRIN 2007FYETBY), Regione Piemonte (E14), “San Paolo foundation”, NATO (CBP.NR.NRCL 983251), RFBR 08-02-00555a Russian Federal Agency for Science and Innovation (Rosnauka) state contract 02.740.11.0223, and the Russian Program for Scientific Schools Support, grant No. NSH-796.2008.2. M. V. Ch. also acknowledges the support of the Lagrange project of the CRT Foundation for her stay at INRIM.

- [1] A. Valencia *et al.*, Phys. Rev. Lett. **99**, 243601 (2007).
- [2] S. Carrasco *et al.*, Opt. Lett. **29**, 2429 (2004).
- [3] S. E. Harris, Phys. Rev. Lett. **98**, 063602 (2007).
- [4] M. B. Nasr *et al.*, Phys. Rev. Lett. **100**, 183601 (2008).
- [5] D. A. Kalashnikov, K. G. Katamadze, and S. P. Kulik, JETP **89**, 224 (2009).
- [6] M. V. Chekhova, JETP Lett. **75**, 225 (2002).
- [7] A. Valencia *et al.*, Phys. Rev. Lett. **88**, 183601 (2002); G. Brida *et al.*, Phys. Rev. Lett. **96**, 143601 (2006).
- [8] D. Strekalov *et al.*, Phys. Rev. A **71**, 041803(R) (2005).
- [9] Sometimes this dependence is called a linear chirp of the inverse grating vector; however, below we keep the term “chirp” for the frequency dependence of the phase.
- [10] A. V. Belinsky and D. N. Klyshko, Laser Phys. **4**, 663 (1994).
- [11] G. Kh. Kitaeva, S. P. Kovalev, and K. A. Kuznetsov, Int. J. Quantum. Inform. **7**, 63 (2009).
- [12] B. Dayan *et al.*, Phys. Rev. Lett. **93**, 023005 (2004); **94**, 043602 (2005).
- [13] V. Dmitriev *et al.*, *Handbook of Nonlinear Optical Crystals*, Springer Series in Optical Sciences Vol 64 (Springer Verlag, Berlin, 1991).
- [14] H. Malitson, J. Opt. Soc. Am. **55**, 1205 (1965).
- [15] G. P. Agrawal, *Nonlinear Fiber Optics* (Academic, San Diego, CA, 2001), 3rd. ed.
- [16] Note that the same technique will be valid if the spectral broadening of two-photon light is caused by any physical mechanism leading to a linear variation of the wave vector mismatch along the crystal, in particular, by temperature gradients.
- [17] A. Meda *et al.* (to be published).