Constraints on Neutron Star Crusts from Oscillations in Giant Flares

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(Received 19 February 2009; published 26 October 2009)

We show that the fundamental seismic shear mode, observed as a quasiperiodic oscillation in giant flares emitted by highly magnetized neutron stars, is particularly sensitive to the nuclear physics of the crust. The identification of an oscillation at $\approx$30 Hz as the fundamental crustal shear mode requires a nuclear symmetry energy that depends very weakly on density near saturation. If the nuclear symmetry energy varies more strongly with density, then lower frequency oscillations, previously identified as torsional Alfvén modes of the fluid core, could instead be associated with the crust. If this is the case, then future observations of giant flares should detect oscillations at around 18 Hz. An accurate measurement of the neutron-skin thickness of lead will also constrain the frequencies predicted by the model.

DOI: 10.1103/PhysRevLett.103.181101 PACS numbers: 26.60.Gj, 21.65.Ef, 97.10.Sj

Giant x-ray flares from highly magnetized neutron stars are powered by catastrophic reconfigurations of the decaying field. Pinning of the rapidly evolving field to the solid crust triggers an associated starquake and generates global seismic vibrations [1], detectable as quasiperiodic oscillations (QPOs) in the x-ray afterglow [2–5].

The current picture is that the QPOs, which range in frequency from 18 up to 1800 Hz, result from torsional (twisting) motions of the star. The lowest (18, 26 Hz) frequencies were initially associated with the magnetized fluid core, while the other (>28 Hz) frequencies were interpreted as shear modes of the solid crust. The problem is actually more complicated, as the fluid core admits an Alfvén continuum, and crust and core are coupled by the strong magnetic field. However, even in this more complex system frequencies close to the pure crustal frequencies can still emerge, from global modes in which crust motion dominates [6–8]. If this picture is correct, then the observed flare QPO frequencies probe the shear properties of the neutron star crust. This is particularly exciting because the frequencies can be measured relatively accurately, to within a few percent or better.

In this Letter, we calculate the frequency of shear oscillations of the neutron star crust, and show that they depend sensitively on a particular aspect of the nuclear physics input: the nuclear symmetry energy. The symmetry energy, the energy cost of creating an isospin asymmetry in nucleonic matter, is one of the most significant uncertainties in the description of the crust [9–11]. The sensitivity of the flare QPO frequency to the symmetry energy implies that they can constrain the properties of the nuclear symmetry energy. We also make a novel connection to the proposed measurement of the neutron-skin thickness of a lead nucleus (the difference between the neutron and proton radii), to be performed next year at Jefferson Lab in the PREX [12,13] experiment.

At lower densities, the outer crust of the neutron star consists of nuclei embedded in a sea of degenerate electrons. As density rises, nuclei become progressively heavier and more neutron rich. Once density increases past the neutron drip point, 4×10^{11} g/cm^3, it becomes energetically favorable for neutrons to drip out of nuclei. This begins the inner crust region of the neutron star, where exotic nuclei are embedded in a sea of superfluid neutrons. At a density of around 1.5×10^{14} g/cm^3, nuclei are no longer favored and dissolve into their constituents.

For the model of the inner neutron star crust, we employ the liquid droplet model from Ref. [11], as updated in Ref. [14]. This model consistently describes the nuclei and the dripped neutrons in the crust with one equation of state for homogeneous nucleonic matter, and is very similar to that employed for the description of matter in core-collapse supernovae [15]. For the homogeneous nucleonic matter EOS, we use the Skyrme model [16], which enables us to describe matter over a large enough density range to obtain masses and radii, and also can describe neutron matter at very low densities in the inner crust.

The determination of the crust-core transition is difficult because the relevant energy surfaces are very flat near the transition, so we choose to fix the transition density at 0.07 fm^{-3}. Increasing this value to 0.1 fm^{-3} decreases the frequency of the radial overtones (which depend strongly on crust thickness), but changes the fundamental crust mode frequency only by a percent or so. We choose to use the outer crust model from Ref. [17], but have checked that using alternate models from Ref. [18] does not change the inferred QPO frequencies (the shear mode frequencies are largely insensitive to uncertainties in the neutron drip point [19]).

The shear modulus of the crust [20] is

$$\mu = \frac{0.1194}{1 + 0.595(\Gamma_0/\Gamma)^2} \frac{n_i(Ze)^2}{a}. \tag{1}$$

Here Z is the atomic number of the ions, n_i is the ion density, and $a = (3/4\pi n_i)^{1/3}$ is the average inter-ion spacing. The parameter $\Gamma = (Ze)^2/ak_BT$, where T is the temperature, measures the ratio of the Coulomb to thermal energies, and we use $\Gamma = \Gamma_0 = 173$ to determine the upper boundary of the crust [21]. Equation (1), derived using a Monte Carlo simulation of the Coulomb interactions in a neutron star crust, assumes that the contribution to the shear modulus from neutron-lattice interactions and neutron-neutron interactions is small.

Neutron stars and their magnetospheres admit many types of vibration, driven by different restoring forces. The identification of the QPOs excited in magnetar flares with crust shear modes is based on several factors. Shear modes have a lower excitation energy (compared to vibrations involving bulk compression), and damp sufficiently slowly to explain the observed QPO durations [22]. Excitation of crust motion is particularly likely if, as argued by [23], the yielding of a strained crust triggers the flares. Coupling of a twisting crust to the external field also provides a plausible mechanism for modulating the x-ray emission [3]. Most importantly the observed QPOs match expectations from models in terms of both frequency and the scaling between different harmonics. Coupling of crust and core by the magnetic field does complicate the system but, as demonstrated by [8], does not prevent the emergence of the natural (uncoupled) frequencies. We can therefore compute crust shear mode frequencies assuming free slip between crust and core.

We employ the simple Newtonian perturbation model for torsional shear oscillations used by [24,25]. This model uses a plane-parallel geometry with constant gravitational acceleration, g, computed beforehand using the TOV equations. The Newtonian equations of hydrostatic equilibrium then determine the crust density profile. Using a slab rather than spherical geometry allows us to incorporate the magnetic field without difficulty; we assume a constant field $\mathbf{B} = B\hat{z}$. In order to correct for the error that this introduces, and to ensure that we recover the correct spherical geometry frequencies in the zero field limit we follow [26] and mimic a spherical geometry by setting $\nabla_\perp^2 \xi = -[(l+2)(l-1)/R^2]\xi$, l being the standard angular quantum number.

For pure toroidal shear modes, which are incompressible and have no vertical component of displacement, the perturbation equations for the horizontal displacement ξ reduce to

$$\frac{(\mu\xi')'}{\rho} + v_A^2\xi'' + \left[\omega^2\left(1 + \frac{v_A^2}{c^2}\right) - \frac{(l^2 + l - 2)\mu}{\rho R^2}\right]\xi = 0, \tag{2}$$

where $v_A = B/(4\pi\rho)^{1/2}$ is the Alfvén speed. The shear speed is $v_s = (\mu/\rho)^{1/2}$. We assume a periodic time dependence $\exp(i\omega t)$, ω being the frequency, and correct for gravitational redshift to obtain observed frequency. Primes indicate derivatives with respect to z.

Using the schematic model of Ref. [11], we can construct a neutron star crust from an arbitrarily chosen symmetry energy. This crust model includes some corrections to the nuclear masses due to the surrounding neutron gas. Figure 1 displays the connection to the nuclear symmetry energy as a function of density, $E_{\rm sym}(n)$, relating the magnitude of the symmetry energy, S, at the nuclear saturation density, $n_0 = 0.16$ fm^{-3}, and a parameter related to the derivative of the symmetry energy, $L = 3n_0 \partial E_{\rm sym}/\partial n|_{n=n_0}$, to the shear speed, v_s. The shear speed is large if the magnitude of the symmetry energy is large and the density dependence is small. Also displayed is the relationship to the neutron-skin thickness of lead (δR), from the Typel-Brown correlation [27], which depends most strongly on L.

For a full oscillation model, we need a complete model for hadronic matter up to the densities in the center of neutron stars. The shear speeds as a function of mass density are given in Fig. 2 for the Skyrme models, BSk14 [28], Gs [29], NRAPR [9], Rs [29], SLy4 [30], SkI6 [31], SkO [32], SkT2 [33]. We also use the model from APR [34], for which δR has been estimated at 0.20 fm [9]. Previous calculations such as those in Refs. [24,26] were based on the crust of Ref. [35]. This also used the Skyrme model SLy4 [30] and is shown as a dotted line, labeled DH01. The difference between that result and our model, derived from the same Skyrme interaction, provides an estimate of the uncertainties in the construction of the masses of the neutron-rich nuclei in the crust, given the

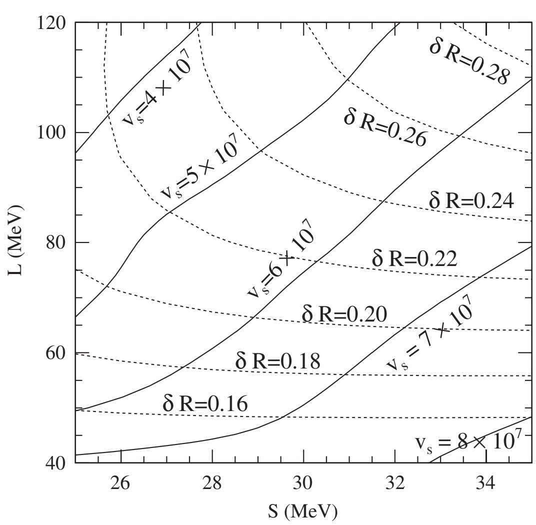

FIG. 1. Shear speeds v_s (solid lines) and the neutron-skin thickness of lead δR (dashed lines) as a function of the symmetry energy. The abscissa is the magnitude of the symmetry energy at the nuclear saturation density and the ordinate is the derivative of the symmetry energy.

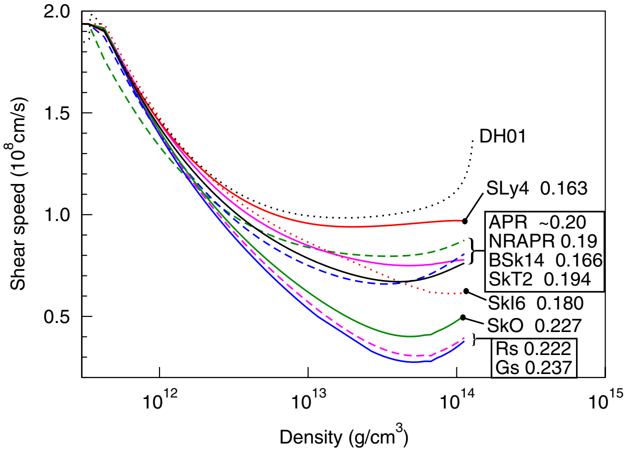

FIG. 2 (color online). The shear speeds as a function of density for the crust models used in this work, and the shear speed of Ref. [35] (labeled DH01), which was based on SLy4. Also shown is the neutron-skin thickness of lead, δR, for each model.

same underlying nucleon-nucleon interaction. That difference is smaller than the variation arising from the nucleon-nucleon interaction itself, which is as much as a factor of 4. This large variation in the shear speed is the source of the sensitivity of the QPO frequencies to the nuclear physics input. As expected from Fig. 1, there is a correlation between the neutron-skin thickness in lead, with smaller skins generally giving larger shear speeds. The upcoming PREX measurement will help constrain the frequencies predicted by the model, reducing the uncertainty originating from our lack of knowledge of the symmetry energy.

Figure 3 shows the frequencies of the fundamental $n = 0$, $l = 2$ crust shear mode and the first ($n = 1$) radial overtone as a function of neutron star mass for the various crust models. Also shown in this figure are some of the QPO frequencies measured during the 2004 hyperflare

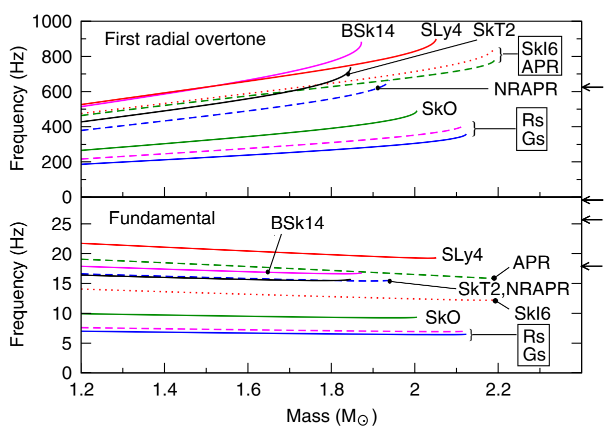

FIG. 3 (color online). The crust oscillation frequencies as a function of neutron star mass, for both the fundamental ($n = 0$, $l = 2$) torsional shear mode and the first radial ($n = 1$) overtone. The curves end at the maximum mass. The arrows on the right indicate QPO frequencies measured during the 2004 hyperflare from SGR 1806-20 [2,4,5].

from SGR 1806-20. The dramatic effect of uncertainty in the symmetry energy on shear speed is evident in the spread of mode frequencies. The frequency of the fundamental mode is larger for smaller neutron-skin thicknesses; however, all of the models have difficulty explaining a fundamental above 22 Hz. Previous studies including [24], which interpreted the 28 Hz QPO in SGR 1900 + 14 and the 29 Hz QPO in SGR 1806-20 as the fundamental, employed the higher shear speed model of [35], thereby yielding higher frequencies. Once the correct l scaling is taken into account, however, even this crust model would necessitate a rather low stellar mass [26]. The models that we have considered, which have a symmetry energy that varies more strongly with density, cannot explain a fundamental frequency at 28–29 Hz. The only observed QPO that would fall into the predicted range is the 18 Hz QPO detected in the SGR 1806-20 hyperflare [2,4] and previously interpreted as a torsional mode of the magnetized fluid core [2].

Relativistic perturbation calculations predict very similar frequencies for the fundamental to those generated by Newtonian calculations [26]. The mode frequencies are also largely insensitive to magnetic field effects, since the electron Fermi momentum in the inner crust is much larger than the energy spacing between Landau levels except at fields larger than the 10^{14}–10^{15} G implied for magnetars. Note also that the fundamental frequency is nearly mass independent, which means that uncertainties in the neutron star mass will not spoil the connection between the frequency and the nuclear symmetry energy. It is not well known how finite-size effects and the presence of extremely deformed nuclei (i.e., nuclear pasta) may affect the shear modulus. Recent calculations by [36], which reexamine Eq. (1) would exacerbate the problem since they indicate that shear modulus is actually lower by $\sim$10%. This would reduce mode frequencies by a further 3%–4%. Finally, the structure of the neutron-rich nuclei is not well understood and this also adds a systematic uncertainty to our results.

Our crust model thus suggests that a revision of the previous interpretation of the 28 Hz mode as the fundamental shear mode may be necessary. One alternative is that the fundamental mode may have a lower frequency (an idea also explored in [37]). For this to be plausible, it should be possible to fit the sequence of QPOs detected in each flare with the expected scaling in l. In the SGR 1806-20 giant flare, a QPO was detected at 18 Hz. If this is the fundamental ($l = 2$), then a reasonable fit to the higher frequencies can be obtained with the following mode identifications: 29 Hz ($l = 3$), 92 Hz ($l = 10$), 150 Hz ($l = 16$). The one QPO that would not fit the sequence is the 26 Hz QPO, but this could also be accommodated if the fundamental were even lower, at $\approx$11 Hz. Such low values are predicted by some of our models, although it might then be difficult to explain the 625 Hz QPO as an $n = 1$ radial overtone. Alternatively, the 26 Hz QPO is a core-dominated mode.

No lower frequency QPOs were detected in the SGR 1900 + 14 hyperflare. However, the sequence of detected QPOs would be compatible with a fundamental at ≈18 Hz with the following mode identifications: 28 Hz ($l = 3$), 53.5 Hz ($l = 6$), 84 Hz ($l = 9$), and 155 Hz ($l = 17$). As for SGR 1806-20, a lower frequency fundamental at ≈11 Hz could also be accommodated. With this in mind, we performed a thorough rotational phase and energy-dependent search of the data from the *Rossi X-ray Timing Explorer* for the SGR 1900 + 14 hyperflare to search for lower frequency QPOs. No detections were made, but due to poorer data quality we could not search for QPOs as weak as those detected in SGR 1806-20. At 18 Hz, for example, we were able to set a 3 sigma upper limit on a QPO amplitude of 7% rms: in the SGR 1806-20 hyperflare the 18 Hz QPO had an amplitude of 4%.

It is interesting that an 18 (or 11) Hz fundamental frequency might be able to explain both mode sequences. If this is a common feature then we would expect to detect such a feature in future giant flare light curves. The lowering of crust shear mode frequencies and their proximity to the expected frequencies of magnetic torsional modes does, however, complicate efforts to use magnetar QPOs to measure interior field strengths [2,37]. A sequence of modes, including overtones, is likely to be necessary to enable correct mode identification. There are several other possible effects on the shear modulus, including that of shell effects, frozen impurities [38,39], superfluidity and entrainment [40], and the magnetic field which we have not addressed. A more definitive mode assignment will have to wait until these effects are better understood, or more data are obtained.

A. W. S. was supported by JINA at MSU under NSF-PFC Grant No. PHY 08-22648, and by NASA ATFP Grant No. NNX08AG76G. A. L. W. thanks JINA for hospitality. We also thank Ed Brown, Lars Samuelsson, and participants of the University of Washington Institute for Nuclear Theory workshop "The Neutron Star Crust and Surface: Observations and Models" for stimulating discussions.

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