Symmetry Induced Four-Wave Capillary Wave Turbulence

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(Received 5 June 2008; published 22 October 2009)

We report theoretical and experimental results on 4-wave capillary wave turbulence. A system consisting of two immiscible and incompressible fluids of the same density can be written in a Hamiltonian way for the conjugated pair (η, Ψ). Adding the symmetry $z \rightarrow -z$, the set of capillary waves display a Kolmogorov-Zakharov spectrum k^{-4} in wave vector space and $f^{-8/3}$ in the frequency domain. The wave system is studied experimentally with two immiscible fluids of almost equal densities (water and silicon oil) where the capillary surface waves are excited by a low-frequency random forcing. The probability density function of the local wave amplitude shows a quasi-Gaussian behavior and the power spectral density is shows a power-law behavior in frequency with a slope of -2.75. Theoretical and experimental results are in fairly good agreement with each other.

DOI: 10.1103/PhysRevLett.103.174503 PACS numbers: 47.35.Pq, 05.45.−a, 47.27.eb, 47.27.nb

Introduction.—Wave turbulence [1] deals with a set of nonlinear random waves in a dispersive medium that, although forced far from thermodynamic equilibrium, can be described statistically. This description is done by means of a kinetic equation for the spectral density distribution $n_{\boldsymbol{k}}$ in $\boldsymbol{k}$ (the wave vector) which evolves through resonant interactions of $\mathcal{N}$ waves. In addition to the equilibrium solutions represented by Rayleigh-Jeans distributions, the kinetic equation can display stationary power-law nonequilibrium solutions $n_{\boldsymbol{k}} \sim k^{-\mu}$, with $|\boldsymbol{k}| = k$ and $\mu > 0$, called Kolmogorov-Zakharov (KZ) spectra describing the energy exchange (or other conserved quantities) between large and small scales. KZ spectra have been predicted theoretically and observed numerically and experimentally in systems such as bending waves in elastic sheets [2,3], Alfvén waves in plasma [4], spin waves in solids [5], and gravity waves in fluids [6–9], to name a few examples. In all these examples, although the theoretical description depends on several constraints (negligible viscosities, density contrasts, aspect ratios, etc. [1,10]), experimental and numerical results corroborate the theoretical prediction of the appearance of power-law nonequilibrium spectra. Still, there are certain features that are yet to be studied and compared among theory, experiments, and numerics, for instance, the non-Gaussianity of the wave amplitudes [11], the nature and existence of intermittency in a wave system [12,13], the role of symmetries and dissipation in the wave interactions [14,15] or the deviations of the exponent μ from its theoretical value.

In this Letter we focus on capillary wave turbulence [16–18] and the effect of symmetries in wave interactions. We study theoretically and experimentally the statistical properties of random waves at the interface between two immiscible and incompressible deep fluids of equal densities (ρ_1 and ρ_2) and depths (h_1 and h_2). Because of these facts the symmetry $z \rightarrow -z$ is forced on the system: the typical 3-wave capillary wave turbulence breaks down and a four-wave resonant interaction appears as the leading order perturbation. We discuss the effect of this symmetry on the nonlinear type of wave interaction and on the KZ spectrum of the wave amplitude. We compare the theoretical prediction with the experimental measurement of the power spectral density (PSD) of the local wave amplitude at a water-oil interface in the limit of the Atwood number $A = (\rho_1 - \rho_2)/(\rho_1 + \rho_2) \rightarrow 0$. Also, the computed probability density function (PDF) for the excited surface wave amplitude shows a highly symmetric quasi-Gaussian shape. The level of agreement between theoretical and experimental results stresses the fact that wave turbulence is a robust phenomenon for nonlinear capillary waves.

Theoretical study.—Let us study the system of potential flows of two incompressible and immiscible fluids in a box of height $2h = h_1 + h_2$, where $\eta(\boldsymbol{r} = (x, y), t)$ corresponds to the surface elevation between them, ρ_1 is the density of the lower fluid ($-h_1 < z < \eta$), ρ_2 the density of the upper fluid ($\eta < z < h_2$), with $\rho_1 > \rho_2$ and σ the surface tension coefficient. The flows are defined by the velocity potentials $\phi_1(\boldsymbol{r}, z, t)$ in the lower fluid and $\phi_2(\boldsymbol{r}, z, t)$ in the upper fluid with $\nabla\phi_1 = \boldsymbol{v}_1$, $\nabla\phi_2 = \boldsymbol{v}_2$. It is possible to prove that the dynamics of the interface has a Hamiltonian structure [10,19], i.e.,

$$\frac{\partial \eta(\boldsymbol{r})}{\partial t} = \frac{\delta H}{\delta \Psi}, \qquad \frac{\partial \Psi(\boldsymbol{r})}{\partial t} = -\frac{\delta H}{\delta \eta}, \tag{1}$$

with $\Psi(r) = \rho_1\phi_1(\boldsymbol{r}, \eta(\boldsymbol{r})) - \rho_2\phi_2(\boldsymbol{r}, \eta(\boldsymbol{r}))$ and $H = K + U$ given by

$$H = \iint_{-h_1}^{\eta} \rho_1 \frac{(\nabla\phi_1)^2}{2} dzdr + \iint_{\eta}^{h_2} \rho_2 \frac{(\nabla\phi_2)^2}{2} dzd\boldsymbol{r} + \int\Big[\frac{g}{2}(\rho_1 - \rho_2)\eta(\boldsymbol{r})^2 + \sigma\big(\sqrt{1 + |\nabla_\perp\eta|^2} - 1\big)\Big]d\boldsymbol{r}, \tag{2}$$

where $\perp$ correspond to the $\boldsymbol{r}$ coordinates. It is easy to see that the Hamiltonian corresponds precisely to the sum of the kinetic energy K (upper line) and potential energy U (lower line) of the system. The system is also constrained to the boundary conditions $\partial_z\phi_1|_{z=-h_1} = \partial_z\phi_2|_{z=h_2} = 0$ (zero normal velocity at the upper and lower walls), $[(\boldsymbol{v}_{2\perp}\nabla_\perp)\eta(\boldsymbol{r}) - v_{2z}]_{z=\eta} = [(\boldsymbol{v}_{1\perp}\nabla_\perp)\eta(\boldsymbol{r}) - v_{1z}]_{z=\eta}$ (continuity of the normal velocity at the interface) and the incompressibility conditions $\nabla^2\phi_1 = \nabla^2\phi_2 = 0$. A formal expression can be found for the Hamiltonian [20]. The original work was presented for the 2D case, but can be easily extended for the 3D case. The kinetic energy can be expressed as

$$K = \frac{1}{2}\int \Psi\hat{G}_2(\eta)[\rho_2\hat{G}_1(\eta) + \rho_1\hat{G}_2(\eta)]^{-1}\hat{G}_1(\eta)\Psi d\boldsymbol{r}, \tag{3}$$

where $\hat{G}_1(\eta)$, $\hat{G}_2(\eta)$ are the Dirichelet-Neumann operators for the fluid domain $-h_1 < z < \eta(\boldsymbol{r})$ and $\eta(\boldsymbol{r}) < z < h_2$, respectively, defined by $\hat{G}_i(\eta)\phi_i(\boldsymbol{r}, \eta(\boldsymbol{r})) = (-1)^i \times [\nabla_{\boldsymbol{r}}\phi_i\nabla_{\boldsymbol{r}}\eta(\boldsymbol{r}) - \frac{\partial\phi_i}{\partial z}]_{z=\eta}$. In 3D systems it does not seem possible to write an explicit Hamiltonian in terms of η and Ψ. This problem is bypassed by using the small angle approximation to write the Hamiltonian as an infinite Fourier series in $\boldsymbol{k}$ space [1,7,17]. In terms of the operators $\hat{G}_1(\eta)$, $\hat{G}_2(\eta)$, it corresponds to find an asymptotic series in term of the small parameter $k\eta_0 \ll 1$, with η_0 the characteristic surface elevation. Adding the symmetry $z \to -z$ to the initial problem (in this case by imposing equal depth $h_1 = h_2 = h$ and density $\rho_1 = \rho_2 = \rho$) the expansion naturally needs to satisfy this constrain. Therefore, the order of the nonlinearity increases from $\mathcal{N} = 3$ to 4, and also the system becomes gravity free. In the Hamiltonian expansion $H = H_2 + H_4\ldots$, one gets

$$H_2 = \frac{1}{2}\int\left[\frac{1}{2\rho}k\tanh[kh]\Psi_{\boldsymbol{k}\boldsymbol{k}}\Psi_{-\boldsymbol{k}} + \sigma k^2\eta_{\boldsymbol{k}}\eta_{-\boldsymbol{k}}\right]d\boldsymbol{k},$$
$$H_4 = \int(T^{(1)}_{1,2;3,4}\Psi_{\boldsymbol{k}_1}\Psi_{\boldsymbol{k}_2}\eta_{\boldsymbol{k}_3}\eta_{\boldsymbol{k}_4} + T^{(2)}_{1,2;3,4}\eta_{\boldsymbol{k}_1}\eta_{\boldsymbol{k}_2}\eta_{\boldsymbol{k}_3}\eta_{\boldsymbol{k}_4}) \times \delta^{(2)}(\boldsymbol{k}_1 + \boldsymbol{k}_2 + \boldsymbol{k}_3 + \boldsymbol{k}_4)d\boldsymbol{k}_{1234}, \tag{4}$$

where $H_3 \equiv 0$, which comes explicitly from the Dirichelet-Neumann operator expansion [20]. Physically, the $z \to -z$ symmetry is not merely imposed by eliminating gravity in the limit $A \to 0$, but also by imposing equal depth of both fluids, making the capillary surface waves unable to distinguish up from down in the vertical direction. Even more, using the same symmetry arguments, every odd term in the expansion of H will be zero in this limit. This fact is confirmed by explicitly calculating the transfer matrixes $T^{(1)}_{1,2;3,4}$ and $T^{(2)}_{1,2;3,4}$, which depend on four wave vectors [21].

Following [1,2], we write $\eta_{\boldsymbol{k}} = \frac{X_k}{\sqrt{2}}\sum_s A^s_{\boldsymbol{k}}$ and $\Psi_k = -i\frac{X_k^{-1}}{\sqrt{2}}\sum_s sA^s_{\boldsymbol{k}}$, in canonical variables $A^s_{\boldsymbol{k}}$ where $s = \pm$ such that $A^+_{\boldsymbol{k}} \equiv A_{\boldsymbol{k}}$, $A^-_{\boldsymbol{k}} \equiv A^*_{-\boldsymbol{k}}$ and $X_k = (\frac{\omega_k}{\sigma k^2})^{1/2}$ where $\omega_{\boldsymbol{k}} = \sqrt{\frac{\sigma}{2\rho}k^3\tanh[kh]}$ is the dispersion relation. From the Hamiltonian evolution of $A^s_{\boldsymbol{k}}$, a hierarchy of linear equations for the averaged moments ($\langle A^{s_1}_{\boldsymbol{k}_1}A^{s_2}_{\boldsymbol{k}_2}\rangle$, $\langle A^{s_1}_{\boldsymbol{k}_1}A^{s_2}_{\boldsymbol{k}_2}A^{s_3}_{\boldsymbol{k}_3}A^{s_4}_{\boldsymbol{k}_4}\rangle$ and so forth) is written. An asymptotic closure of these equations can be given when the system is regarded as homogeneous in space and there is separation of linear and nonlinear time scales due to the weak nonlinear interactions [13]. This closure is given by the evolution of the wave spectrum $n_{\boldsymbol{k}}$, that comes from the second order moment $\langle A_{\boldsymbol{k}_1}A^*_{\boldsymbol{k}_2}\rangle = n_{\boldsymbol{k}_1}\delta^{(2)}(\boldsymbol{k}_1 + \boldsymbol{k}_2)$. It satisfies a Boltzmann-type kinetic equation describing the slow evolution of the wave spectrum through a four-wave resonant process:

$$\frac{d}{dt}n_{\boldsymbol{k}} = 12\pi\sum_{s_1,s_2,s_3}\int|L^{-1,s_1,s_2,s_3}_{-\boldsymbol{k},\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3}|^2 n_{\boldsymbol{k}_1}n_{\boldsymbol{k}_2}n_{\boldsymbol{k}_3}n_{\boldsymbol{k}}\left(\frac{1}{n_{\boldsymbol{k}}} + s_1\frac{1}{n_{\boldsymbol{k}_1}} + s_2\frac{1}{n_{\boldsymbol{k}_2}} + s_3\frac{1}{n_{\boldsymbol{k}_3}}\right) \times \delta(\omega_{\boldsymbol{k}} + s_1\omega_{\boldsymbol{k}_1} + s_2\omega_{\boldsymbol{k}_2} + s_3\omega_{\boldsymbol{k}_3})\delta^{(2)}(\boldsymbol{k} + \boldsymbol{k}_1 + \boldsymbol{k}_2 + \boldsymbol{k}_3)d\boldsymbol{k}_{123}, \tag{5}$$

where $L^{-s_1,s_2,s_3,s_4}_{-\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3,\boldsymbol{k}_4} = -is_1\frac{1}{24}\mathcal{P}_{1234}[s_1s_2\frac{X_{k_3}}{X_{k_1}}\frac{X_{k_4}}{X_{k_2}}T^{(1)}_{1,2;3,4} - X_{k_1}X_{k_2}X_{k_3}X_{k_4}T^{(2)}_{1,2;3,4}]$ is the scattering matrix, $\mathcal{P}_{1234}[\cdot]$ is the sum over the 12 possible permutations of 1, 2, 3 , and 4. With this kinetic equation, we seek isotropic nonequilibrium distribution solutions [22]. Despite some differences with the usual kinetic equation, the method of Zakharov can be applied here as in [2]. In the deep fluid limit ($hk \to \infty$), one finds that the scattering matrix L and frequency $\omega_{\boldsymbol{k}} = \sqrt{\frac{\sigma}{2\rho}k^3}$ of capillary waves are homogenous functions of degree $\beta = 3$ and $\alpha = 3/2$, respectively, i.e., $L^{s,s_1,s_2,s_3}_{\lambda\boldsymbol{k},\lambda\boldsymbol{k}_1,\lambda\boldsymbol{k}_2,\lambda\boldsymbol{k}_3} = \lambda^\beta L^{s,s_1,s_2,s_3}_{\boldsymbol{k},\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3}$ and $\omega_{\lambda k} = \lambda^\alpha\omega_{\boldsymbol{k}}$. Looking for a power-law solution of the form $n_k = \Lambda k^{-\mu}$ with Λ a constant, it is possible to perform the Zakharov transformation over the right-hand side of Eq. (5), called collisional term [22]. In such a way one gets a stationary out-of-equilibrium spectrum with $\mu = 4$ that represents a constant energy flux solution. If we consider a flux of energy per unit of mass P through the big scales towards the small scales, one can find an explicit expression for Λ that leads to $n_k = CP^{1/3}\rho k^{-4}$. C is a pure real number that depends on some integrals directly related with the collisional term that can, in principle, be computed numerically. No inverse cascade is allowed in this system due to the $3 \leftrightarrow 1$ wave interaction process, as it was already reported in [2]. As we are considering a statistically homogenous system in space, it is natural to compute the moments by taking space averages. Nevertheless, from the experimental point of view, taking space averages is quite a difficult task. On the contrary, time averages of local properties are much more accessible. It is possible to relate both for stationary solutions in the linear regime as $n_T(\omega)d\omega \propto n(k)k^{d-1}dk$, where $n_T(\omega)$ is the time averaged wave number, in frequency domain. Using ω_k of capillary waves one gets $n_T(\omega) \propto P^{1/3}\omega^{-7/3}$. Thus we obtain the spectrum for local surface elevation $\langle|\eta_\omega|^2\rangle_T \propto P^{1/3}\omega^{-8/3}$.

Experimental study.—A Plexiglas container (height $h = 60$ mm, length $l = 100$ mm, depth $d = 80$ mm) is half filled with distilled water (density $\rho_1 = 1.00\ \mathrm{g/cm^3}$, kinematic viscosity $\nu_1 = 0.01\ \mathrm{cm^2/s}$) and half filled with silicon oil (PDMS AB112153 from ABCR, density $\rho_2 = 0.93\ \mathrm{g/cm^3}$, kinematic viscosity $\nu_2 = 0.07\ \mathrm{cm^2/s}$). The interfacial tension coefficient $\sigma \sim 30$ mN/m [23]. The equilibrium interface position is measured at 35 mm. Capillary surface waves are excited by a wave maker that plunges completely into the upper fluid, oscillating vertically. The wave maker is driven by an electromagnetic vibration exciter via a power amplifier. The random forcing, supplied by the source output of a dynamical spectrum analyzer, is low-pass filtered between $0 - f_{\mathrm{driv}} = 3$ Hz. The excited surface wave amplitude η is locally measured 4 cm away from the container walls by means of a wire capacitive gauge of 0.1 mm in diameter. The measured capacitive fluctuations are proportional to the local wave amplitude ones. They are sampled at 800 Hz during 300 s, and low-pass filtered numerically at 500 Hz to avoid aliasing. We have checked the wire probe's linear response in η by changing fluid depths and its constant frequency response (in magnitude) in a frequency band between 1 to 100 Hz. A noteworthy difference between our setup and the one of [8] is that in our case both dielectrics are liquids of similar densities and similar viscosities.

With the acquired data, the probability distribution function (PDF) of η, normalized by its rms fluctuations σ_η, is calculated, as shown in Fig. 1 (main). Notice that $\langle\eta\rangle \sim 0$ and that its fluctuations are close to being symmetric with respect to $\eta = 0$. No exponential tails are found. The kurtosis is slightly larger than 3, but not large enough to exclude Gaussianity. For comparison, we show in Fig. 1 (inset) the PDF of η/σ_η when gravity-capillary wave turbulence develops. We see a clear asymmetric tail (positive skewness) as it is shown elsewhere [8]. This contrast is a clear indication of the imposed symmetry in the system: there is no external field (gravity) that breaks the $z \rightarrow -z$ parity so the surface perturbations are symmetric with respect to $\eta = 0$. It is unclear if the fluctuations are indeed Gaussian: resolution of large events could not be made in the present experimental setup. The wave system has a very low Atwood number $A = (\rho_1 - \rho_2)/(\rho_1 + \rho_2) \simeq 0.04$, reducing the effective gravity drastically. One finds that in this system the capillary length $l_c = 2\pi\sqrt{A\sigma/g(\rho_1 + \rho_2)}$ where the crossover from gravity to capillary regime takes place is an order of magnitude larger than in a liquid-air interface problem [24]. The frequency crossover between

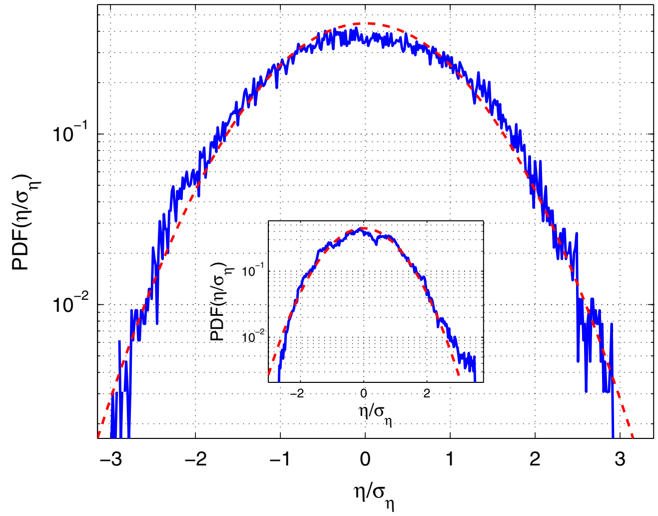

FIG. 1 (color online). Probability density function (PDF) of the normalized local wave amplitude η/σ_η at the interface of two immiscible fluids with $A \simeq 0.04$ (full line) and a parabolic fit (dashed line). Inset: PDF of η/σ_η at the interface of a water-air interface for $A = 1$ (full line) and a parabolic fit (dashed line).

gravity and capillary regimes $f_c = \omega_c/2\pi = \pi\sqrt{Ag/2l_c}$ will be obtained at a frequency close to 3–4 Hz. Therefore, when the frequency cutoff of the forcing is larger than f_c, the only KZ-type spectrum we can observe is the capillary one.

In Fig. 2 we show both the pure capillary $A \simeq 0.04$ (main) and the gravity-capillary $A \simeq 1$ (inset) spectra. For pure capillary waves, as the forcing amplitude is increased, low-frequency normal modes and harmonics of f_{driv} disappear and a power-law spectrum develops. Only one

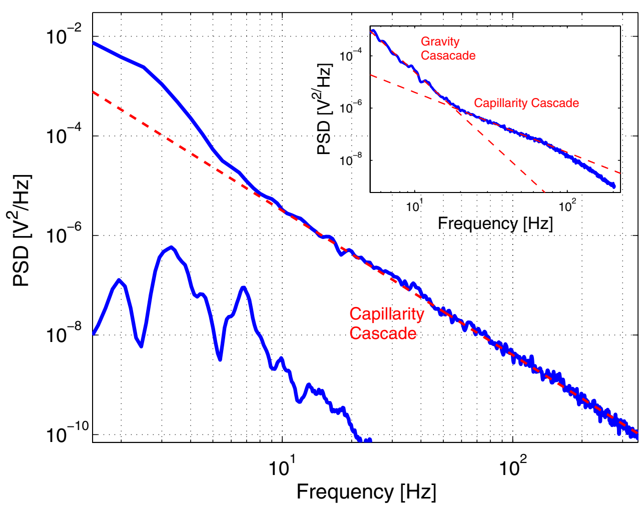

FIG. 2 (color online). Power spectral densities (PSD) of the local wave amplitude η at the interface of two immiscible fluids with $A \simeq 0.04$ (full line) for low (bottom) and high (top) forcing amplitudes. Best fit slope -2.75 (dashed line). Inset: PSD of η at a water-air interface for $A = 1$ (full line) and best fit KZ spectra (dashed line) for gravity (-5.35) and capillary (-2.52) waves.

scale-invariant spectrum with slope -2.75 ± 0.05 appears in the capillary-driven transparency window (for frequencies larger than the characteristic frequencies of the broad-band forcing), which is within the experimental error from the theoretical $f^{-8/3}$. In this regime, no cusps over the wave crests were observed, which sustains the assumption $k\eta_0 \ll 1$ and eliminates the possibility of singularities polluting the spectral content of the signal. The gravity-capillary spectra of Fig. 2 (inset) is calculated from the local elevation of surface waves when the lighter fluid (PDMS) is removed. Waves are excited using a wavemaker plunging in water driven by a low-frequency ($f_{\rm driv} = 3$ Hz) random forcing similar to [8,12]. Both gravity $f^{-5.35}$ and capillary $f^{-2.52}$ wave spectra are within the experimental range of [8].

Conclusions.—In this Letter we develop a wave turbulence approach for the interface fluctuations between two immiscible and incompressible fluids in the limiting case of equal depths and $A \to 0$. A 4-wave capillary wave turbulence is found due to an imposed spatial symmetry, where the theoretical wave spectrum of the amplitude fluctuations behaves as $f^{-8/3}$ (k^{-4}) in frequency (wave vector) domain that represents a stationary energy cascade. Our theoretical predictions are supported by experimental results and data from which we have computed the PDF and PSD of the local amplitude fluctuations η at the interface of an oil-water mixture. The PDF has a Gaussian form, and no exponential tails where found [8]. The PSD shows a power-law behavior $\sim f^{-2.75}$ within experimental error of the expected theoretical slope for the 4-wave interaction process. It must be noticed that the computed spectrum is also within experimental error of the theoretical $f^{-17/6}$ of the 3-wave interaction process. Because the difference between slopes of both spectra is small and since the theoretical conditions of this regime are not fully satisfied, the PSD alone does not guarantee a pure 4-wave regime and possibly 3-wave processes may not be completely suppressed. Nevertheless, their effects should be small in magnitude. For instance, a small difference in depth introduces a negligible correction in the limit $kh \gg 1$, because of the exponential dependence in kh. The effect of the small but nonzero Atwood number will be discussed elsewhere, but in principle, it can always be discarded in a frequency region of the spectrum. Despite the small difference between the 3-waves and 4-waves exponents, the highly symmetric PDF gives strong evidence of the $z \to -z$ symmetry as it has been observed in other systems [3] and shows that the asymmetry associated with the gravity-capillary regime is demonstrably suppressed. These experimental results give strong evidence how the system is tending towards a 4-wave capillary wave turbulence regime described by our theoretical developments.

The authors would like to acknowledge the support of Asefe and CONICYT and fruitful discussions with S. Fauve, F. Petrélis, E. Falcon, S. Rica, and A. M. Cazabat. C. F. acknowledges FONDECYT project 11090049.

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