Electric-Magnetic Duality and Topological Insulators

A. Karch
Department of Physics, University of Washington, Seattle, Washington 98195-1560, USA
(Received 22 July 2009; published 19 October 2009)

We work out the action of the $SL(2,\mathbb{Z})$ electric-magnetic duality group for an insulator with a nontrivial permittivity, permeability, and θ angle. This theory has recently been proposed to be the correct low-energy effective action for topological insulators. As applications, we give manifestly $SL(2,\mathbb{Z})$ covariant expressions for the Faraday rotation at orthogonal incidence at the interface of two such materials, as well as for the induced magnetic and electric charges, slightly clarifying the meaning of expressions previously derived in the literature. We also use electric-magnetic duality to find a gravitational dual for a strongly coupled version of this theory using the gauge/gravity correspondence.

DOI: 10.1103/PhysRevLett.103.171601 PACS numbers: 11.15.Tk, 11.25.Tq, 13.40.−f, 73.25.+i

Introduction.—The Maxwell-Lagrangian of classical electromagnetism can be modified by including a term proportional to $\theta\vec{E}\cdot\vec{B}$. This "axionic electrodynamics" was studied two decades ago to describe how standard electrodynamics would be modified in the presence of a nontrivial background for a putative axion field. Several novel effects that would result from such a term have been uncovered: a magnetic monopole surrounded by a small bubble of vacuum with no axion but an otherwise constant axion field throughout space would pick up a nontrivial electric charge [1] giving a nice realization of the Witten effect [2]. Magnetic charges induce electric mirror charges and vice versa in the presence of a planar domain wall across which θ jumps [3]. Reflection off such a domain wall induces a nontrivial rotation of the polarization of the field [4].

Recently, interest in this theory has seen a sudden revival as it was argued in [5] that it is also the low-energy effective action for so-called "topological insulators." In this context, one fascinating property of an interface with a nontrivial jump in θ has been emphasized [6]: an electric charge close to the interface induces a mirror *magnetic* charge. While this was originally derived in [3], [6] generalized the calculation to materials with nontrivial permittivity and permeability and, of course, emphasized the potential experimental relevance in the context of topological insulators.

Motivated by this observation, we take a look at the action of electric-magnetic duality in this theory. $U(1)$ gauge theories with nonzero θ are well known to exhibit an $SL(2,\mathbb{Z})$ duality group which strongly constrains the quantum physics, see, e.g., [7]. In this Letter, we work out the action of $SL(2,\mathbb{Z})$ on an insulator with nontrivial permittivity, permeability, and θ angle. This allows us to write the formula for the induced mirror charge as well as the one for the Faraday effect in a manifestly $SL(2,\mathbb{Z})$ covariant form. Along the way, we also clarify the role of the mirror charges calculated in [6]. While the mirror charges calculated in [6] allow for a correct determination of the electric and magnetic fields that are sourced by the charge next to the interface, what appears to be an electric mirror charge in that work is in fact a dyonic mirror charge.

Electric-magnetic duality maps interfaces between two ordinary insulators into an interface between an ordinary and a topological insulator in the dual theory. We use this fact to construct gravitational duals for topological insulators in strongly coupled $\mathcal{N}=4$ Super-Yang-Mills (SYM) theory via the gauge/gravity correspondence.

Electric-magnetic duality in matter.—Let us begin with standard Maxwell's equations for electromagnetism in matter in Gaussian units. They derive from a Lagrangian

$$S_0=\int d^3xdtL_0=\frac{1}{8\pi}\int d^3xdt\Big(\epsilon\vec{E}^2-\frac{1}{\mu}\vec{B}^2\Big). \quad (1)$$

ϵ and μ are the permittivity and permeability of the medium. The corresponding vacuum quantities ϵ_0, μ_0 in Gaussian units are set to 1, see, e.g., [8]. We also set c_0, the speed of light in vacuum, to 1. In matter, electromagnetic waves propagate with a velocity of light that is $c=(\mu\epsilon)^{-1/2}$. With these conventions, Maxwell's equations read

$$\begin{aligned}\vec{\nabla}\cdot\vec{D}&=4\pi\rho_e, &\quad \vec{\nabla}\times\vec{H}&=\frac{\partial\vec{D}}{\partial t}+4\pi\vec{j}_e,\\ \vec{\nabla}\cdot\vec{B}&=4\pi\rho_m, &\quad -\vec{\nabla}\times\vec{E}&=\frac{\partial\vec{B}}{\partial t}+4\pi\vec{j}_m,\end{aligned} \quad (2)$$

with the constitutive relations

$$\vec{D}=4\pi\frac{\delta L_0}{\delta\vec{E}}=\epsilon\vec{E}, \qquad \vec{H}=-4\pi\frac{\delta L_0}{\delta\vec{B}}=\frac{\vec{B}}{\mu}. \quad (3)$$

$\rho_{e,m}$ and $\vec{j}_{e,m}$ are the electric (magnetic) charge and current densities, respectively. In vacuum, these equations are invariant under duality rotations

$$\begin{pmatrix}\vec{D}\\ N\vec{B}\end{pmatrix}=\mathcal{R}_\xi\begin{pmatrix}\vec{D}'\\ N\vec{B}'\end{pmatrix}, \qquad \begin{pmatrix}N\vec{E}\\ \vec{H}\end{pmatrix}=\mathcal{R}_\xi\begin{pmatrix}N\vec{E}'\\ \vec{H}'\end{pmatrix}, \quad (4)$$

where

$$\mathcal{R}_\xi=\begin{pmatrix}\cos\xi & \sin\xi\\ -\sin\xi & \cos\xi\end{pmatrix}$$

is a standard rotation matrix and N an arbitrary normalization constant. In the presence of charges, this duality is still a symmetry of the classical equations of motion as long as the charges transform under the duality as well

$$\begin{pmatrix}\rho_e\\ N\rho_m\end{pmatrix}=\mathcal{R}_\xi\begin{pmatrix}\rho'_e\\ N\rho'_m\end{pmatrix},\qquad \begin{pmatrix}\vec{j}_e\\ N\vec{j}_m\end{pmatrix}=\mathcal{R}_\xi\begin{pmatrix}\vec{j}'_e\\ N\vec{j}'_m\end{pmatrix}. \quad (5)$$

Quantum mechanically, the total electric charge $q_e = \frac{1}{4\pi}\int d\vec{S}\cdot\vec{D}$ (S being any closed surface) has to be an integer multiple of the electron charge e, whereas the magnetic charge $q_m = \frac{1}{4\pi}\int d\vec{S}\cdot\vec{B}$ by the Dirac quantization condition should be an integer multiple of $g = \frac{e}{2\alpha}$, where $\alpha = \frac{e^2}{\hbar c}$ is the fine-structure constant. Only the special case of a duality transformation with $\xi = \pi/2$ and $N = 2\alpha$ is consistent with leaving this requirement invariant. This transformation is typically referred to as the S generator of electric-magnetic duality. In addition to its action on the fields, it only leaves the constitutive relations invariant if we exchange ϵ/N and $N\mu$. The speed of light $(\mu\epsilon)^{-1/2}$ is invariant under S duality.

For a topological insulator (or electromagnetism in the presence of a constant axion field), the effective action contains a second term. That is, $S = S_0 - S_\theta$, where

$$\begin{aligned} S_\theta &= \frac{\theta}{2\pi}\frac{\alpha}{16\pi}\int d^3x dt \epsilon^{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma}\\ &= \frac{\theta}{2\pi}\frac{\alpha}{2\pi}\int d^3x dt \vec{E}\cdot\vec{B}. \end{aligned} \quad (6)$$

Maxwell's equations are untouched by this addition. However, the constitutive relations are modified to

$$\vec{D} = \epsilon\vec{E} - \frac{\theta}{2\pi}(2\alpha\vec{B}), \qquad \vec{H} = \frac{\vec{B}}{\mu} + \frac{\theta}{2\pi}(2\alpha\vec{E}). \quad (7)$$

Classically, these equations are invariant under shifts of θ by any constant, $\theta = \theta' + C$, together with

$$\begin{aligned} &\begin{pmatrix}\vec{D}\\ 2\alpha\vec{B}\end{pmatrix}=\Lambda\begin{pmatrix}\vec{D}'\\ 2\alpha\vec{B}'\end{pmatrix},\qquad \begin{pmatrix}2\alpha\vec{E}\\ \vec{H}\end{pmatrix}=(\Lambda^T)^{-1}\begin{pmatrix}2\alpha\vec{E}'\\ \vec{H}'\end{pmatrix},\\ &\begin{pmatrix}\rho_e\\ 2\alpha\rho_m\end{pmatrix}=\Lambda\begin{pmatrix}\rho'_e\\ 2\alpha\rho'_m\end{pmatrix},\qquad \begin{pmatrix}\vec{j}_e\\ 2\alpha\vec{j}_m\end{pmatrix}=\Lambda\begin{pmatrix}\vec{j}'_e\\ 2\alpha\vec{j}'_m\end{pmatrix}, \end{aligned} \quad (8)$$

where

$$\Lambda = \begin{pmatrix}1 & -\frac{C}{2\pi}\\ 0 & 1\end{pmatrix}$$

and hence

$$(\Lambda^T)^{-1} = \begin{pmatrix}1 & 0\\ \frac{C}{2\pi} & 1\end{pmatrix}.$$

Classically, we can always use this symmetry to set θ to zero. Quantum mechanically, however, shifts in θ are only a symmetry if C is an integer multiple of 2π. While shifts in θ do leave the equations of motion invariant, they change the action and hence the weight $e^{iS/\hbar}$ in the path integral. As long as electric and magnetic fluxes are properly quantized, $S_\theta/\hbar$, however, is an integer multiple of θ (see, e.g., [7]), and so shifts of θ by integer multiples of 2π leave the path integral invariant. Consequently, values of θ between 0 and 2π are physically distinct. Only $\theta = 0$ and $\theta = \pi$ give a time-reversal symmetric theory. Time reversal takes θ into $-\theta$, so $\theta = 0$ is time reversal invariant as it stands, whereas $\theta = \pi$ is invariant after a shift of θ by 2π. A shift of θ by 2π is typically referred to as the T generator of electric-magnetic duality.

The full quantum mechanical duality group is obtained by repeated application of the T and S generators. The two together generate an $SL(2,\mathbb{Z})$ symmetry which acts on the fields as in Eq. (8) with a general matrix

$$\Lambda = \begin{pmatrix}\mathtt{a} & \mathtt{b}\\ \mathtt{c} & \mathtt{d}\end{pmatrix}$$

with integers $\mathtt{a}$, $\mathtt{b}$, $\mathtt{c}$, and $\mathtt{d}$ satisfying $\mathtt{ad} - \mathtt{bc} = 1$.

To determine the action of a general $SL(2,\mathbb{Z})$ transformation on the three parameters in the constitutive relation ϵ, μ, and θ note that the constitutive relation can be written in the compact form

$$\begin{pmatrix}\vec{D}\\ 2\alpha\vec{B}\end{pmatrix} = \mathcal{M}\begin{pmatrix}2\alpha\vec{E}\\ \vec{H}\end{pmatrix}, \quad (9)$$

with

$$\mathcal{M} = \frac{1}{c}\frac{2\alpha}{c\epsilon}\begin{pmatrix}\frac{\theta^2}{4\pi^2} + (\frac{c\epsilon}{2\alpha})^2 & -\frac{\theta}{2\pi}\\ -\frac{\theta}{2\pi} & 1\end{pmatrix}, \quad (10)$$

where we have replaced $1/\mu = c^2\epsilon$. The transformation equation (8) is a symmetry as long as $\mathcal{M}$ transforms as

$$\mathcal{M} = \Lambda\mathcal{M}'\Lambda^T. \quad (11)$$

$1/c^2 = \det(\mathcal{M})$ is duality invariant. This transformation on the matrix $\mathcal{M}$ can also be written as a transformation $\tau' = \frac{a\tau + b}{c\tau + d}$ for the complexified parameter $\tau = \frac{\theta}{2\pi} + i\frac{c\epsilon}{2\alpha}$. The duality transformation of μ then follows from the invariance of the speed of light, $1/\mu' = c^2\epsilon'$.

Duality covariant expression for mirror charges.—With the formalism we have set up in the previous section, it is easy to redo the calculation of [6] in a manifestly duality covariant fashion. The goal is to calculate the electric and magnetic fields resulting from a single test charge with charge

$$\vec{q} = \begin{pmatrix}q_e\\ 2\alpha q_m\end{pmatrix}$$

at a distance d from a planar interface (which we take to be the $z = 0$ plane) between two materials characterized by $\mathcal{M}_{1,2}$, that is, with different μ_i, ϵ_i, and θ_i ($i = 1, 2$). Of particular interest is the case with $\theta_1 = 0$ and $\theta_2 = \pi$, the interface between an ordinary and a topological conductor. But here our goal is to work out the generic case.

For a static electromagnetism problem in the absence of currents, the easiest way to proceed is to introduce electric and magnetic potentials $\Phi_{e,m}$ with $\vec{D} = -\vec{\nabla}\Phi_e$ and $\vec{B} = -\vec{\nabla}\Phi_m$. Above the interface, they are given by

$$\Phi_e^{\rm I} = \frac{q_e}{R_1} + \frac{q_e^{(2)}}{R_2}, \qquad \Phi_m^{\rm I} = \frac{q_m}{R_1} + \frac{q_m^{(2)}}{R_2} \qquad (12)$$

and below by

$$\Phi_e^{\rm II} = \frac{q_e}{R_1} + \frac{q_e^{(1)}}{R_1}, \qquad \Phi_m^{\rm II} = \frac{q_m}{R_1} + \frac{q_m^{(1)}}{R_1}, \qquad (13)$$

where $q_{e,m}^{(1,2)}$ are the mirror charges located a distance d above (for $q_{e,m}^{(1)}$) or below (for $q_{e,m}^{(2)}$) the interface. $R_1^2 = x^2 + y^2 + (d - z)^2$ and $R_2^2 = x^2 + y^2 + (d + z)^2$. Maxwell's equations in the absence of surface currents or charges as usual demand continuity of the perpendicular components of

$$\begin{pmatrix} \vec{D} \\ 2\alpha\vec{B} \end{pmatrix}$$

and the parallel components of

$$\begin{pmatrix} 2\alpha\vec{E} \\ \vec{H} \end{pmatrix}$$

or, in other words,

$$\vec{q}^{\,(1)} = -\vec{q}^{(2)}, \qquad (\vec{q} + \vec{q}^{(2)}) = \mathcal{T}(\vec{q} + \vec{q}^{(1)}), \qquad (14)$$

where $\mathcal{T} = \mathcal{M}_1\mathcal{M}_2^{-1}$. As the transformation properties of $\mathcal{M}$ imply that

$$\mathcal{T} = \Lambda\mathcal{T}'\Lambda^{-1}, \qquad (15)$$

these expressions are manifestly $SL(2,\mathbb{Z})$ covariant. They can be solved by matrix multiplication

$$\vec{q}^{\,(2)} = -\vec{q}^{(1)} = (\mathcal{T} + 1)^{-1}(\mathcal{T} - 1)\vec{q}. \qquad (16)$$

For the magnetic charges $q_m^{(1,2)}$, this expression is in perfect agreement with the corresponding expressions for $g_{1,2}$ in [6]. For the electric charges, there seems to be a disagreement with the values $q_{1,2}$ obtained in [6]. In particular, our mirror electric charges are equal opposite whereas theirs are equal. This apparent discrepancy can quickly be resolved by noting that $q_{1,2}$ in [6] are not actually the induced electric mirror charges. Proper electric charges (where physical charges are quantized in units of the electron charge e) are the sources of $\vec{D}$ flux. The q_i of that work were introduced as giving the sources of $\vec{E}$ flux instead. Because of the modified constitutive relations in the presence of the θ terms, these two differ by a multiple of the magnetic charge. As the mirror charges are simply a useful technical construct to obtain the correct electric and magnetic fields (and are not quantized in any case), this does not in any way alter the exciting findings of [6], which were mostly concerned with the induced magnetic charge. The proper mirror electric charges are related to the quantities calculated in [6] by $(q + q_e^{(1)} + q_m^{(1)}\alpha\theta_2/\pi)/\epsilon_2 = (q/\epsilon_1 + q_1)$ and $(q_e^{(2)} + q_m^{(2)}\alpha\theta_2/\pi)/\epsilon_1 = q_2$. After this substitution, their results are seen to be in full agreement with the expressions derived here.

Faraday effect in reflection.—Our tools can also be used to calculate reflection and refraction of light on interfaces between two materials with different ϵ, μ, and θ in a duality covariant framework. Snell's law is unmodified in the presence of a jump in θ, so the only task is to calculate the reflected and transmitted amplitudes. The full expressions have been worked out before in [9] and all we want to do here is exhibit $SL(2,\mathbb{Z})$ covariance. This is most easily done in the case of orthogonal incidence to which we will restrict ourselves from now on. In the presence of a nontrivial θ angle, Maxwell's equations still allow propagating wave solutions with $\omega = ck$, all fields orthogonal to the direction of propagation, $\hat{k}$, and

$$\begin{pmatrix} 2\alpha\vec{E} \\ \vec{H} \end{pmatrix} = c\hat{k} \times \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} \vec{D} \\ 2\alpha\vec{B} \end{pmatrix}. \qquad (17)$$

That is, $\vec{E}$ is still orthogonal to $\vec{B}$ and so is $\vec{H}$ to $\vec{D}$. However, the zweibein defined by $\hat{H}$, $\hat{D}$ is no longer aligned with the $\hat{E}$, $\hat{B}$ zweibein. For orthogonal incidence, all fields are parallel to the interface and the only boundary condition is continuity of

$$\vec{\mathcal{E}} = \begin{pmatrix} 2\alpha\vec{E} \\ \vec{H} \end{pmatrix}.$$

So we directly get $\vec{\mathcal{E}}_{\rm in} + \vec{\mathcal{E}}_R = \vec{\mathcal{E}}_T$, where the subscripts in R and T stand for the incoming, reflected, and transmitted fields, respectively. As $\vec{E}$ and $\vec{H}$ are not independent for a wave solution, we get a second equation from Eq. (17). Together with our constitutive relation equation (9) and the fact that for orthogonal incidence we simply have $\hat{k}_{\rm in} = \hat{k}_T = -\hat{k}_R$, we get for a wave incoming from medium 2

$$c_2\mathcal{M}_2(\vec{\mathcal{E}}_{\rm in} - \vec{\mathcal{E}}_R) = c_1\mathcal{M}_1\vec{\mathcal{E}}_T = c_1\mathcal{M}_1(\vec{\mathcal{E}}_{\rm in} + \vec{\mathcal{E}}_R). \quad (18)$$

As for the mirror charge calculation above, this can now be solved by matrix multiplication in terms of $\tilde{\mathcal{T}} = \frac{c_1}{c_2}\mathcal{M}_2^{-1}\mathcal{M}_1$

$$\vec{\mathcal{E}}_R = (1 + \tilde{\mathcal{T}})^{-1}(1 - \tilde{\mathcal{T}})\vec{\mathcal{E}}_{\rm in}, \qquad (19)$$

which is in nice agreement with the formulas in [9]. Note, in particular, that for an incoming wave with

$$\vec{\mathcal{E}}_{\rm in} = \begin{pmatrix} 2\alpha\vec{E} \\ c_2\epsilon_2\hat{k} \times \vec{E} + 2\alpha\theta_2/(2\pi)\vec{E} \end{pmatrix},$$

the reflected wave has a component in the $\hat{k} \times \vec{E}$ direction. That is, the polarization got rotated by an angle β with

$$\tan(\beta) = 4\alpha\frac{(\theta_2 - \theta_1)}{2\pi}\frac{\mu_1\sqrt{\epsilon_2\mu_2}}{\epsilon_2\mu_1 - \mu_2\epsilon_1} + \mathcal{O}(\alpha^2). \qquad (20)$$

Like the induced magnetic mirror charge, this nontrivial Faraday angle is a direct consequence of having a jump in θ. This formula is in perfect agreement with [3,5].

Dynamical axion.—Another interesting property of topological insulators recently uncovered in [10] is that once the θ angle is promoted to a dynamical axion field, for example, by considering antiferromagnetic long range order, a new mode called an "axionic polariton" appears in the low-energy spectrum. In the presence of a background magnetic field, the axionic polariton is a coupled mode of light and axion field. Its dispersion relation exhibits two branches, separated by a gap whose magnitude can be dialed by dialing the background magnetic field. It is interesting to ask if this phenomenon can also be described in our $SL(2,\mathbb{Z})$ covariant framework. As $SL(2,\mathbb{Z})$ maps θ into ϵ, promoting just the axion to a dynamical field is inconsistent with duality. Duality can be preserved by also promoting ϵ and with it $\mu^{-1} = c^2\epsilon$ into a dynamical field, the dilaton. One can ask whether the diable gap persists in the $SL(2,\mathbb{Z})$ invariant system with a dynamical axion-dilaton pair. Unfortunately, with a dynamic dilaton, a constant background magnetic field is no longer a stationary background solution. The $\vec{B}^2/\mu$ term in the Lagrangian acts as a source for the dilaton. A constant B field at time $t = 0$ will lead to a time-dependent dilaton solution that runs away to weak coupling.

Gauge/gravity realization via Janus.—Using $SL(2,\mathbb{Z})$, we can also construct a strongly coupled analog of the interface between topological and regular insulator using the gauge/gravity correspondence. Note that an $SL(2,\mathbb{Z})$ duality transformation takes a theory in which only ϵ and μ jump across the interface into one that also has a nontrivial jump in θ. There is a well-known example of an gauge/gravity setup that corresponds to an interface with jump in μ and ϵ, the Janus solution [11]. The analog of electromagnetism here is $\mathcal{N} = 4$ SYM theory, which also mediates a Coulomb force between charge carriers. The theory has a dual supergravity description in the limit that the analog of α is very large. In addition, the number of degrees of freedom N_c is large in the supergravity limit so that the theory is effectively classical. The Janus solution describes an interface in this theory across which ϵ and μ jump but with a uniform speed of light, that is, $c_1 = c_2$ [11,12]. The dual supergravity description has the full $SL(2,\mathbb{R})$ duality of the classical theory. New axionic Janus solutions with a nontrivial jump in θ can be produced by applying an $SL(2,\mathbb{R})$ transformation on the original Janus solution. One obtains a supergravity solution in which, in addition to the dilaton that already had a nontrivial profile in the original Janus solution, the axion pseudoscalar is also turned on. It was shown in [13] that the most general supergravity solution with only the dilaton, axion, and metric turned on and preserving the same global symmetry as the original Janus solution can be obtained by an $SL(2,\mathbb{R})$ transform of Janus.

This way, we can find supergravity solutions realizing any change in θ we wish. As the speed of light is duality invariant, all of these axionic Janus solutions have the property that $c_1 = c_2$. It is easy to see why the Janus ansatz does not allow for interfaces across which c jumps: in the Janus construction, one is looking for solutions that preserve the $2+1$-dimensional relativistic conformal symmetry, $SO(3,2)$. This includes the $(2+1)$D Lorentz group as a subgroup. In a theory with a varying speed of light, this is not a good symmetry. To look for generalized Janus solutions realizing an interface with a generic jump in μ, ϵ, and θ, one has to look for metrics in which g_{tt}/g_{xx} has a nontrivial dependence on the radial coordinate. Unlike in the Janus case, where the $SO(3,2)$ symmetry assured that Einstein's equations boil down to simple ordinary differential equations, these generalized Janus solutions have to be solutions to partial differential equations in two variables. It would be nice to see if they can be constructed explicitly as they would be new examples of singularity free solutions of type IIB supergravity. For the purpose of realizing interfaces with a nontrivial jump in θ, it is sufficient to work with $c_1 = c_2$.

Conclusions.—In this letter we have laid out the action of the $SL(2,\mathbb{Z})$ electric-magnetic duality group in the effective theory proposed to describe topological insulators. We demonstrated that several classical phenomena in these unusual materials, like an induced magnetic mirror charge and a nontrivial Faraday effect, can be compactly worked out in a manifestly $SL(2,\mathbb{Z})$ covariant way. We also used the fact that this duality relates the interface between two ordinary insulators to an interface between a topological and an ordinary insulator to identify the supergravity description of a strongly coupled analog of topological insulators. We hope that in the future electric-magnetic duality will be useful to unravel the properties of these interesting materials.

Thanks to S. Hartnoll, C. Herzog, K. Jensen, D. Son, and especially L. Yaffe for useful discussions. This work was supported in part by U.S. Department of Energy under Grant No. DE-FG02-96ER40956. I would also like to thank the KITP in Santa Barbara for hospitality during the final stages of this work. There this research was supported in part by the National Science Foundation under Grant No. PHY05-51164.

[1] F. Wilczek, Phys. Rev. Lett. **58**, 1799 (1987).
[2] E. Witten, Phys. Lett. **86B**, 283 (1979).
[3] P. Sikivie, Phys. Lett. **137B**, 353 (1984).
[4] M. C. Huang and P. Sikivie, Phys. Rev. D **32**, 1560 (1985).
[5] X.-L. Qi, T. Hughes, and S.-C. Zhang, Phys. Rev. B **78**, 195424 (2008).
[6] X.-L. Qi, R. Li, J. Zang, and S.-C. Zhang, Science **323**, 1184 (2009).
[7] E. Witten, Selecta Mathematica Sovietica **1**, 383 (1995).
[8] J. D. Jackson, *Classical Electrodynamics* (Wiley, New York, 1998), 3rd ed.
[9] Y. N. Obukhov and F. W. Hehl, Phys. Lett. A **341**, 357 (2005).
[10] R. Li, J. Wang, X. Qi, and S.-C. Zhang, arXiv:0908.1537v1.
[11] D. Bak, M. Gutperle, and S. Hirano, J. High Energy Phys. 05 (2003) 072.
[12] A. B. Clark, D. Z. Freedman, A. Karch, and M. Schnabl, Phys. Rev. D **71**, 066003 (2005).
[13] E. D'Hoker, J. Estes, and M. Gutperle, Nucl. Phys. **B757**, 79 (2006).