Liquid Crystal Cells with "Dirty" Substrates

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We explore liquid crystal order in a cell with a "dirty" substrate imposing a random surface pinning. Modeling such systems by a random-field *xy* model with *surface* heterogeneity, we find that orientational order in the three-dimensional system is marginally unstable to such surface pinning. We compute the Larkin length scale, and the corresponding surface and bulk distortions. On longer scales we calculate correlation functions using the functional renormalization group and matching methods, finding a universal logarithmic and double-logarithmic roughness in two and three dimensions, respectively. For a finite thickness cell, we explore the interplay of homogeneous-heterogeneous substrate pair and detail crossovers as a function of disorder strength and cell thickness.

Recently there has been considerable progress in understanding the phenomenology of ordered states subject to random heterogeneities present in real materials [1]. While the focus has been on *bulk* heterogeneity, there are realizations in which random pinning is confined to a *surface* of the sample. A technologically relevant example (inset of Fig. 1) is that of a liquid-crystal cell (e.g., of a laptop display), where a "dirty" substrate imposes random pinning, that competes with liquid-crystal ordering. Manifestations of heterogeneous surface anchoring include common schlieren textures and multistability effects [2] observed in nematic cells, as well as layer distortions in smectic cells [3].

In this Letter we study such *surface* randomly pinned systems modeling them via a *d*-dimensional *xy* model with the heterogeneity confined to a $(d - 1)$-dimensional surface. One might *a priori* expect surface pinning effects to be vanishingly weak compared to the ordering tendency of the homogeneous bulk, and thus the *xy* order to be stable to weak surface disorder in *any* dimension.

Our finding contrasts sharply with this intuition. Namely, our key qualitative observation is that the *xy* order on the surface of such *d*-dimensional system with $d \leq d_{lc} = 3$ is always destabilized by arbitrary weak random surface pinning [4].

Our finding can be understood from a generalization of the bulk Imry-Ma argument [5,6] to the surface pinning problem. For an ordered region of size L, the surface random field can lower the energy by $E_{pin} \sim V_p \sqrt{N_p} \sim \Delta_f^{1/2} L^{(d-1)/2}$, where V_p is a typical pinning strength with zero mean and variance $\Delta_f \approx V_p^2 / \xi_0^{d-1}$ (ξ_0 is the pinning correlation length) and N_p is the number of surface pinning sites. Since surface distortions on scale L extend a distance L into the bulk, the corresponding elastic energy cost scales as $E_e \sim KL^{d-2}$, where K is the elastic stiffness. For $d < 3$ and $L > \xi_L \sim (K^2/\Delta_f)^{(1/(3-d))}$ the surface heterogeneity dominates over the elastic energy, and thus on these long scales always destroys long-range *xy* order for

arbitrarily weak surface pinning [4]. A more detailed 3D analysis gives

$$\xi_L \approx ae^{cK^2/\Delta_f}, \quad \text{for } d = 3, \tag{1}$$

where $c = 8\pi^3$, and a is a microscopic cutoff, set by liquid-crystal molecular size of a few nanometers.

Although these (heterogeneous) substrate-induced distortions from short scales $x < \xi_L$ only penetrate a distance ξ_L into the bulk, decaying exponentially beyond this scale, the bulk ordered state never recovers. That is, strikingly, we find that logarithmically rough distortions, persist an arbitrary long distance z from the dirty substrate, as summarized by $C(\mathbf{x}, z, z') = \overline{\langle (\phi(\mathbf{x}, z) - \phi(0, z'))^2 \rangle}$, with (for $x \gg \xi_L$ in 2D)

$$C_{2D}^{(\infty)}(\mathbf{x}, z, z) \approx b_2 e^{-2z/\xi_L} + \frac{\pi^2}{9} \ln\left[1 + \frac{x^2}{(2z + \xi_L)^2}\right], \tag{2}$$

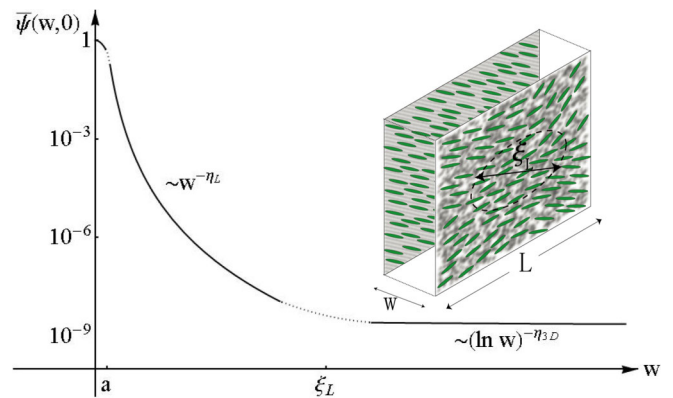(b_2 is a weak function of $2z/\xi_L$), and in 3D given by a double-logarithmic asymptotics (12).

FIG. 1 (color online). Orientational order parameter $\overline{\psi}(w, 0)$ (controlling light transmission through a liquid-crystal cell) at the random pinning substrate of a 3D Dirichlet cell of thickness w, illustrated in the inset.

With an eye to liquid-crystal cell applications [2,3], we extend our analysis to a nematic cell of finite thickness w, with a heterogeneous front substrate, and a back substrate with a homogeneous Dirichlet ($\mathcal{D}$) or Neumann ($\mathcal{N}$) boundary condition, that can be imposed by substrate treatments, for example, surface polymer coating and rubbing. For the $\mathcal{D}$-cell long-range orientational order is stable to weak pinning, but with a value of *surface* orientational order parameter $\overline{\psi}$ that exhibits a strong crossover as a function of w/ξ_L as shown in Fig. (1), that in 3D is given by

$$
\overline{\psi} \approx \begin{cases} \left(\frac{a}{w}\right)^{\eta_L}, & \text{thin cell,} \quad a \ll w \ll \xi_L, \\ e^{-\alpha \left[\frac{\ln(\xi_L/a)}{\ln(w/a)}\right]\eta_{3D}}, & \text{thick cell,} \quad w \gg \xi_L, \end{cases} \tag{3}
$$

where $\eta_{3D} = \pi^2/18$ is a universal exponent and $\alpha = 2\pi^2$, $\eta_L = 2\pi^2/\ln(\xi_L/a)$ are nonuniversal constants. In contrast, the long-range orientational order is unstable for the $\mathcal{N}$ cell, with the ratio w/ξ_L determining the rate of decay with the transverse cell size, L.

We now outline the analysis that leads to above results. As a "toy" model of a nematic liquid-crystal cell [7] with a dirty substrate and thickness w, we employ a d-dimensional *surface* random-field xy model

$$
H = \int_{-\infty}^{\infty} d^{d-1}x \int_0^w dz \left[\frac{K}{2}(\nabla\phi(\mathbf{r}))^2 - V[\phi(\mathbf{r}), \mathbf{x}]\delta(z) \right]. \tag{4}
$$

In the equation above $\phi(\mathbf{r})$ is the xy-field distortion (azimuthal nematic director angle in the plane of the substrate) at a point $\mathbf{r} \equiv (\mathbf{x}, z)$, the random surface pinning potential $V[\phi(\mathbf{x}, z), \mathbf{x}]\delta(z)$, characterized by a Gaussian distribution with a variance $\overline{V(\phi, \mathbf{x})V(\phi', \mathbf{x}')} = R(\phi - \phi')\delta^{d-1}(\mathbf{x} - \mathbf{x}')$, is confined to a front substrate at $z = 0$, and at the back homogeneous substrate at $z = w$ we impose either a Dirichlet $[\phi(\mathbf{x}, z)|_{z=w} = 0]$ or a Neumann $[\partial_z\phi(\mathbf{x}, z)|_{z=w} = 0]$ boundary condition for the $\mathcal{D}$ and $\mathcal{N}$ cells, respectively. The long-scale behavior of the periodic (period 2π) variance function $R(\phi)$ characterizes low-temperature properties of the system.

Because the random pinning potential in (4) is confined to the front substrate at $z = 0$, no nonlinearities appear in the bulk ($0 < z < w$) of the cell. Consequently, it is convenient to exactly eliminate the bulk harmonic degrees of freedom $\phi(\mathbf{x}, z)$ in favor of the random substrate field $\phi_0(\mathbf{x}) \equiv \phi(\mathbf{x}, z = 0)$ [8]. This can be done via a constrained path integral by integrating out $\phi(\mathbf{x}, z)$ with a constraint $\phi(\mathbf{x}, z = 0) = \phi_0(\mathbf{x})$, thereby obtaining an effective $(d-1)$-dimensional Hamiltonian for $\phi_0(\mathbf{x})$ [8]. Equivalently (for $T = 0$ properties), we can eliminate $\phi(\mathbf{x}, z)$ by solving the bulk Euler-Lagrange equation $\nabla^2\phi(\mathbf{r}) = 0$. To this end, we solve $\partial_z^2\phi(\mathbf{q}, z) - q^2\phi(\mathbf{q}, z) = 0$, for $\phi(\mathbf{q}, z) = \int d^{d-1}x\phi(\mathbf{x}, z)e^{-i\mathbf{q}\cdot\mathbf{x}}$. The solutions for boundary conditions of interest are summarized by $\phi^{(a)}(\mathbf{q}, z) = \phi_0(\mathbf{q})\varphi^{(a)}(q, z)$, with mode functions

$\varphi^{(a)}(q, z)$ given by e^{-qz} ($a = \infty$), $\sinh[q(w - z)]/\sinh(qw)$ ($a = \mathcal{D}$), $\cosh[q(w - z)]/\cosh(qw)$ ($a = \mathcal{N}$).

Substituting these into (4) we obtain a $(d-1)$-dimensional Hamiltonian $H_s = \int \frac{d^{d-1}q}{(2\pi)^{d-1}} \frac{1}{2}\Gamma_q^{(a)}|\phi_0(\mathbf{q})|^2 - \int d^{d-1}xV[\phi_0(\mathbf{x}), \mathbf{x}]$, confined to the random substrate at $z = 0$, with elastic kernels given by

$$
\Gamma_q^{(\infty)} = Kq, \qquad w \to \infty, \tag{5}
$$

$$
\Gamma_q^{(\mathcal{D})} = Kq\coth(qw), \qquad \text{Dirichlet,} \tag{6}
$$

$$
\Gamma_q^{(\mathcal{N})} = Kq\tanh(qw), \qquad \text{Neumann.} \tag{7}
$$

The finite thickness $\Gamma_q^{(\mathcal{D})}$ and $\Gamma_q^{(\mathcal{N})}$ reduce to Kq for $w \to \infty$, as required. The q nonanalyticity and long wavelength stiffening (compared to the bulk Kq^2 elasticity) of the latter arises due to a mediation of surface distortions by long-range deformations in the bulk of the cell. In the opposite limit of a thin cell and long scales, the Dirichlet kernel reduces to a "massive" one, K/w, and the Neumann kernel simplifies to Kwq^2 of an ordinary surface (without a contact with the bulk) xy model.

For convenience we use the replica "trick" [9] to work with a translationally invariant theory, at the expense of introducing n replica fields (with the $n \to 0$ limit taken at the end). The disorder-averaged free energy is given by $\overline{F} = -T\overline{\ln Z} = -T\lim_{n\to 0}(\overline{Z^n} - 1)/n$, with $\overline{Z^n} = \int[d\phi_0^\alpha]e^{-H_s^{(r)}[\phi_0^\alpha]/T}$, where the effective translation invariant replicated Hamiltonian $H_s^{(r)}[\phi_0^\alpha]$ is given by

$$
H_s^{(r)} = \sum_\alpha^n \int \frac{d^{d-1}q}{(2\pi)^{d-1}} \frac{1}{2}\Gamma_q^{(a)}|\phi_0^\alpha(\mathbf{q})|^2
$$
$$
- \frac{1}{2T}\sum_{\alpha,\beta}^n \int d^{d-1}xR[\phi_0^\alpha(\mathbf{x}) - \phi_0^\beta(\mathbf{x})]. \tag{8}
$$

The advantage of this dimensional reduction is that the problem becomes quite similar to that of the bulk random pinning [10–12] in one lower dimension and with a modified elasticity encoded in $\Gamma_q^{(a)}$, (5)–(7).

The importance of surface pinning can be assessed by computing distortions $\overline{\langle\phi^2\rangle}$ within the Larkin random torque approximation [5] $\tau(\mathbf{x}) = \partial_{\phi_0}V[\phi_0(\mathbf{x}), \mathbf{x}]|_{\phi_0=0}$ to the random potential $V(\phi_0, \mathbf{x})$, with inherited Gaussian statistics and variance $\overline{\tau(\mathbf{x})\tau(\mathbf{x}')} = -R''(0)\delta^{d-1}(\mathbf{x} - \mathbf{x}') \equiv \Delta_f\delta^{d-1}(\mathbf{x} - \mathbf{x}')$.

Simple analysis gives $\overline{\langle\phi_0^2(\mathbf{x})\rangle}^{(a)} \approx \int_\mathbf{q} \frac{\Delta_f}{[\Gamma_q^{(a)}]^2}$, diverging with surface extent L for an infinitely thick cell, $w \to \infty$ for $d \leq d_{\text{lc}} = 3$, and for an $\mathcal{N}$ cell of thickness w for $d \leq d_{\text{lc}}^{(\mathcal{N})} = 5$. The latter is consistent with the $d_{\text{lc}}^{\text{bulk}} = 4$ [5,6] for a $(d-1)$-dimensional random-field xy model, to which a thin $\mathcal{N}$ cell reduces. In contrast, for a $\mathcal{D}$ cell these orientational root-mean-squared (rms) fluctuations are finite, but grow with cell thickness w.

We identify the substrate extent L_* at which these rms fluctuations grow to order 2π with the Larkin length scale $\xi_L^{(a)}$ [5]. For an infinite cell thickness we find ξ_L given by Eq. (1). For a finite thickness $\mathcal{D}$ cell we find

$$\xi_L^{(\mathcal{D})} \approx \begin{cases} \xi_L, & \xi_L \ll w, \\ \frac{c_d w^{\nu_d+1}}{(\xi_L^c - \xi_L)^{\nu_d}}, & \xi_L \approx \xi_L^c, \end{cases} \qquad (9)$$

with $\xi_L^c = a_d w$ the "critical" Larkin length, $c_2 = 1$, $a_2 \approx 1.71$, $\nu_2 = 1$, and $c_3 \approx 0.79$, $a_3 \approx 1.23$, $\nu_3 = 1/2$. Thus for a cell thicker than the Larkin length, ξ_L, the back $\mathcal{D}$ substrate has weak influence on the range of the xy order, dominated by the random front substrate. However, for a thin cell, such that ξ_L spans the cell thickness, the $\mathcal{D}$ substrate effectively enforces the xy-order alignment across cell, suppressing ϕ_0^{rms} below 2π and thereby driving the cell Larkin scale, $\xi_L^{(\mathcal{D})}$ to diverge.

However, this divergence is *not* an indication of a sharp transition. Rather, it is a signal of a crossover from a weakly xy-ordered state (at strong disorder and thick cell) for $\xi_L \ll w$ to a strongly xy-ordered state (at weak disorder and thin cell) for $\xi_L \gg w$. In both limits the aligning $\mathcal{D}$ substrate dominates over the random one, leading to a long-range xy order. For the $\mathcal{N}$ cell we instead find

$$\xi_L^{(\mathcal{N})} \sim \begin{cases} \xi_L, & w \gg \xi_L, \\ w^{\gamma_d} \begin{cases} \sqrt{\ln(\xi_L/w)}, & d = 3, \\ (\xi_L)^{((3-d)/(5-d))}, & d < 3, \end{cases} & w \ll \xi_L, \end{cases} \qquad (10)$$

with $\gamma_d = 2/(5-d)$, lower limit corresponding to a thin $(d-1)$-dimensional "film" pinned by $(d-1)$-dimensional "bulk" disorder.

On length scales longer than the crossover scale $\xi_L^{(a)}$, ϕ_0 distortions grow into a nonlinear regime, where random torque model is inadequate, and the effects of the random potential $V[\phi_0(\mathbf{x}), \mathbf{x}]$ must be treated nonperturbatively. This can be done systematically using a functional renormalization group (FRG) analysis [11,12] in an expansion in $\epsilon = d_{\mathrm{lc}} - d = 3 - d$.

To this end we employ the standard momentum-shell FRG [11–13], integrating out perturbatively (to one-loop) in $R[\phi_0^\alpha(\mathbf{x}) - \phi_0^\beta(\mathbf{x})]$ the short-scale modes with support in an infinitesimal momentum shell $\Lambda/b < q < \Lambda \equiv 1/a$, with $b = e^{\delta\ell}$. Taking $w(b) = b^{-1}w$, for $T = 0$ we obtain

$$\partial_\ell \hat{R}_a(\phi) = \epsilon^{(a)}(\ell)\hat{R}_a(\phi) + \frac{1}{2}(\hat{R}_a''(\phi))^2 - \hat{R}_a''(\phi)\hat{R}_a''(0), \qquad (11)$$

with effective eigenvalues given by $\epsilon^{(\mathcal{D},\mathcal{N})}(\ell) = \epsilon \mp \frac{4\Lambda w(\ell)}{\sinh[2\Lambda w(\ell)]}$ and dimensionless disorder variance functions defined by $\hat{R}_{\mathcal{D},\mathcal{N}}(\phi) \equiv \frac{C_{d-1}\Lambda^{d-3}}{K^2 \coth^{\pm 2}(\Lambda w)} R(\phi)$ for the $\mathcal{D}$ and $\mathcal{N}$ choices of boundary conditions on the back substrate. In Eq. (11), the prime indicates a partial derivative with respect to ϕ, and $C_d = 1/[\Gamma(d/2)2^{d-1}\pi^{d/2}]$.

For $w \to \infty$, aside from the Gaussian eigenvalue of $\epsilon = 3 - d$ (rather than $4 - d$), the flow of $\hat{R}$ for our *surface* pinning problem reduces to that of the *bulk* pinning problem [4,11,12,14]. In 3D the disorder is marginally irrelevant, vanishing according to $\hat{R}''(\phi, \ell) = \frac{1}{\ell}[-\frac{1}{6}(\phi - \pi)^2 + \frac{\pi^2}{18}]$, and flows to a fixed point $\hat{R}_*''(\phi) = \epsilon[-\frac{1}{6}(\phi - \pi)^2 + \frac{\pi^2}{18}]$ for $d < 3$ [4,14]. In 2D T is no longer irrelevant leading to a qualitatively distinct behavior dominated by a single harmonic of the random potential, which in turn leads to a finite T super-roughening transition [13,15]. In the opposite extreme of a microscopically thin cell, such that $w \ll a$, the eigenvalues reduce to $\epsilon^{(\mathcal{D})}(\ell) \approx 1 - d$ and $\epsilon^{(\mathcal{N})}(\ell) \approx 5 - d$. These correspond to flows of a $(d-1)$-dimensional *bulk* random-field xy model, with the $\mathcal{D}$ cell effectively subjected to a uniform external field due to the back substrate, dominating over the random pinning in the physical dimensions ($d = 2, 3$).

For a more realistic case of a finite cell thickness w, there is a crossover at scale $b_w^* \equiv w/a$ from a thick d dimensional cell at small ℓ such that $\Lambda w(\ell) \gg 1$ to an effective $(d-1)$-dimensional "film" for $\Lambda w(\ell) \ll 1$. Another crossover scale encoded in flow equations, Eq. (11), is set by a scale b_L^* at which the nonlinear terms become comparable to the linear ones, where the flow leaves the vicinity of the Gaussian fixed point. This latter scale is simply set by the Larkin length, with $b_L^* = \xi_L/a$. As we will see blow, the nature of distortions strongly depends on the relative size of these two crossover scales and on the type of boundary condition on the uniform substrate. We naturally designate the two cases, $w \ll \xi_L$ and $w \gg \xi_L$, as thin and thick cells, respectively.

We can now utilize these FRG flows to compute the orientational correlation function $C^{(a)}$ by establishing a relation for it at different scales: $C^{(a)}[\mathbf{q}, K, w, \hat{R}_a] = e^{(d-1)\ell}C^{(a)}[\mathbf{q}e^\ell, K(\ell), w(\ell), \hat{R}_a(\ell)]$, where the prefactor comes from the dimensional rescalings, keeping T fixed. Choosing ℓ_* such that $qe^{\ell_*} = \Lambda$, allows us to reexpress ℓ_* inside $C^{(a)}$ in terms of q. This gives $C^{(a)}[\mathbf{q}, K, w, \hat{R}_a] \approx -(\frac{\Lambda}{q})^{d-1}\frac{R''(0, \ell_*)}{[\Gamma_\Lambda^{(a)}(\ell_*)]^2}$, which explicitly requires an analysis of the flow for specific boundary conditions.

For an infinitely thick cell ($w \to \infty$) and $q < 1/\xi_L$ this gives in $d < 3$: $C_*^{(\infty)}[\mathbf{q}, K, \infty, \hat{R}] \approx \frac{1}{q^{d-1}}\frac{(3-d)\pi^2}{9C_{d-1}}$, and in 3D: $C_*^{(\infty)}[\mathbf{q}, K, \infty, \hat{R}] \approx \frac{-1}{q^2\ln(qa)}\frac{\pi^2}{9C_2}$, from which real space correlations on in-plane scales $x > \xi_L$ (inaccessible within Larkin approximation) can be computed. Fourier-transforming $C^{(\infty)}(\mathbf{q})$ we find $C^{(\infty)}(\mathbf{x}, z, z) \approx C_L^{(\infty)}(\mathbf{x}, z) + C_*^{(\infty)}(\mathbf{x}, z)$, where $C_L^{(\infty)}(\mathbf{x}, z)$ is the contribution from short scales, $\xi_L^{-1} < q < a^{-1}$, where random torque model is valid, given by $C_L^{(\infty)}(\mathbf{x}, z, z) = (3-d)8\pi^2(\frac{2z}{\xi_L})^{3-d}\Gamma(d-3, 2z/\xi_L, 2z/a) \approx b_d e^{-2z/\xi_L}$ with $\Gamma(p, z_1, z_2) = \int_{z_1}^{z_2} t^{p-1}e^{-t}dt$ the generalized incomplete Gamma function. The second long-scale part, $C_*^{(\infty)}(\mathbf{x}, z)$ is a universal contribution, a Fourier transform of $C_*^{(\infty)}[\mathbf{q}, K, \infty, \hat{R}]$,

which when combined with $C_L^{(\infty)}$ at low T, in 2D gives Eq. (2). In 3D this matching procedure gives [4]

$$C_{3D}^{(\infty)}(\mathbf{x}, z, z) \sim \frac{2\pi^2}{9} \begin{cases} \ln[\frac{\ln(x/a)}{\ln(\xi_L/a)}], & 2z \ll \xi_L \ll x, \\ \ln[\frac{\ln(x/a)}{\ln(2z/a)}], & \xi_L \ll 2z \ll x, \\ \frac{x^2}{16z^2}\frac{1}{\ln(2z/a)}, & \xi_L \ll x \ll 2z. \end{cases} \quad (12)$$

Another quantity of interest is the average orientational order parameter, $\overline{\psi}(z) = \overline{\langle e^{i\phi}\rangle} \approx e^{-\overline{\langle \phi^2\rangle}/2}$, where somewhat crudely we approximated it by assuming Gaussian-correlated $\phi(\mathbf{r})$.

From $C^{(a)}(\mathbf{q})$ for $w \to \infty$ and for arbitrarily thick $\mathcal{N}$ cell we find that $\phi_{\text{rms}}^2(L, z) = \overline{\langle \phi^2\rangle}$ grows without bound with planar cell extent L. Thus for $a = \infty$ and $a = \mathcal{N}$ cells in $d \leq 3$ the orientational order parameters, $\overline{\psi}^{(\infty, \mathcal{N})}$ vanish in the thermodynamic limit for arbitrarily weak surface heterogeneity.

In contrast, in a $\mathcal{D}$ cell the growth of $\phi_{\text{rms}}^2(L, z)$ is suppressed by the aligning homogeneous $\mathcal{D}$ substrate. Thus the order is stable for an arbitrarily thick cell, characterized by $\overline{\psi}(w, z)$ that is a strong function of w/ξ_L, computable via above matching analysis. For a thin cell ($w \ll \xi_L$), $w(\ell) = e^{-\ell}w$ reaches the microscopic scale a at $e^{\ell_w^*} = w/a$ and therefore $\epsilon^{(\mathcal{D})}(\ell > \ell_w^*) \approx \epsilon - 2 = 1 - d$ before $e^{\ell_L^*} = \xi_L/a$. Since beyond ℓ_w^*, $\epsilon^{(\mathcal{D})}(\ell > \ell_w^*) < 0$, pinning is irrelevant with its growth cutoff at scale $e^{\ell_w^*}$. In this case, scales beyond ξ_L are not probed (the flow never leaves the vicinity of the Gaussian fixed point) and $\phi_{\text{rms}}(w, z)$ can be accurately computed within the random torque model, cut off by w. Thus, utilizing our earlier definition of ξ_L, for a *thin* $\mathcal{D}$ cell in $d < 3$, we find $\phi_{\text{rms}}^2(w, 0) \approx 4\pi^2(\frac{w}{\xi_L})^{3-d}$.

For a thick $\mathcal{D}$ cell ($w \gg \xi_L$), the flow instead crosses over to the vicinity of the nontrivial fixed point $R_*(\phi)$ (leaves the Gaussian fixed point) *before* it is cut off by the finite w. Thus on these intermediate scales (defined by $w/a \equiv e^{\ell_w^*} > e^\ell > e^{\ell_L^*} \equiv \xi_L/a$) despite being ultimately cut off by w, ϕ_0 distortions are large, requiring a nonperturbative treatment of surface pinning. As for above computation of $C^{(\infty)}(\mathbf{x})$, in this regime $\overline{\psi}(w, 0) = e^{-\phi_{\text{rms}}^2(w,0)/2}$ can be well estimated by the matching FRG analysis.

Thus, neglecting the subdominant contribution from scales longer than w, approximating the ϕ_0 correlator $C(\mathbf{q})$ by its fixed-point value, valid for $w \gg \xi_L$ [such that the flow approaches the vicinity of the nontrivial fixed point, $R_*(\phi, \ell)$], and approximating $C(\mathbf{q})$ by its Gaussian fixed-point expression, valid for $\xi_L^{-1} < q < a^{-1}$, for a thick cell $w \gg \xi_L$ and $d < 3$, we find $\overline{\langle \phi_0^2\rangle} \approx \frac{\epsilon\pi^2}{9} \times \ln(w/\xi_L) + 4\pi^2$. Putting these crossovers together for $d < 3$ we find

$$\overline{\psi}(w, 0) \approx \begin{cases} e^{-2\pi^2(w/\xi_L)^{3-d}}, & \text{thin cell, } w \ll \xi_L, \\ e^{-2\pi^2(\frac{\xi_L}{w})^{\eta_d^*}}, & \text{thick cell, } w \gg \xi_L. \end{cases} \quad (13)$$

In 3D such matching analysis gives $\overline{\phi_0^2} \approx 4\pi^2\frac{\ln(w/a)}{\ln(\xi_L/a)}$ for a thin cell ($w \ll \xi_L$), and $\overline{\phi_0^2} \approx \frac{\pi^2}{9}\ln[\frac{\ln(w/a)}{\ln(\xi_L/a)}] + 4\pi^2$ for a thick cell ($w \gg \xi_L$), leading to $\overline{\psi}(w, 0)$ in (3).

To conclude, above we have studied stability of an ordered *xy* model to random *surface* pinning and discussed these results in the context of a nematic liquid-crystal cell with a "dirty" nonrubbed substrate. We found that for a thick 3D cell, at long scales, the nematic order is marginally unstable to such surface pinning, and computed the extent of the orientational order in cells of finite thickness with a second homogeneous substrate. We expect these predictions to be testable via polarizer-analyzer transmission and confocal microscopies, and through birefringence response to an in-plane electric or magnetic field.

We leave the challenging question of topological defects proliferation, generalizations to other interesting (e.g., smectic [3]) states, and glassy nonequilibrium relaxation (memory effects, aging, etc., all expected in our system) to future studies.

Note added.—While our Letter was under review, we learned of an earlier seminal work [4], with significant overlap with our infinite cell thickness limit. Where overlap exists, our results are in complete agreement.

[1] D. S. Fisher, G. M. Grinstein, and A. Khurana, Phys. Today **41**, No. 12, 56 (1988).

[2] N. Aryasova *et al.*, Mol. Cryst. Liq. Cryst. **412**, 351 (2004).

[3] C. D. Jones and N. A. Clark, Bull. Am. Phys. Soc. **49**, 307 (2004).

[4] D. Feldman and V. Vinokur, Phys. Rev. Lett. **89**, 227204 (2002).

[5] A. Larkin, Sov. Phys. JETP **31**, 784 (1970).

[6] Y. Imry and S. K. Ma, Phys. Rev. Lett. **35**, 1399 (1975).

[7] A realistic model must include *nonplanar* director distortions, as well as point and line (1/2-disclinations) defects allowed by the headless nematic director.

[8] L. Radzihovsky, Phys. Rev. B **73**, 104504 (2006).

[9] S. F. Edwards and P. W. Anderson, J. Phys. F **5**, 965 (1975).

[10] T. Nattermann, Phys. Rev. Lett. **64**, 2454 (1990).

[11] D. S. Fisher, Phys. Rev. B **31**, 7233 (1985).

[12] T. Giamarchi and P. Le Doussal, Phys. Rev. B **52**, 1242 (1995).

[13] L. Radzihovsky and Q. Zhang (to be published).

[14] R. Chitra *et al.*, Phys. Rev. B **59**, 4058 (1999).

[15] J. Toner and D. P. DiVincenzo, Phys. Rev. B **41**, 632 (1990).