Universality of Solar-Wind Turbulent Spectrum from MHD to Electron Scales

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To investigate the universality of magnetic turbulence in space plasmas, we analyze seven time periods in the free solar wind under different plasma conditions. Three instruments on Cluster spacecraft operating in different frequency ranges give us the possibility to resolve spectra up to 300 Hz. We show that the spectra form a quasiuniversal spectrum following the Kolmogorov's law $\sim k^{-5/3}$ at MHD scales, a $\sim k^{-2.8}$ power law at ion scales, and an exponential $\sim \exp[-\sqrt{k\rho_e}]$ at scales $k\rho_e \sim [0.1, 1]$, where ρ_e is the electron gyroradius. This is the first observation of an exponential magnetic spectrum in space plasmas that may indicate the onset of dissipation. We distinguish for the first time between the role of different spatial kinetic plasma scales and show that the electron Larmor radius plays the role of a dissipation scale in space plasma turbulence.

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Space plasmas are usually in a turbulent state, and the solar wind is one of the closest laboratories of space plasma turbulence, where *in situ* measurements are possible thanks to a number of space missions [1]. These measurements obtain time series which provide access to frequency spectra or to spectra of wave vectors along the flow. It is well established that at MHD scales (below $\sim$0.3 Hz, at 1 A.U.), the solar-wind turbulent spectrum of magnetic fluctuations follows Kolmogorov's spectrum $\sim f^{-5/3}$. However, the characteristics of turbulence in the vicinity of the kinetic plasma scales (such as the inertial lengths $\lambda_{i,e} = c/\omega_{pi,e}$, with c being the speed of light and $\omega_{pi,e}$ the plasma frequencies of ions and electrons, respectively, the Larmor radii $\rho_{i,e}$, and the cyclotron frequencies $\omega_{ci,e} = eB/m_{i,e}$) are not well known experimentally and are a matter of debate. It was shown that at ion scales, the turbulent spectrum has a break and steepens to $\sim f^{-s}$, with a spectral index s that is clearly nonuniversal, taking on values in the range of 2–4 [2,3]. These indices were obtained from data that enabled a rather restricted range of scales above the break to be investigated, up to $\sim$3 Hz. It is not known whether such indices persist at higher frequencies. At electron scales, the observations are difficult and our knowledge is very poor. Denskat *et al.* [4], using Helios data, obtained high resolution magnetic spectra at two distances from the Sun: up to 50 Hz at 1 A.U. and up to 470 Hz at 0.3 A.U. However, in both cases, the electron characteristic scales were not reached. It was only with Cluster observations that these electron scales were reached. For the solar-wind downstream of the Earth's bow shock, it was shown that the turbulence spectrum changes its shape around $k\lambda_e \simeq k\rho_e \sim 1$ [5]. This result was recently confirmed in the upstream solar wind magnetically connected to the bow shock [6]. However, in both studies the plasma β (the ratio between plasma and magnetic pressures) was $\sim$1 and so it was not possible to separate the roles of ρ_e and λ_e.

Measurements of solar-wind turbulent spectra in the vicinity of ion and electron plasma scales may clarify our understanding of the processes of dissipation (or dispersion) of turbulent energy in collisionless plasmas. A number of processes may be considered at these scales: cyclotron damping at f_{ci} and f_{ce} of Alfvén and whistler waves, respectively [7], scattering of oblique whistler waves at $f_{ci} < f < f_{ce}$ [8], and linear dissipation of kinetic Alfvén waves at $1/\rho_i < k < 1/\rho_e$ [6,9,10].

In this Letter, we use the Cluster spacecraft [11] data to analyze the free solar wind of different origin, fast and slow, and under different plasma conditions. While Sahraoui *et al.* [6] use the FluxGate Magnetometer (FGM) [12] and Spatiotemporal Analysis of Field Fluctuation experiment/Search Coil (STAFF-SC) [13] at the burst mode, which allow them in principle to investigate turbulence spectra up to 180 Hz, we complete these instruments with the Spatiotemporal Analysis of Field Fluctuation experiment/Spectrum Analyzer (STAFF-SA), which provides 4 s averages of the power spectral density of the magnetic fluctuations at 27 logarithmically spaced frequencies, between 8 and 4 kHz. Above $\sim$100 Hz, however, the instrument noise becomes a significant issue, which we take into account in our analysis.

Spacecraft data are a superposition of physical signal and an instrumental noise. As suggested in [13], we use measurements in the magnetospheric lobe (precisely, the data on 5 April 2001, 06:00–07:00 UT) as the noise level of the instrument. The final STAFF-SA spectra were obtained by subtraction of the noise spectrum from the initial solar-wind spectra, as was done in [14]. The maximal frequency in our analysis is defined as the highest frequency where the corrected spectrum remains above the noise.

We select seven time intervals of 42 min when Cluster was at apogee (19 Earth radii) and spent 1 h or more in the free solar wind: the electric field data at the electron plasma frequency show no evidence of magnetic connection to the bow shock [15]. In Table I, the dates of the intervals are shown as day/month/year, and their starting times are denoted by t_i. Average plasma parameters for the selected intervals are also given in this table. Magnetic field measurements were obtained from Cluster 1. Ion moments (density N, velocity V, and perpendicular temperature $T_{\perp i}$) are measured by the Cluster Ion Spectroscopy/Hot Ion Analyzer (CIS/HIA) experiment [16] on Cluster 1. The ion parallel temperatures are not properly determined in the solar wind by the CIS instrument [17]. Electrons are measured by the PEACE instrument [18], mostly on Cluster 2. One can see from Table I that the angle Θ_{BV} between **B** and **V** is always larger than 60°. Other plasma parameters are rather variable: V varies from $\sim$360 km/s to 670 km/s, the total perpendicular plasma beta, $\beta_{i\perp} + \beta_{e\perp} = 2\mu_0 nk(T_{i\perp} + T_{e\perp})/B^2$, varies between 0.6 and 3.3, and the Alfvén speed $V_a \in [30, 130]$ km/s. $V_{thi,e} = \sqrt{kT_{\perp i,e}/m_{i,e}}$ are the ion and electron perpendicular thermal speeds, respectively; $\rho_{i,e} = V_{thi,e}/\omega_{ci,e}$ are the corresponding Larmor radii. During these seven intervals, we never observed quasiparallel whistler waves, characterized by a quasicircular right-hand polarization which can be captured by the STAFF-SA instrument. The two intervals 3 and 5 display the most intense spectra and are observed in the fast solar wind, a few hours downstream of an interplanetary shock.

Figure 1 (top) shows the magnetic spectrum $P(f)$ for interval 5. Up to 10 Hz, it is calculated using the Morlet wavelet transform, as was done in [19]. One can clearly recognize in $P(f)$ two power laws and an exponential range: At low frequencies, the spectrum is $\sim f^{-1.7}$, consistent with Kolmogorov's law. Between f_{ci} and $f_{\lambda_i} \simeq f_{\rho_i}$ (where $f_{\lambda_i} = V/2\pi\lambda_i$ and $f_{\rho_i} = V/2\pi\rho_i$), the first break appears. At higher frequencies, the spectrum follows an $\sim f^{-2.8}$ law. However, at $10 \leq f \leq f_{\lambda_e} = 85$ Hz, where the instrumental noise is not yet important, the spectrum is no longer a power law, but follows approximatively an exponential function $\exp[-a(f/f_0)^{0.5}]$. At higher frequencies, $f > f_{\lambda_e}$, the spectrum is too close to the noise level (see the dotted line) to draw any firm conclusions.

To demonstrate the above scaling laws, Fig. 1 (bottom) shows compensated energy spectra. The low frequency part of the spectrum is well compensated by $f^{1.7}$ (solid line), the middle range by $f^{2.8}$ (dashed line), and the high frequency part up to f_{λ_e} by $\exp[a(f/f_0)^{0.5}]$ (dash-dotted line). The combined compensated spectrum is indeed very flat up to f_{λ_e}.

The spectra for the seven intervals are presented in Fig. 2(a). Horizontal bars indicate the spread of $f_{ci,e}$ among these seven independent observations. One can see that the spectra have similar shapes. Their intensity is, however, different. To superpose the spectra, we begin by applying Taylor's hypothesis, which should be valid for the whole frequency range as far as quasiparallel whistler waves are not observed during selected intervals (as mentioned above). Thus, we assume that the frequency spectra are indeed Doppler-shifted k spectra $P(k) = P(f)V/2\pi$ with $k = 2\pi f/V$. Then, we determine a relative intensity of the jth spectrum, $P_j(k)$ with $j = 1, \ldots, 7$, as $P_0(j) = \langle P_j(k)/P_1(k)\rangle$, where $P_1(k)$ is a reference spectrum and $\langle \ldots \rangle$ indicates a mean over the range of wave vectors $10^{-5} < k < 10^{-1}$ km^{-1}. With this normalization, the re-

TABLE I. Solar-wind parameters for selected time periods.

Number	1	2	3	4	5	6	7
Day/Month/Year	05/04/2001	19/02/2002	18/02/2003	31/12/2003	22/01/2004	27/01/2004	12/01/2005
t_i (UT)	22:36	01:48	00:18	10:48	05:03	00:36	02:00
B (nT)	7.26	6.98	15.5	10.9	15.5	9.52	13.6
N (cm^{-3})	2.9	29	6.7	22	20	8.4	33
$T_{\perp i}$ (eV)	17	7.3	40	10	61	10	14
$T_{\perp e}$ (eV)	19	6.7	13	15	26	13	16
V (km/s)	540	370	670	430	630	430	440
Θ_{BV} (°)	86	65	78	74	83	79	86
$\beta_{\perp i}$	0.4	1.7	0.5	0.8	2.0	0.4	1.0
$\beta_{\perp e}$	0.4	1.6	0.1	1.1	0.8	0.5	1.1
f_{ci} (Hz)	0.11	0.11	0.24	0.17	0.24	0.15	0.21
f_{ce} (Hz)	203	195	435	306	434	267	379
λ_i (km)	130	43	88	48	52	78	40
λ_e (km)	3.1	1.0	2.1	1.1	1.2	1.8	0.9
ρ_i (km)	59	40	42	30	51	35	28
ρ_e(km)	1.4	0.9	0.6	0.8	0.8	0.9	0.7
V_a (km/s)	93	28	130	51	77	72	52
V_{thi} (km/s)	41	27	62	32	76	32	36
$V_{the} 10^3$ (km/s)	1.8	1.1	1.5	1.6	2.1	1.5	1.7

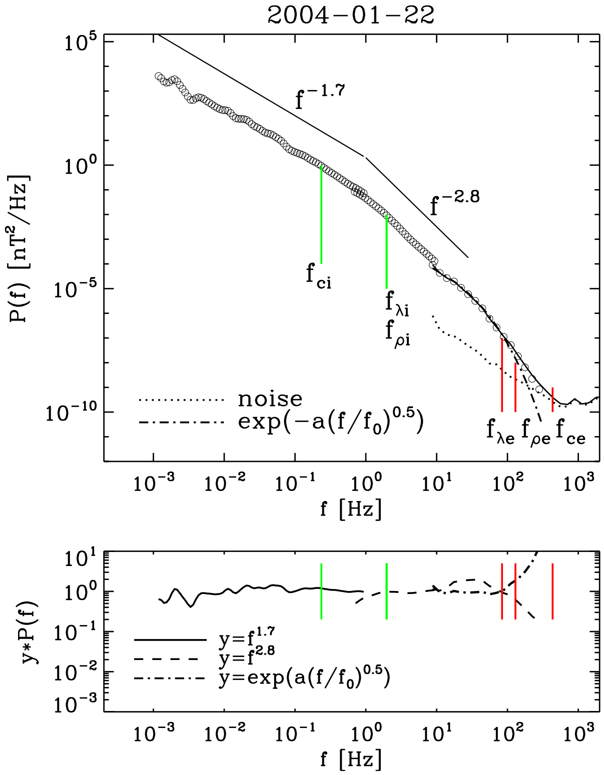

FIG. 1 (color online). Top: magnetic power spectral density for interval 5, measured by three instruments of Cluster in the solar wind: FGM (up to 1 Hz), STAFF-SC (up to 10 Hz), and STAFF-SA ($f \geq 8$ Hz, solid line: initial spectrum, open circles: spectrum after the noise subtraction). Vertical bars indicate plasma kinetic scales, where $f_{\lambda_{i,e}}$ correspond to the Doppler-shifted $\lambda_{i,e}$ and $f_{\rho_{i,e}}$ to $\rho_{i,e}$. Power laws $f^{-1.7}$ and $f^{-2.8}$ are shown. The dash-dotted line indicates exponential fit $\sim \exp[-a(f/f_0)^{0.5}]$, with $f_0 = f_{\rho_e}$ and the constant $a \simeq 9$. Bottom: compensated spectrum by $f^{1.7}$ (solid line), $f^{2.8}$ (dashed line), and by the exponential (dash-dotted line).

scaled spectra may be nearly superposed as shown in Fig. 2(b).

One expects that the spectral level, P_0, depends on the solar-wind kinetic, thermal, or magnetic energy. The scatter plots shown in Figs. 3(a) and 3(b) indicate a clear power-law dependence of P_0 on the magnetic energy and a less clear dependence on the kinetic energy (and thermal energy, not shown).

To understand the meaning of the observed dependence on the magnetic energy, one may use a Kolmogorov-like phenomenology. Suppose first that the solar-wind magnetic turbulence dissipates through an effective diffusion mechanism of $\sim \eta \Delta B$ (η being a probably turbulent magnetic diffusivity) and second that the observed turbulence is quasistationary. In such a case, there is a balance between the energy input from nonlinear interactions at large scales and the energy drain from the dissipation at small scales. This implies that the energy transfer rate ϵ depends on the dissipation scale ℓ_d as $\epsilon = \eta^3 \ell_d^{-4}$; thus $P_0 \sim \epsilon^{2/3} \sim \ell_d^{-8/3}$. The dependences observed in Figs. 3(c) and 3(d),

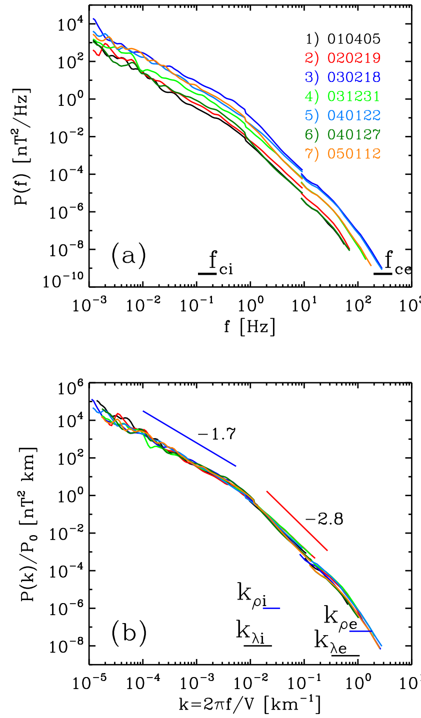

FIG. 2 (color online). (a) Magnetic spectra for seven time periods of 42 min; spread of $f_{ci,e}$ for the seven intervals is shown. (b) k spectra normalized over P_0; characteristic wave numbers, $k_{\rho_i} = 1/\rho_i$, etc., are shown.

$P_0 \sim (1/f_{ci})^{-2.8}$ and $P_0 \sim \rho_e^{-3.2}$, are close to the prediction of this phenomenological model. More statistics are needed to confirm the observed exponents. We can state, however, that the observed dependences imply that ρ_e and/or f_{ci} and/or f_{ce} play an important role in the dissipation processes in collisionless plasmas. Let us now confirm these results.

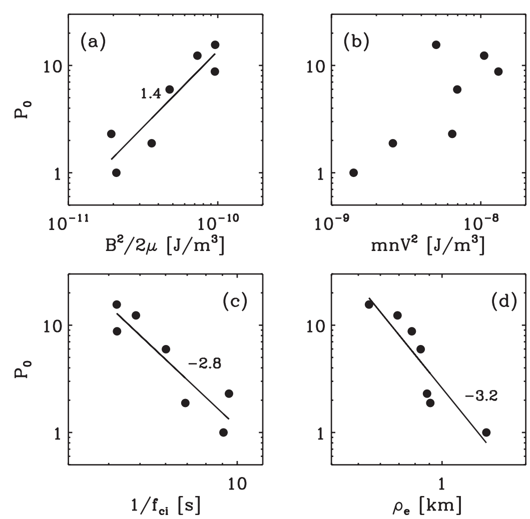

FIG. 3. Relative spectral intensity P_0 as a function of (a) magnetic and (b) kinetic energies; (c) P_0 as a function of the ion cyclotron period and (d) the electron gyroradius. Linear fits with corresponding slopes are shown by solid lines.

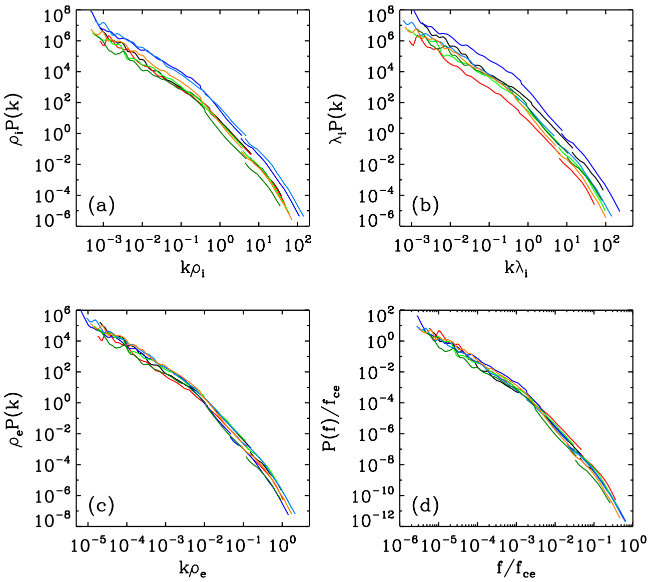

FIG. 4 (color online). Universal Kolmogorov function $\propto \ell_d E(k)$ for hypothesized dissipation scales ℓ_d as a function of (a) $k\rho_i$, (b) $k\lambda_i$, (c) $k\rho_e$, and (d) f/f_{ce}.

From the balance between the energy input and the dissipation, for the Kolmogorov's spectrum $E(k)$, it follows as well that $E(k)\ell_d/\eta^2$ is a universal function of $k\ell_d$ [20,21]. Figure 4 tests which of the kinetic scales is to be used as ℓ_d to recover a universal function from the observed spectra. We assume for simplicity that η is constant despite the varying plasma conditions. One can see that the ρ_i and λ_i normalizations are not efficient to collapse the spectra together. Normalization on λ_e gives the same result as for λ_i. At the same time, the normalization on ρ_e and f_{ce} bring the spectra close to each other. This confirms that the electron gyroradius ρ_e and/or cyclotron periods of the particles are important in the dissipation.

With the present observations, it is not possible to distinguish between ρ_e and cyclotron periods as far as there is a correlation between ρ_e and B. We can argue, however, that if the cyclotron period had been the only dissipation scale, the turbulent cascade would have stopped by the cyclotron damping of Alfvén waves at f_{ci}, showing an exponential cutoff at this scale [22]. Solar-wind observations show the contrary: the turbulent spectrum continues up to electron scales. Thus, we conclude that ρ_e is the dissipation scale of magnetic turbulence in the solar wind, but we cannot exclude that at f_{ci} and f_{ce} there is a partial dissipation by cyclotron damping.

In the present Letter, we analyzed high resolution magnetic spectra from MHD to electron scales. We show here for the first time that whatever the plasma conditions and the solar-wind regime, slow or fast, the magnetic spectra have similar shape. This indicates a certain universality, at least for the quasiperpendicular configuration between **B** and **V**. Such a quasiuniversal spectrum consists of three parts: two power laws and an exponential domain. At MHD scales it follows a Kolmogorov's $\sim k^{-5/3}$ spectrum, in agreement with previous observations. Between f_{ci} and Doppler-shifted λ_i and ρ_i, a spectral break is observed. Above the break, it follows a $k^{-2.8}$ power law. At smaller scales, for a broad range $k\rho_e \sim [0.1, 1]$, the spectrum is no longer a power law, but it follows an exponential $\sim \exp[-a\sqrt{k\rho_e}]$. This is the first observation of an exponential magnetic spectrum in space plasmas. Such spectra were predicted by the anisotropic dissipation model of Gogoberidze [8]. The author suggests that small scale fluctuations with oblique **k** are diffused on oblique fluctuations from the inertial range. This diffusion is anisotropic and it gives an $\sim \exp(-k^{\Delta\alpha/2})$ spectrum in the dissipation range, where $\Delta\alpha = \alpha_\perp - \alpha_\parallel$ is the difference between the energy diffusion scaling perpendicular and parallel to **B**.

It is a long-standing problem to distinguish between the role of different kinetic scales in space plasmas. For the first time, we clearly demonstrate that the role of dissipation scale in space plasma turbulence is played by the electron gyroradius as argued by several previous authors [6,10,23].

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[1] R. Bruno and V. Carbone, Living Rev. Solar Phys. **2**, 4 (2005).
[2] R. J. Leamon *et al.*, J. Geophys. Res. **103**, 4775 (1998).
[3] C. W. Smith *et al.*, Astrophys. J. **645**, L85 (2006).
[4] K. U. Denskat *et al.*, J. Geophys. Z. Geophys. **54**, 60 (1983).
[5] O. Alexandrova, C. Lacombe, and A. Mangeney, Ann. Geophys. **26**, 3585 (2008).
[6] F. Sahraoui, M. L. Goldstein, P. Robert, and Y. V. Khotyaintsev, Phys. Rev. Lett. **102**, 231102 (2009).
[7] S. P. Gary, *Theory of Space Plasma Microinstabilities* (Cambridge University Press, Cambridge, 1993).
[8] G. Gogoberidze, Phys. Rev. E **72**, 046407 (2005).
[9] S. D. Bale, P. J. Kellogg, F. S. Mozer, T. S. Horbury, and H. Reme, Phys. Rev. Lett. **94**, 215002 (2005).
[10] A. A. Schekochihin *et al.*, Astrophys. J. Suppl. Ser. **182**, 310 (2009).
[11] C. P. Escoubet *et al.*, Space Sci. Rev. **79**, 11 (1997).
[12] A. Balogh *et al.*, Ann. Geophys. **19**, 1207 (2001).
[13] N. Cornilleau-Wehrlin *et al.*, Ann. Geophys. **21**, 437 (2003).
[14] N. Lin *et al.*, J. Geophys. Res. **103**, 12 023 (1998).
[15] P. Canu (private communication).
[16] H. Rème *et al.*, Ann. Geophys. **19**, 1303 (2001).
[17] I. Dandouras (private communication).
[18] A. D. Johnstone *et al.*, Space Sci. Rev. **79**, 351 (1997).
[19] O. Alexandrova *et al.*, Astrophys. J. **674**, 1153 (2008).
[20] U. Frisch, *Turbulence: The Legacy of A. N. Kolmogorov* (Cambridge University Press, Cambridge, 1995).
[21] P. A. Davidson, *Turbulence: An Introduction for Scientists and Engineers* (Oxford University Press, Oxford, 2004).
[22] O. Stawicki *et al.*, J. Geophys. Res. **106**, 8273 (2001).
[23] R. J. Leamon *et al.*, J. Geophys. Res. **104**, 22 331 (1999).