Roles of Dark Energy Perturbations in Dynamical Dark Energy Models: Can We Ignore Them?

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We show the importance of properly including the perturbations of the dark energy component in the dynamical dark energy models based on a scalar field and modified gravity theories in order to meet with present and future observational precisions. Based on a simple scaling scalar field dark energy model, we show that observationally distinguishable substantial differences appear by ignoring the dark energy perturbation. By ignoring it the perturbed system of equations becomes inconsistent and deviations in (gauge-invariant) power spectra depend on the gauge choice.

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The high-z type Ia supernovae (SNIa) luminosity-distance relation suggests that the expansion rate of our Universe is currently under acceleration [1]. The cosmological constant is readily (re)introduced to explain the observation theoretically. Theoretical studies of the large-scale structure formation process imprinted in the matter power spectrum [2,3] and the cosmic microwave background radiation (CMB) power spectrum [4] also favor the presence of a substantial amount of agent with repulsive nature like the cosmological constant. With the advent of the recent acceleration, the long lasting age problem of the world model, which has persisted ever since the first observation of the expansion of the Universe, has now evaporated from the cosmological scene.

The nature of the agent causing the acceleration, however, is still unknown and it is one of the fundamental mysteries in the present day theoretical cosmology. Although the cosmological constant is a historically well known possibility, it also has two well appreciated problems: the cosmological constant (why so small) problem and the coincidence (why now or fine-tuning) problem. Much literature has been devoted to addressing these problems, especially the latter one, by introducing dynamical agents, often termed the dark energy. As far as we can tell the fine-tuning problem has not been properly addressed even using the dynamical dark energy. Introduction of the dynamic possibility of the dark energy, however, has opened a whole new arena for cosmological research based on variety of possibilities using field, fluid, modified gravity, other dimensions, etc.

In the case of the cosmological constant as the dark energy, due to its constant nature (both in time and space) its contribution directly appears *only* in the background world model. However, when we consider the dynamical dark energy we should pay attention to its dynamical roles not only in the background world model but also in the structure formation process. Here, we address the importance of properly including the role of dark energy perturbation (DEP) imprinted in the large-scale matter and the CMB anisotropies power spectra, and the perturbation growth process especially in the context of present and future observations with due precision.

Recent dramatic progress made in observational cosmology opens the possibility to constrain the character of dark energy, and calls for equally precise theoretical tools in the cosmic structure formation process. The expansion history based on the SNIa, the matter power spectrum, the CMB anisotropy power spectra, and the perturbation growth factor provide four domains where theories meet with observations. The relevant present and future observation programs in the CMB, SNIa, and the large-scale clustering include the WMAP (Wilkinson Microwave Anisotropy Probe) and the Planck missions, 2dFGRS (two-degree-field galaxy redshift survey), SDSS (Sloan digital sky survey), to mention a few. The large-scale clustering can be probed by diverse observations: weak lensing, Lyman-α and hydrogen 21 cm tomography, x-ray galaxy clustering mass function, galaxy redshift-space distortion, integrated Sachs-Wolfe effect, etc. In the following we will compare our results with SDSS DR7 (seventh data release) for the matter power spectrum [3], WMAP 5-year data for the CMB spectrum [4], and the future x-ray and weak lensing observations of clusters using x-ray surveys for the perturbation growth factor [5].

Our study is motivated by often used practices in the literature which ignore the DEP even in the case of dark energy models using the scalar field or modified gravity theories, see [6]. That is, in the presence of a dynamical dark energy it is *not* guaranteed to use the following conventionally known equation [7]

$$\ddot{\delta}_b + 2H\dot{\delta}_b - 4\pi G\delta\rho_b = 0, \tag{1}$$

which is true *only* for the cosmological constant as the dark energy; $\delta_b \equiv \delta\rho_b/\rho_b$ is the relative density fluctuation of baryon component, $H \equiv \dot{a}/a$, a is the cosmic scale factor, and an overdot denotes a time derivative. In the presence of dynamical dark energy we have contributions from the DEP in the right-hand side which are accompanied by a second-order differential equation describing the equation of motion of the perturbed dark energy. Even in modified gravity context, in the literature, we often notice a similar equation replacing G by some effective $G_{\rm eff}$. Without proper (perhaps numerical) verification such a simplification is hardly allowed mathematically because it corresponds to replacing a second-order differential equation by an algebraic coefficient (which is zero in the above case); as we have $G_{\rm eff} = G$ in Einstein's gravity limit, if such an approximation of ignoring the DEP is not allowed in Einstein's gravity the same is true even in modified gravity context.

Indeed it is always prudent and correct to include the DEP in principle, but more a relevant issue would be whether we could ignore such accompanied fluctuations in practice. In this Letter, by using a simple dynamical dark energy model based on a scalar field we will show that the answer is negative even in Einstein's gravity; for related works, see [8].

As a simple dynamical model of dark energy we consider a minimally coupled scalar field with a double exponential potential (we set $c \equiv 1 \equiv \hbar$) $V(\phi) = V_1 e^{-\lambda_1\phi} + V_2 e^{-\lambda_2\phi}$, where ϕ is the scalar field. The background evolution was investigated previously by Bassett *et al.* in [9], and we consider the background parameters of the scalar field suggested in that work: we call it a ϕCDM (cold dark matter) model. As a fiducial model we take a flat ΛCDM Universe with parameters $\Omega_m = 0.274$ ($\Omega_c = 0.2284$ and $\Omega_b = 0.0456$), $\Omega_\Lambda = 0.726$, $h = 0.705$, $n_s = 0.960$, $\sigma_8 = 0.812$, $T_0 = 2.725$ K, $Y_{\rm He} = 0.24$, $N_\nu = 3.04$ based on the WMAP 5-year observations [4], but without reionization. Evolution of the background world models is presented in Fig. 1. For all ϕCDM models we take $V_1 = 10^{-56}$ and $\lambda_1 = 9.43$, and from red to violet curves, $\lambda_2 = 1.0, 0.5, 0.0, -0.2, -1.0, -10$, and -30. For each model, V_2 parameter has been determined to have the present dark energy density parameter equal to $\Omega_\phi = 0.726$. The initial dark energy density parameter $\Omega_{\phi i}$ is determined by the parameter λ_1; i.e., $\Omega_{\phi i} = 3(1+w)/\lambda_1^2 = 0.045$ during the radiation domination with $w = \frac{1}{3}$, see Eq. (4) in [10].

Our dark energy model allows exact scaling during the radiation and matter dominated eras (provided by λ_1 term) and behaves as the dark energy in the present epoch (provided by λ_2 term). Following [9] we consider the initial contribution from the dark energy to be close to a maximum amount allowed by the big bang nucleosynthesis (BBN) calculation $\Omega_{\phi i} < 0.045$ [11]. The parameters used in our dark energy model are consistent with currently known cosmological constraints from the BBN and the high-z SNIa observations, see Fig. 1.

In order to calculate the matter and CMB power spectra, and evolution of the baryon density perturbation we solve a system composed of matter (dust and CDM), radiation (handled using the Boltzmann equation or tight coupling approximation), together with the cosmological constant or the scalar field as the dark energy. Our set of equations and the numerical methods are presented in [12]. As the initial conditions for perturbation variables we use the scaling

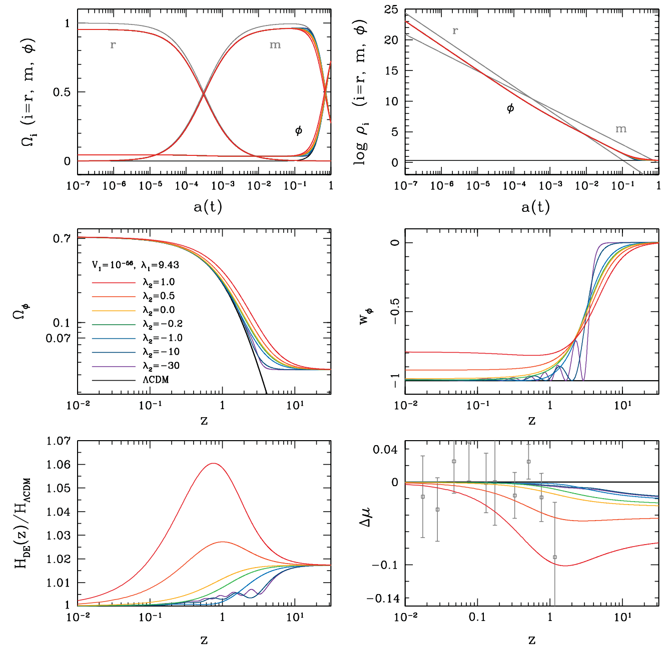

FIG. 1 (color). Top panels: Evolution of Ω_i and ρ_i as a function of scale factor $a(t)$ in the ϕCDM Universes with scalar field potential parameters set by Basset *et al.* [9] (colored curves), where $i = r, m, \phi$ indicates radiation, matter (baryon + CDM), and scalar field, respectively. Black curves represent those of ΛCDM model. Middle and bottom panels: Evolution of Ω_ϕ, w_ϕ, $H_{\rm DE}(z)/H_{\Lambda{\rm CDM}}$, and $\Delta\mu(z) = \mu_{\rm DE}(z) - \mu_{\Lambda{\rm CDM}}(z)$ for the same set of ϕCDM models. In the $\Delta\mu$-plot, the gray open squares with error bars represent the deviation of SNIa data points from the ΛCDM model considered here. The binned SNIa data are based on the Union sample [15].

solutions derived in [10]. We solved the system in three different gauge conditions: the CDM-comoving gauge (CCG), the uniform-expansion gauge (UEG), and the uniform-curvature gauge (UCG); the CCG, the UEG, and the UCG, respectively, set the velocity of the CDM, the perturbed expansion of normal frame vector (or the perturbed trace of extrinsic curvature), and the perturbed part of intrinsic scalar curvature equal to zero as the temporal gauge condition; all perturbation variables we use are spatially gauge invariant [13]. The CCG is the same as the synchronous gauge without the gauge mode. Each of these gauge conditions fixes the gauge degrees of freedom completely; thus, any variable in these gauge conditions is equivalent to a unique gauge-invariant combination of variables. The value of any gauge-invariant variables evaluated in the three gauges should coincide exactly. We used this to check the consistency of the calculation and the numerical accuracy.

In Fig. 2 we present the matter power spectrum and the CMB temperature and polarization anisotropy power spectra based on the same parameters used in the background world model. The CMB temperature and polarization anisotropies are naturally gauge invariant, and for the matter power spectrum we present the power spectrum of density

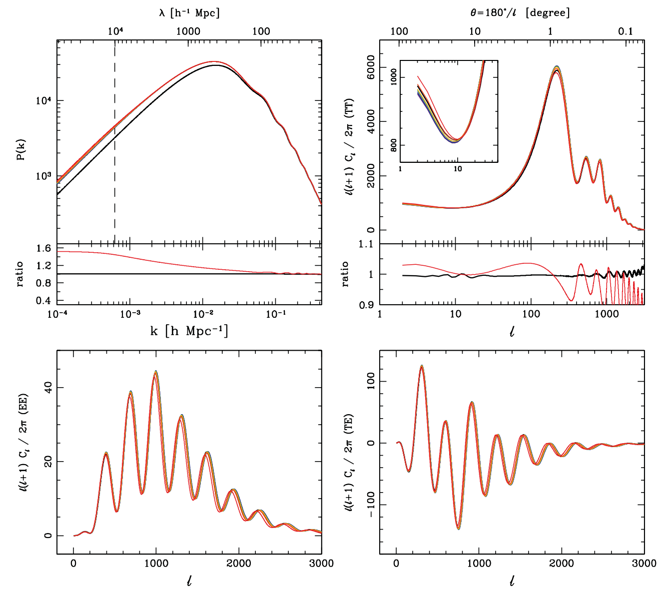

FIG. 2 (color). The matter power spectrum (top left), and CMB TT (top right), EE (bottom left), TE (bottom right) power spectra of ϕCDM Universe with scalar field potential parameters used in Fig. 1, with the same colored code. Predictions of ΛCDM model are shown as black curves. The vertical line in the top-left panel indicates the present horizon size ($10081h^{-1}$ Mpc) of the ΛCDM Universe. The small box in the top-right panel magnifies the CMB TT powers at low ℓ's. All calculations are made in three different gauge conditions (CCG, UEG, and UCG), where evolution of perturbation of the dark energy scalar field has been properly considered. The results in the three gauges coincide exactly. The matter and CMB power spectra of the ΛCDM model have been normalized with σ_8 and COBE spectrum, respectively. For comparison, all the ϕCDM power spectra have been normalized with the ΛCDM ones at small scales, $\ell = 700$ for CMB and $k = 0.3h$ Mpc^{-1} for matter ones. For a ϕCDM with $\lambda_2 = 1.0$ that is most deviated from the ΛCDM prediction, the ratios of its powers to our ΛCDM predictions are also shown in the bottom region of top panels; as an indication of numerical accuracy of our code "the CMBFAST-derived power spectra [16] divided by our result for ΛCDM model" is represented as a black curve.

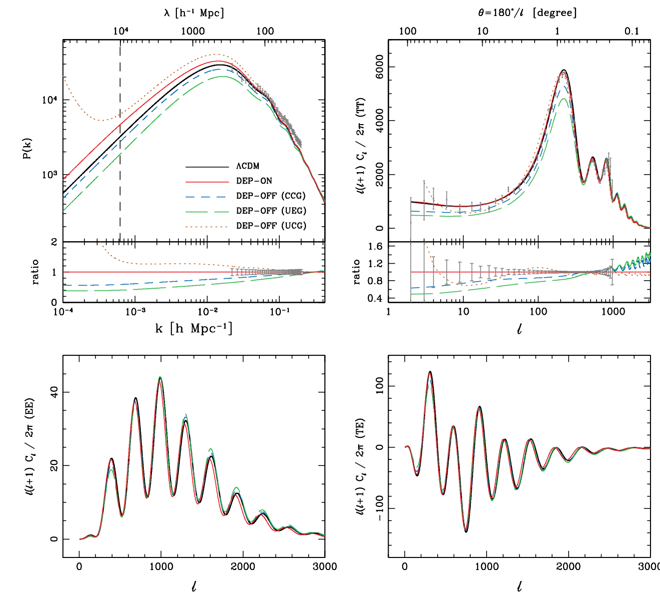

FIG. 3 (color). The same as Fig. 2 now ignoring the DEP (DEP-OFF) in the CCG (blue, dashed), the UEG (green, long dashed), and the UCG (brown, dotted curves) for $\lambda_2 = 1.0$. Red solid curves represent power spectra with proper DEP (DEP-ON). The power spectra ignoring the DEP apparently depend on the gauge choice which reflects internal inconsistency of the system. For matter and CMB TT power spectra, recent measurements from SDSS DR7 LRG [3] and WMAP 5-year [4] data (including the cosmic variance) have been added (gray dots with error bars), and power ratios between cases ignoring and considering DEP are also shown for CCG, UEG, and UCG, together with fractional errors of observed spectra.

perturbation based on the CCG which is also a gauge-invariant concept; i.e., density perturbation in the CCG is the same as a unique gauge-invariant combination between the density perturbation and the velocity perturbation of the CDM component. Despite the variety of outcome in the redshift-distance relation in the parameters used (see right-bottom panel in Fig. 1) the matter power spectra of the ϕCDM models are all similar with some tilt relative the fiducial ΛCDM model, whereas the differences in the CMB power spectra are less distinguished. Figure 2 shows that when we properly include the DEP the three gauges give identical results both for the ΛCDM and ϕCDM cases.

Now, in Fig. 3 we ignored (set equal to zero by hand) the perturbed part of dark energy. Apparently, the results depend on the gauge conditions used. As the values of gauge-invariant variables depend on the gauge conditions used in the calculation this alarms inconsistency of the system. Such differences are expected because by ignoring the DEP the perturbed system of equations becomes inconsistent. The presence of fluctuations in the matter and metric simultaneously and inevitably excites fluctuations in the dark energy. And it is not allowed to turn off the DEP by hand. The issue we would like to address, however, is whether we could ignore the DEP in practice. Our result in Fig. 3 shows that ignoring the DEP easily leads to observationally significant deviations in the power spectra which are even gauge dependent.

In our normalization the matter power spectrum shows about $-20\%/-34\%/+20\%$ ($-10\%/-19\%/+8.9\%$) error caused by ignoring the DEP at $k \simeq 0.022h$ Mpc^{-1} in the CCG/UEG/UCG; the values inside the parentheses are for $\Omega_{\phi i} = 0.0225$ which is one half of the value used in our figures. The current observation from SDSS DR7 LRG (luminous red galaxies) shows 11% (correlated) error at the same scale, which is already smaller than the deviations caused by ignoring the DEP in all gauges. The CMB temperature power spectrum shows about $-9.8\%/-18\%/+.64\%$ ($-6.0\%/-10\%/+.63\%$) error caused by ignoring the DEP at $\ell = 200$; the WMAP 5-yr data in this scale have about 2% (binned) error (mostly due

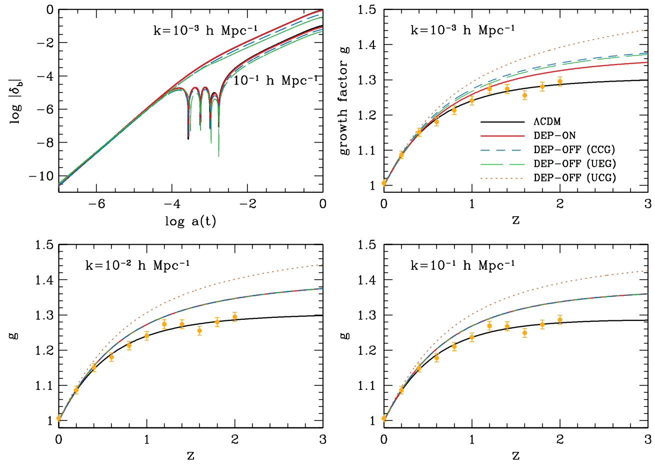

FIG. 4 (color). Evolution of baryon density perturbation (top left), and the normalized perturbation growth factor $g \equiv (\delta_b/a)$ in three different scales for $\lambda_2 = 1.0$; due to strong deviations we omit UCG cases in top-left panel. As we normalize g to unity at present, the effect of DEP appears only in the large scale (top right), and except for the UCG, it has no effects in the two small scales (bottom panels). We add 1% error bar expected from future x-ray and weak lensing observations [5].

to the cosmic variance); as the figure shows the small error in the UCG is only due to a coincidence in this scale.

Thus, deviations depend directly on the amount of $\Omega_{\phi i}$ in the early scaling era. In our scaling dark energy model by reducing $\Omega_{\phi i}$ the deviations caused by ignoring the DEP become proportionally smaller. However, this does not imply that our model is an extreme example in the effects of DEP. In fact, we can easily introduce models theoretically (i.e., not by hand) where Ω_ϕ is negligible during the nucleosynthesis era but becomes significant during later radiation and early matter eras and then reduces to the dark energy in recent era so that the rest of the cosmological effects are indistinguishable but the resulting power spectra in the matter and CMB are substantially different in diverse ways, see [14].

This still implies that the substantial deviations in the power spectra due to DEP are mainly caused during the scaling era. This is partly supported by studying the baryon density perturbation growth factor g in the recent past which provides another domain where theory meets with observation, see Fig. 4. In the CCG and the UEG cases the DEPs do not cause a difference in observationally relevant small scales. Although the observationally distinguishable substantial deviations in the UCG case can be regarded as exceptional peculiarity of that gauge choice, this still indicates the inconsistency of equations without DEP and potential danger of ignoring the DEP without proper confirmation. That is, by ignoring the DEP the system of equations becomes inconsistent and even (gauge-invariant) observable results depend on the gauge choice; thus, Fig. 4 shows the particular importance of taking proper gauge in the absence of the DEP. Notice that in our realistic situation with early radiation era the growth factor shows scale dependence.

In this Letter we investigated the roles of DEP in a dynamical dark energy model based on the scalar field. The moral is that when we consider dynamical dark energy it is essentially important to take into account the fluctuating aspects of dark energy properly. When one ignores DEP it is important to show that one can do that without causing observationally significant differences. Our model shows an example where it is crucially important to include the DEP. Otherwise, the system of equations becomes inconsistent, and the consequent results are not reliable compared with currently available observations.

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