Anomalous Josephson Current through a Spin-Orbit Coupled Quantum Dot

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For a general model of a mesoscopic multilevel quantum dot, we determine the necessary conditions for the existence of an anomalous Josephson current with spontaneously broken time-reversal symmetry. They correspond to a finite spin-orbit coupling, a suitably oriented Zeeman field, and the dot being a chiral conductor. We provide analytical expressions for the anomalous supercurrent covering a wide parameter regime.

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Introduction.—The Josephson effect, where an equilibrium supercurrent flows through a junction between two superconductors held at phase difference ϕ, is of fundamental importance in condensed matter physics, quantum information science, microelectronic applications, and metrology [1]. It has recently attracted renewed interest in mesoscopic and nanoscale junctions after the experimental demonstration of gate-tunable Josephson currents through junctions with just a few relevant electronic levels (“quantum dot”) in a variety of material systems, e.g., InAs nanowires [2], the 2D electron gas in semiconductors [3], and carbon nanotubes [4]. One important novel aspect arises because the spin-orbit interaction (SOI) strength α due to structural and bulk inversion asymmetries is often significant [5,6], and theoretical work has therefore started to address SOI effects on the Josephson current in such devices [7–13]. So far this activity has mainly focused on 0 and π junctions (positive or negative critical current I_c, respectively). A remarkable prediction concerns the possibility for an anomalous supercurrent I_a, flowing even at zero phase difference ($\phi = 0$) if both a (suitably oriented) Zeeman field $\boldsymbol{b}$ and the SOI are present [8,12,13]. For $\phi = 0$, the Hamiltonian is invariant under time-reversal symmetry (TRS) even when $\alpha \neq 0$ and $\boldsymbol{b} \neq 0$, and the anomalous supercurrent thus spontaneously breaks TRS, which otherwise enforces $I(\phi) = -I(-\phi)$ and hence $I_a = 0$ [1]. Anomalous supercurrents were first predicted in unconventional superconductors [14], but were never observed there. In the tunneling limit, where the conventional current-phase relation (CPR) reads $I(\phi) = I_c \sin\phi$, this is equivalent to a phase shift ϕ_0, i.e., $I(\phi) = I_c \sin(\phi + \phi_0)$ and thus $I_a = I_c \sin\phi_0$. The phase shift in such a “ϕ_0 junction” could be observed in a SQUID containing one 0 and one ϕ_0 junction via the shift of the diffraction pattern, or as spontaneous current in a superconducting ring containing a ϕ_0 junction. Both effects are tunable by external gate voltages (affecting the SOI), Zeeman fields, and by an orbital magnetic flux. Moreover, a ϕ_0 junction can also act as a superconducting rectifier [12].

Mesoscopic systems contacted by conventional s-wave BCS superconductors could then yield a new class of systems with spontaneously broken TRS, and therefore exhibit anomalous supercurrents. Recent works have started to address this point. First, Ref. [8] considered a long ballistic one-dimensional Rashba quantum wire, where $I_a \neq 0$ is tied to the Zeeman effect and to the difference between the velocities of right- and left-moving electrons. However, for physically realizable α, the reported I_a values turn out to be extremely small, $I_a \propto \alpha^4$, or are most likely inaccessible in experiments. In a mainly numerical study [12], the anomalous Josephson effect was also found for a multichannel spin-polarizing quantum point contact. Finally, Buzdin [13] reported $I_a \neq 0$ in junctions containing a noncentrosymmetric ferromagnet as a weak link. There is clearly a need to systematically classify all ingredients necessary to observe the anomalous Josephson effect. In this work, we compute the Josephson current through a generic phase-coherent mesoscopic system (an arbitrary multilevel quantum dot). Analytical predictions for I_a are provided for a physically important parameter regime. In addition, these results are supported by numerics. A necessary condition for $I_a \neq 0$ emerges from our study: the quantum dot must be a chiral conductor, see Eq. (13) below. This requirement was implicit in Ref. [8] but is apparently violated [15] in Ref. [13].

Model and exact solution.—As a generic model for the Josephson current through a quantum dot, we consider $H = H_L + H_R + H_T + H_D$, with two identical s-wave BCS superconductors ($H_{j=L/R}$) of gap Δ held at phase difference ϕ and tunnel-coupled (H_T) to the quantum dot. With lead fermion operators $c_{L/R,k\sigma}$ for spin $\sigma = \uparrow, \downarrow$ and momentum $\boldsymbol{k}$, the BCS Hamiltonian reads (we often put $\hbar = 1$ and, for simplicity, take the zero-temperature limit)

$$H_j = \sum_{\boldsymbol{k}\sigma} \frac{k^2}{2m} c^\dagger_{jk\sigma} c_{jk\sigma} + \sum_{\boldsymbol{k}} (\Delta e^{\mp i\phi/2} c^\dagger_{jk\uparrow} c^\dagger_{j(-\boldsymbol{k})\downarrow} + \text{H.c.}).$$

For $\alpha = 0$ and $\boldsymbol{b} = 0$, the closed dot can be described in terms of real-valued eigenfunctions $\chi_n(\boldsymbol{r})$ [with $\boldsymbol{r} = (x, y, z)$] for eigenenergy ϵ_n, where $n = 1, \ldots, N$ labels the relevant dot orbitals. Using fermion operators $d_{n\sigma}$ for these orbitals and disregarding strong correlation effects, the dot Hamiltonian in the presence of SOI and Zeeman field can be written in the form (the Pauli matrices $\sigma_{x,y,z}$ act in spin space, spin indices are kept implicit, and we absorb the magnetic factor $g\mu_B/2$ into $\boldsymbol{b}$)

$$H_D = \sum_n d_n^\dagger [\epsilon_n + \boldsymbol{b} \cdot \boldsymbol{\sigma}] d_n - i \sum_{nn'} d_n^\dagger \boldsymbol{a}_{nn'} \cdot \boldsymbol{\sigma} d_{n'}. \quad (1)$$

We consider both the Rashba and Dresselhaus SOI due to structural and bulk inversion asymmetry, respectively, with the electric field pointing in z direction. The SOI is encoded in the antisymmetric matrices $\boldsymbol{a}_{x,y}$ forming the vector [10]

$$\boldsymbol{a}_{nn'} = \frac{\alpha}{m} \int d\boldsymbol{r} \chi_n(\boldsymbol{r}) \begin{pmatrix} \sin(\theta)\partial_x + \cos(\theta)\partial_y \\ -\cos(\theta)\partial_x - \sin(\theta)\partial_y \end{pmatrix} \chi_{n'}(\boldsymbol{r}). \quad (2)$$

The overall SOI strength is represented by the characteristic inverse length α, while the angle $\theta = 0$ corresponds to a pure Rashba and $\theta = \pi/2$ to a pure Dresselhaus case [5]. The tunneling Hamiltonian is

$$H_T = \sum_{j=L/R, \boldsymbol{k} n\sigma} t_{jn} c_{j\boldsymbol{k}\sigma}^\dagger d_{n\sigma} + \text{H.c.}, \quad (3)$$

where we make the inessential assumption of $\boldsymbol{k}$-independent tunneling matrix elements. It is useful to define the Hermitian $N \times N$ hybridization matrices (in dot level space),

$$\boldsymbol{\Gamma}_{j=L/R, nn'} = \pi\nu t_{jn}^* t_{jn'}, \quad (4)$$

where ν is the normal-state density of states in the leads. We find that the anomalous supercurrent is most pronounced if the Zeeman field points along a direction defined by the SO angle θ, namely $\boldsymbol{b} = B(\cos\theta, -\sin\theta, 0)^T$. When the magnetic field is orthogonal to this, the anomalous supercurrent is absent. The above model ignores orbital magnetic fields. For the important case of a 2D dot, this is justified by choosing an in-plane magnetic field as above.

Integrating out all fermionic degrees of freedom, the Josephson current for arbitrary system parameters is obtained:

$$I(\phi) = -\frac{2e}{h} \int_0^\infty d\omega \partial_\phi \mathrm{tr} \ln \boldsymbol{S}(\omega) \quad (5)$$

with a $4N \times 4N$ matrix $\boldsymbol{S}(\omega)$. Using auxiliary Pauli matrices $\tau_{x,z}$, the matrix $\boldsymbol{S}$ can be expressed as [10]

$$\boldsymbol{S} = -i\omega\left(1 + \frac{\boldsymbol{\Gamma}_L + \boldsymbol{\Gamma}_R}{\sqrt{\omega^2 + \Delta^2}}\right) + \boldsymbol{E}\sigma_z\tau_z + \boldsymbol{Z} + \frac{\Delta}{\sqrt{\omega^2 + \Delta^2}} \times [(\boldsymbol{\Gamma}_L + \boldsymbol{\Gamma}_R)\cos(\phi/2)\sigma_x\tau_z + (\boldsymbol{\Gamma}_L - \boldsymbol{\Gamma}_R) \times \sin(\phi/2)\sigma_y],$$

where $\boldsymbol{E} \equiv \mathrm{diag}(\tilde{\epsilon}_1, \ldots, \tilde{\epsilon}_N)$ with $\tilde{\epsilon}_n = \epsilon_n - \alpha^2/2m$ contains the dot level energies. The Zeeman field and the SOI are encoded in the ω-independent $4N \times 4N$ matrices

$$\boldsymbol{Z} = (i\boldsymbol{A}_x\sigma_x + \boldsymbol{B}_y\sigma_y)\tau_x + (i\boldsymbol{A}_y\sigma_x - \boldsymbol{B}_x\sigma_y)\tau_y + \boldsymbol{B}_z\tau_z + i\boldsymbol{A}_z\sigma_z, \quad (6)$$

with the vector of real antisymmetric matrices [16]

$$\boldsymbol{A}_{nn'} = \frac{\alpha}{m} \int d\boldsymbol{r} \chi_n \partial_y \chi_{n'} \begin{pmatrix} -\cos\theta[1 - 2\cos(2\theta)\sin^2(\alpha x)] \\ \sin\theta[1 + 2\cos(2\theta)\sin^2(\alpha x)] \\ \cos(2\theta)\sin(2\alpha x) \end{pmatrix}. \quad (7)$$

The matrices $\boldsymbol{B}_{x,y,z}$ are real and symmetric, and for the above Zeeman field given by

$$\boldsymbol{B}_{nn'} = B \int d\boldsymbol{r} \chi_n \chi_{n'} \begin{pmatrix} \cos\theta\cos^2(\alpha x) - \cos(3\theta)\sin^2(\alpha x) \\ -\sin\theta\cos^2(\alpha x) - \sin(3\theta)\sin^2(\alpha x) \\ -\cos(2\theta)\sin(2\alpha x) \end{pmatrix}. \quad (8)$$

One can verify from the above expressions that no anomalous supercurrent can exist in the absence of either the SOI or the Zeeman field.

Analytical approach.—We now derive the anomalous supercurrent I_a in the most relevant limit of weak SOI and weak Zeeman field, where $I_a \propto \alpha B$ is small. Moreover, the derivation below assumes that the off-diagonal entries in the $\boldsymbol{\Gamma}_{L/R}$ matrices (4) are small against the diagonal entries. This condition is met in most cases of practical interest, e.g., if one dot level is resonantly coupled to the leads and all other levels are only weakly coupled, or when quasirandom phase shifts between different t_{jn} have to be taken into account. We then consider the limit $\phi \to 0$, where under the above conditions, it makes sense to write $\boldsymbol{S} = \boldsymbol{S}_0 + \boldsymbol{S}_1$ in Eq. (5). The "leading" part is diagonal in dot level space,

$$\boldsymbol{S}_0 = -i\omega + \boldsymbol{E}\sigma_z\tau_z + \frac{-i\omega + \Delta\sigma_x\tau_z}{\sqrt{\omega^2 + \Delta^2}} \boldsymbol{\Gamma}_0, \quad (9)$$

with $\boldsymbol{\Gamma}_0 = \mathrm{diag}(\boldsymbol{\Gamma}_L + \boldsymbol{\Gamma}_R)$. Similarly, using $\boldsymbol{\Gamma}_1 = \boldsymbol{\Gamma}_L + \boldsymbol{\Gamma}_R - \boldsymbol{\Gamma}_0$, the "perturbation" part is

$$\boldsymbol{S}_1 = \boldsymbol{Z} + \frac{(\phi/2)\Delta(\boldsymbol{\Gamma}_L - \boldsymbol{\Gamma}_R)\sigma_y + [-i\omega + \Delta\sigma_x\tau_z]\boldsymbol{\Gamma}_1}{\sqrt{\omega^2 + \Delta^2}}, \quad (10)$$

with $\boldsymbol{Z}$ given in Eq. (6). The anomalous supercurrent then follows by expansion of the tracelog in Eq. (5),

$$I_a = \frac{2e}{h} \sum_{n=1}^\infty \frac{(-1)^n}{n} \int_0^\infty d\omega \partial_\phi \mathrm{tr}(\boldsymbol{S}_0^{-1}\boldsymbol{S}_1)^n. \quad (11)$$

Using Eq. (6), straightforward but lengthy algebra shows that both the $n = 1$ and $n = 2$ contributions always vanish. The leading contribution to I_a then comes from $n = 3$, where the part $\propto \boldsymbol{\Gamma}_1$ in $\boldsymbol{S}_1$, see Eq. (10), does not contribute at all. In the end, we arrive at the analytical expression

$$I_a = \frac{8e\Delta^2}{h}\int_0^\infty \frac{d\omega}{\omega^2+\Delta^2}\mathrm{tr}_d\Big([\boldsymbol{\Gamma}_R, \boldsymbol{\Gamma}_L]_- \boldsymbol{D}^{-1}(\boldsymbol{b}\cdot\boldsymbol{A}) \times \boldsymbol{D}^{-1}\Big[1 - 4\omega^2\Big(1+\frac{\boldsymbol{\Gamma}_0}{\sqrt{\omega^2+\Delta^2}}\Big)\boldsymbol{D}^{-1}\Big]\Big), \tag{12}$$

with the diagonal matrix $\boldsymbol{D} = \omega^2 + \boldsymbol{E}^2 + \boldsymbol{\Gamma}_0^2 + \frac{2\omega^2}{\sqrt{\omega^2+\Delta^2}}\boldsymbol{\Gamma}_0$. The trace operation tr_d extends over dot level space only. Since $\boldsymbol{b}\cdot\boldsymbol{A} \propto \alpha B$, we see that $I_a \propto \alpha B$ as expected. Apart from having a finite SOI and an appropriately oriented Zeeman field, an additional condition must be satisfied in order to have $I_a \neq 0$:

$$[\boldsymbol{\Gamma}_R, \boldsymbol{\Gamma}_L]_- \neq 0, \tag{13}$$

which implies chirality for transport through the quantum dot. From numerical studies of the full current (5), we find that Eq. (13) is a necessary condition for $I_a \neq 0$ for arbitrary other parameters, and hence is not restricted by the conditions under which Eq. (12) has been obtained. As a consequence, a single-level dot ($N = 1$) can never allow for an anomalous supercurrent, and at least two relevant orbitals are required. Finally, we remark that Eq. (12) can be further simplified for $\max(\boldsymbol{\Gamma}_{nn}) \gg \Delta$, where we obtain

$$I_a = \frac{2e\Delta}{\hbar}\mathrm{tr}_d((\boldsymbol{\Gamma}_0^2 + \boldsymbol{E}^2)^{-1}[\boldsymbol{\Gamma}_R, \boldsymbol{\Gamma}_L]_-(\boldsymbol{\Gamma}_0^2 + \boldsymbol{E}^2)^{-1}\boldsymbol{b}\cdot\boldsymbol{A}). \tag{14}$$

Basic explanation.—Having established that spontaneously broken TRS is indeed possible in this system, see Eq. (12), we now give an intuitive physical argument to understand the origin of this effect. Consider the transfer of a Cooper pair through a quantum dot with $N = 2$ levels for $\phi = 0$, schematically shown in Fig. 1. The Cooper pair has amplitude $t_{L\to R}$ ($t_{R\to L}$) for transfer from left to right (right to left). For simplicity, we put $\theta = 0$ and assume real tunneling amplitudes t_{jn} in Eq. (3). To lowest order in αB, SOI and Zeeman field combine to the term $H' = (i\boldsymbol{A}_x + \boldsymbol{B}_x)\sigma_x$, with 2×2 matrices in dot level space, $\boldsymbol{A}_x = A\begin{pmatrix}0 & 1\\ -1 & 0\end{pmatrix}$, where $A \propto \alpha$, and $\boldsymbol{B}_x = B\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$. To lowest order in the t_{jn} and in H', processes like the one sketched in Fig. 1(a) generate a contribution to $t_{L\to R}$. Specifically, the process in Fig. 1(a) yields

$$\delta t_{L\to R} = (t_{L1}t_{R1})(t_{L1}iABt_{R2}), \tag{15}$$

where the $\downarrow$ electron causes the first factor while the $\uparrow$ electron switches levels via the product of $\boldsymbol{A}_x$ and $\boldsymbol{B}_x$ processes. In effect, the SOI and the Zeeman field conspire to produce an effective orbital magnetic field via such processes, which then explains the broken TRS. With the group velocity v, the anomalous supercurrent contribution from Eq. (15) is $\delta I_a \propto vAB\,\boldsymbol{\Gamma}_{L,11}\boldsymbol{\Gamma}_{R,12}$. Consider next the reverse process, see Fig. 1(b), which yields $\delta t_{R\to L} = (t_{R2}B(-iA)t_{L1})(t_{R1}t_{L1})$, and therefore causes the same current contribution $\delta I_a \propto (-v)\cdot(-A)B\boldsymbol{\Gamma}_{L,11}\boldsymbol{\Gamma}_{R,12}$. Summing up all relevant processes, we find

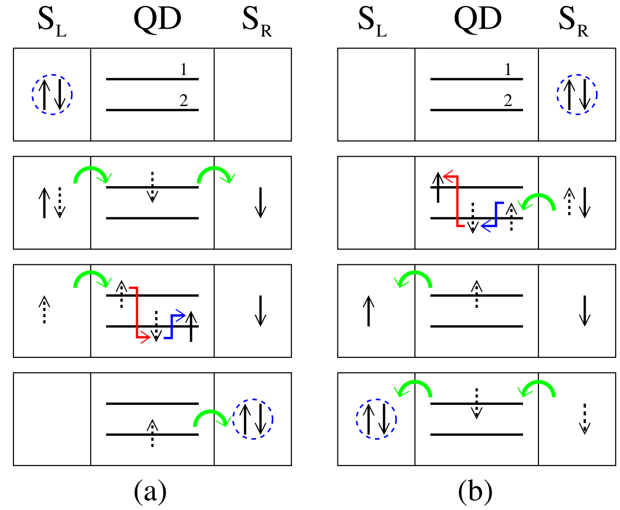

FIG. 1 (color online). Schematic picture of transfer of a Cooper pair through a two-level dot. (a) Contribution to $t_{L\to R}$ yielding an anomalous supercurrent. (b) Reverse process contributing to $t_{R\to L}$. Top and bottom panels represent initial and final states, respectively, which are connected by a sequence of the intermediate virtual states. Solid arrows indicate transitions due to tunneling (green or light gray, connecting leads and dot), spin-orbit (red or gray, connecting different dot levels) and Zeeman (blue or dark gray) coupling. For details see text.

$$I_a \propto B(t_{L1}t_{R1} + t_{L2}t_{R2})(t_{L1}At_{R2} + t_{L2}(-A)t_{R1}) = AB[(\boldsymbol{\Gamma}_{L,11} - \boldsymbol{\Gamma}_{L,22})\boldsymbol{\Gamma}_{R,12} - (\boldsymbol{\Gamma}_{R,11} - \boldsymbol{\Gamma}_{R,22})\boldsymbol{\Gamma}_{L,12}]. \tag{16}$$

Note that $I_a \neq 0$ precisely when Eq. (13) holds, as follows by explicitly computing the commutator in Eq. (13) for $N = 2$. The above arguments resemble the justification for π-junction behavior in quantum dots with Coulomb blockade [17], and can in fact be applied to show that $I_a \neq 0$ under the specified conditions even in the presence of interactions. Interactions give no qualitative change to our results since they do not alter the relevant symmetries. For large $\boldsymbol{\Gamma}_{L/R}$, charging effects are smeared out in any case. For small $\boldsymbol{\Gamma}_{L/R}$ and very strong interactions, the result basically equals the noninteracting one (up to a sign reversal of I_a for an occupied dot).

Numerical analysis for $N = 2$.—We have numerically computed the full current (5) for a two-level dot, taking the wave functions for a harmonic transverse and hard-wall longitudinal confinement as in Ref. [10]. Denoting the distance between the tunnel contacts by L, the Rashba coupling is kept fixed at a moderately small value, $\alpha L = 0.4$, while the remaining parameters were taken both inside and outside the parameter regime where the restrictions needed to derive Eq. (12) apply. The resulting CPRs are shown in the main panel of Fig. 2, where the supercurrent is plotted for increasing values of the Zeeman field. These plots show that a large value for the anomalous current I_a can be obtained with reasonable parameters. The

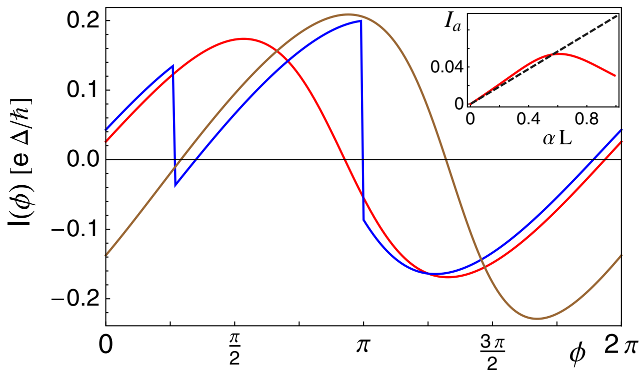

FIG. 2 (color online). Numerical results for the CPR of a two-level dot with $\theta = 0$ for $b_x = 0.3\Delta$, 0.7Δ and 0.9Δ (red or gray, blue or dark gray, brown or light gray curves, respectively, with increasing $|I_a|$). Parameters: $\alpha L = 0.4$, $\Gamma_{L,11} = 2\Delta$, $\Gamma_{L,22} = 0$, $\Gamma_{R,11} = \Gamma_{R,22} = \Delta/2$, $\epsilon_1 = -\epsilon_2 = 0.65\Delta$, $L = 25$ nm, $\Delta = 1$ meV, and $m = 0.035 m_e$. Inset: I_a from numerics (red or gray solid curve) and from Eq. (12) (black dashed curve) vs αL, with $b_x = 0.5\Delta$ and same other parameters as in main panel.

hybridization matrices have been chosen to optimize the commutator in Eq. (13) while satisfying $\Gamma_{L/R,12} = \sqrt{\Gamma_{L/R,11}\Gamma_{L/R,22}}$. The CPR is either continuous or exhibits jumps associated with a change of the ground state, i.e., with the different occupation of the relevant Andreev levels. Such jumps are a common feature in the presence of Zeeman fields already without SOI [1]. In the CPRs obtained by numerics from Eq. (5), we find that the maximal (positive) current can differ from the minimal (negative) current in magnitude. The existence of such two critical currents and its potential for rectification behavior have been discussed in Ref. [12]. The inset of Fig. 2 shows a comparison between the full numerical results for I_a and the analytical predictions deduced from Eq. (12) as a function of α. The comparison is done for an intermediate value of the Zeeman field, $b_x = 0.5\Delta$, and the agreement is good even for rather large I_a.

Conclusions.—We have shown that a generic model for superconducting transport through a quantum dot exhibits spontaneously broken TRS, leading to an anomalous supercurrent appearing at zero phase difference between the superconductors. The effect occurs in the presence of spin-orbit interaction and a suitably oriented Zeeman field, provided that the left and right contact hybridization matrices do not commute. This implies that at least two dot levels must be involved. The wide availability of mesoscopic systems holds the promise to experimentally observe this remarkable effect in the near future.

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[1] A. A. Golubov, M. Yu. Kupriyanov, and E. Il'ichev, Rev. Mod. Phys. **76**, 411 (2004).
[2] Y. J. Doh, J. A. van Dam, A. L. Roest, E. P. A. M. Bakkers, L. P. Kouwenhoven, and S. De Franceschi, Science **309**, 272 (2005); J. A. van Dam, Yu. V. Nazarov, E. P. A. M. Bakkers, S. De Franceschi, and L. P. Kouwenhoven, Nature (London) **442**, 667 (2006); T. Sand-Jespersen *et al.*, Phys. Rev. Lett. **99**, 126603 (2007).
[3] H. Takayanagi, T. Akazaki, and J. Nitta, Phys. Rev. Lett. **75**, 3533 (1995); Th. Schäpers *et al.*, Phys. Rev. B **67**, 014522 (2003); M. Ebel, C. Busch, U. Merkt, M. Grajcar, T. Plecenik, and E. Il'ichev, *ibid.* **71**, 052506 (2005); J. Xiang, A. Vidan, M. Tinkham, R. M. Westervelt, and C. M. Lieber, Nature Nanotech. **1**, 208 (2006); S. Tirelli, A. M. Savin, C. P. Garcia, J. P. Pekola, F. Beltram, and F. Giazotto, Phys. Rev. Lett. **101**, 077004 (2008).
[4] For recent work and references to previous nanotube experiments, see A. Eichler *et al.*, Phys. Rev. B **79**, 161407(R) (2009). In nanotubes, the effects described in this paper are expected to be very small because the SOI is weak and the tunnel contacts often preserve valley pseudospin.
[5] R. Winkler, *Spin-Orbit Coupling Effects in Two-Dimensional Electron and Hole Systems* (Springer, New York, 2003).
[6] J. Nitta, T. Akazaki, H. Takayanagi, and T. Enoki, Phys. Rev. Lett. **78**, 1335 (1997); G. Engels, J. Lange, T. Schäpers, and H. Lüth, Phys. Rev. B **55**, R1958 (1997).
[7] E. V. Bezuglyi, A. S. Rozhavsky, I. D. Vagner, and P. Wyder, Phys. Rev. B **66**, 052508 (2002).
[8] I. V. Krive, L. Y. Gorelik, R. I. Shekhter, and M. Jonson, Fiz. Nizk. Temp. **30**, 535 (2004) [Low Temp. Phys. **30**, 398 (2004)]; I. V. Krive, A. M. Kadigrobov, R. I. Shekhter, and M. Jonson, Phys. Rev. B **71**, 214516 (2005).
[9] O. V. Dimitrova and M. V. Feigel'man, JETP **102**, 652 (2006).
[10] L. Dell'Anna, A. Zazunov, R. Egger, and T. Martin, Phys. Rev. B **75**, 085305 (2007).
[11] B. Béri, J. H. Bardarson, and C. W. J. Beenakker, Phys. Rev. B **77**, 045311 (2008).
[12] A. A. Reynoso, G. Usaj, C. A. Balseiro, D. Feinberg, and M. Avignon, Phys. Rev. Lett. **101**, 107001 (2008).
[13] A. Buzdin, Phys. Rev. Lett. **101**, 107005 (2008).
[14] V. B. Geshkenbein and A. I. Larkin, Pis'ma Zh. Eksp. Teor. Fiz. **43**, 306 (1986) [JETP Lett. **43**, 395 (1986)]; S. Yip, Phys. Rev. B **52**, 3087 (1995); Y. Tanaka and S. Kashiwaya, *ibid.* **56**, 892 (1997); Y. Asano, Y. Tanaka, M. Sigrist, and S. Kashiwaya, *ibid.* **71**, 214501 (2005).
[15] The prediction $I_a \neq 0$ in Ref. [13] is an artifact of an approximation violating a gauge invariance of the system. This problem can be traced to an inconsistent use of the quasiclassical approximation. For a correct treatment, one should proceed along the lines of Ref. [7].
[16] The difference to Eq. (2) arises because a gauge transformation is necessary to obtain the exact solution [10].
[17] B. I. Spivak and S. A. Kivelson, Phys. Rev. B **43**, 3740 (1991).