Narrow Spin Resonance Width and Spin Flip with an rf-Bunched Deuteron Beam

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We used an rf solenoid to study the widths of rf spin resonances with both ***bunched*** and ***unbunched*** beams of 1.85 GeV/c polarized deuterons stored in the COSY synchrotron. With the ***unbunched*** beam at different fixed rf-solenoid frequencies, we observed only partial depolarization near the resonance. However, the ***bunched*** beam's polarization was almost fully flipped; moreover, its resonance was much narrower. We then used Chao's recent equations to explain this behavior and to calculate the polarization's dependence on various rf-solenoid and beam parameters. Our data and calculations indicate that a ***bunched*** deuteron beam's polarization can behave as if the beam has zero momentum spread.

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Stored polarized beams allow one to study the spin dependence of hadronic interactions in the 1 GeV/c to 1 TeV/c region [1–5]. Understanding a ring's spin dynamics and spin resonances is needed to preserve and precisely control the beam's polarization during acceleration and storage. rf magnets can induce rf spin resonances that allow one to manipulate the beam's polarization; they also allow detailed spin resonance studies and beam diagnostics. Running an rf magnet at different fixed frequencies near a spin resonance produces a resonance map. This map can precisely measure the resonance's properties, such as its strength, width, central frequency, and frequency spread, as well as the beam's properties, such as its energy and momentum spread. It was earlier noted by Orlov *et al.* [6,7] and Bukin *et al.* [8–10] that rf bunching could reduce resonance widths to allow more precise measurements of muon g-2 and meson resonances, respectively. We recently made a detailed experimental study of resonance narrowing by measuring resonance maps with both ***bunched*** and ***unbunched*** beams of 1.85 GeV/c polarized deuterons stored in the COSY synchrotron.

In flat circular rings, each beam particle's spin normally precesses around the vertical fields of the ring's dipole magnets. The spin tune $\nu_s = G\gamma$ is the number of spin precessions during one turn around the ring; $G = (g-2)/2$ is the particle's gyromagnetic anomaly, and γ is its Lorentz energy factor. A horizontal magnetic field can perturb the particle's stable vertical polarization creating a spin resonance [11–13]. rf magnets can induce rf spin resonances [14–19]. A deuteron's rf-induced spin resonance's frequency is

$$f_r = f_c(k \pm G_d\gamma), \tag{1}$$

where f_c is the deuteron's circulation frequency, k is an integer, and $G_d = -0.142\,987$.

The deuteron's vector polarization can be described by a two-component complex spinor [11–13] in the frame rotating about the vertical axis with frequency $G\gamma$ [20]:

$$\psi(t) = \begin{bmatrix} h(t) \\ g(t)e^{i2\pi f_c \int_0^t \alpha(t)dt} \end{bmatrix}. \tag{2}$$

The quantities $h \equiv h(t)$ and $g \equiv g(t)$ determine the magnitude of the polarization P_V at time t:

$$P_V/P_V^i = |h(t)|^2 - |g(t)|^2 = 2|h(t)|^2 - 1, \tag{3}$$

where P_V^i is the initial polarization. The $\alpha(t)$ is the f_c-normalized difference between the magnet's rf frequency $f(t)$ and the particle's spin resonance frequency $f_r(t)$:

$$\alpha(t) \equiv [f(t) - f_r(t)]/f_c. \tag{4}$$

Near a single spin resonance of strength $\boldsymbol{\varepsilon}(t)$, a spinor's behavior is governed by the differential equations [20]

$$\frac{1}{2\pi f_c}\frac{dh}{dt} = -i\frac{\boldsymbol{\varepsilon}(t)}{2}g, \tag{5}$$

$$\frac{1}{2\pi f_c}\frac{dg}{dt} = -i\alpha(t)g - i\frac{\boldsymbol{\varepsilon}^*(t)}{2}h. \tag{6}$$

For an ***unbunched*** beam, each particle's resonance frequency $f_r(t)$ in Eq. (4) is constant in time and is given by

$$f_r = f_r^c + \Delta, \tag{7}$$

where Δ is its distance from the resonance's central frequency f_r^c. Using our experimental parameters, we calculated a deuteron's P_V/P_V^i for many Δ values by numerically solving Eqs. (5) and (6) and then applying Eq. (3). We next found the ***unbunched*** beam's average polarization $\langle P_V\rangle_{\rm unb}$ by folding these $P_V(\Delta)$ results together with an assumed Gaussian probability distribution $\rho(\Delta)$, which has an rms spread σ_Δ

$$\langle P_V\rangle_{\rm unb} = \int_{-\infty}^{\infty} P_V(\Delta)\rho(\Delta)d\Delta. \tag{8}$$

For a ***bunched*** beam, each particle's $f_r(t)$ in Eq. (4) oscillates with a synchrotron frequency f_s:

$$f_r(t) = f_r^c + A_s \cos(2\pi f_s t + \varphi_s), \quad (9)$$

where A_s is the synchroton oscillaton's amplitude and φ_s is its phase. For a uniform φ_s distribution, the A_s probability distribution [21] is

$$\rho(A_s) = \frac{A_s}{\sigma_A^2} \exp\left\{-\frac{A_s^2}{2\sigma_A^2}\right\}, \quad (10)$$

where σ_A is the rms spread of A_s.

Using our experimental parameters, we calculated a deuteron's P_V/P_V^i for many A_s values by numerically solving Eqs. (5) and (6) and then applying Eq. (3). Next, we obtained the ***bunched*** beam's average polarization by folding these $P_V(A_s)$ results together with the A_s distribution in Eq. (10):

$$\langle P_V\rangle_{\text{bun}} = \int_0^\infty P_V(A_s)\rho(A_s)dA_s. \quad (11)$$

The experiment's apparatus, including the COSY storage ring [22–25], the EDDA detector [26,27], the electron Cooler [28], the low energy polarimeter (LEP) [29], the injector cyclotron, and the polarized ion source [30–32] were shown in Fig. 4 of Ref. [33]. The polarized D^- ion source beam was accelerated by the cyclotron to 75.7 MeV and injected into COSY. Before injection, the LEP measured the D^- beam's polarization to monitor its stability.

The 20.6 keV electron Cooler reduced the beam's momentum spread $\Delta p/p$ by cooling it for 14 s at the deuterons' 75.7 MeV injection energy, both longitudinally and transversely. The deuterons were then accelerated to 1.85 GeV$/c$, where the rf acceleration cavity was either ***OFF*** during COSY's flat-top giving an ***unbunched*** beam, or ***ON*** giving a ***bunched*** beam.

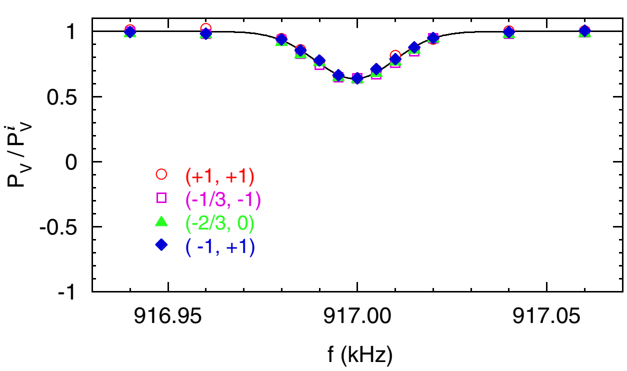

FIG. 1 (color online). Polarization ratio P_V/P_V^i, measured at 1.85 GeV$/c$ with an ***unbunched*** deuteron beam, plotted vs the rf solenoid's fixed frequency f for the four indicated spin states. P_V and P_V^i are the final and initial vertical vector polarizations. The rf solenoid's ramp-up and ramp-down times t_R were 200 ms; its on-time t_{ON} at full $\boldsymbol{\varepsilon}$ was 500 ms. The curve is a fit to Eq. (8) using these rf-solenoid parameters and the ***unbunched*** beam's circulation frequency f_c of 1.147 43 MHz. The errors are purely statistical and are smaller than the symbol size.

A spin resonance was induced using an rf-solenoid magnet [34]; it was a 25-turn air-core water-cooled copper coil, of length 57.5 cm and average diameter 21 cm. Its inductance was 41 ± 3 μH. Its RLC resonant circuit, typically operated near 917 kHz and 5.7 kV rms. The longitudinal rf magnetic field at its center was about 1.17 mT rms, giving an rf $\int Bdl$ of 0.67 ± 0.03 T $\cdot$ mm rms.

The cylindrical EDDA polarimeter [26,27] measured the beam's polarization in COSY. We reduced its systematic errors by repeatedly cycling the polarized deuteron ion source beam through five spin states with nominal vector P_V and tensor P_T vertical polarizations: $(\boldsymbol{P_V}, \boldsymbol{P_T}) = (\mathbf{0}, \mathbf{0}), (+\mathbf{1}, +\mathbf{1}), (-\frac{\mathbf{1}}{\mathbf{3}}, -\mathbf{1}), (-\frac{\mathbf{2}}{\mathbf{3}}, \mathbf{0}), (-\mathbf{1}, +\mathbf{1})$. The measured (0, 0) state polarization was subtracted from the other four measured polarizations to correct for detector efficiencies and beam motion asymmetries.

In the COSY ring, the deuterons' average circulation frequency f_c was 1.147 43 MHz at 1.85 GeV$/c$, where their Lorentz energy factor was $\gamma = 1.4046$. For these parameters, the spin tune $\nu_s = G\gamma$ was $-0.200\,84$. Thus, Eq. (1) gave that the $k = 1$ spin resonance's central frequency should be near $f_r^c = (1 + G\gamma)f_c = 917.0$ kHz.

A resonance map was first obtained with the beam ***unbunched***. For different fixed rf-solenoid frequencies f near 917 kHz, we linearly ramped the rf-solenoid's strength from zero to full $\boldsymbol{\varepsilon}$ during $t_R = 200$ ms; we then held $\boldsymbol{\varepsilon}$ fixed during $t_{\text{ON}} = 500$ ms; next, we linearly ramped it to zero during $t_R = 200$ ms. The resulting measured polarization ratios P_V/P_V^i are plotted in Fig. 1 for the four nonzero spin states; note that their values agree well with each other. We fit these ***unbunched*** P_V/P_V^i data to Eq. (8) with f_r^c, σ_Δ, and $\boldsymbol{\varepsilon}$ as fit parameters. The fit's minimum $\chi^2/(N-3)$ of 3.6 occurred at f_r^c of $916\,999 \pm 2$ Hz, σ_Δ of 10.3 ± 0.7 Hz, and $\boldsymbol{\varepsilon}$ of $(1.057 \pm 0.005) \times 10^{-5}$. The errors include our χ^2-estimated systematic errors. An independent measurement [35] gave $\boldsymbol{\varepsilon}$ of $(1.067 \pm 0.003) \times 10^{-5}$.

We obtained a similar resonance map using a ***bunched*** beam with synchrotron frequency f_s of 215 Hz; these

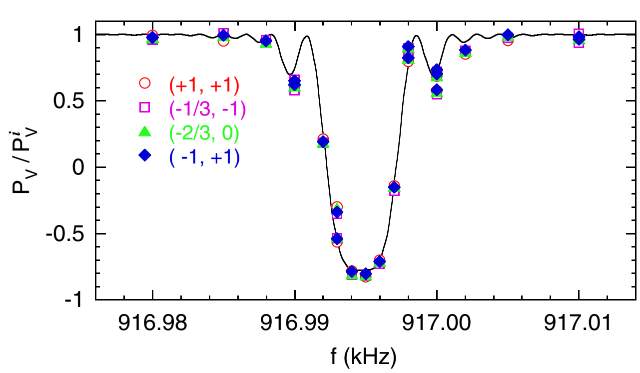

FIG. 2 (color online). Vertical vector polarization ratio P_V/P_V^i, measured at 1.85 GeV$/c$ with a ***bunched*** deuteron beam, plotted vs the rf-solenoid's fixed frequency f for the four indicated spin states. The curve is a fit to Eq. (11) for the parameters in Fig. 1 caption; the synchrotron frequency f_s was 215 Hz.

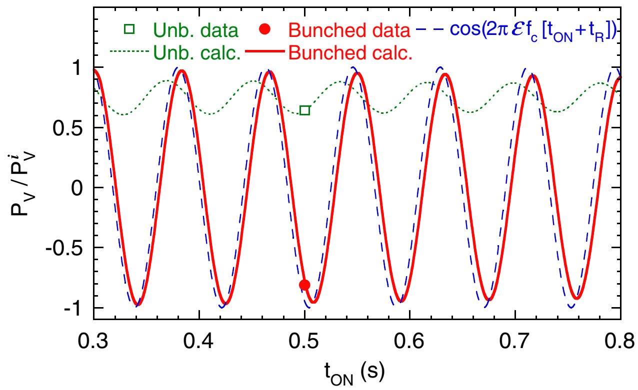

FIG. 3 (color online). ***Unbunched*** and ***bunched*** spin-averaged vertical vector polarization ratios P_V/P_V^i at the spin resonance's center $f = f_r^c$ in Figs. 1 and 2, respectively, plotted, together with calculations, vs the rf solenoid's on-time t_{ON}; t_R was 200 ms. The ***unbunched*** and ***bunched*** calculations used the parameters in the Fig. 1 and 2 captions, respectively. The dashed curve is $\cos(2\pi\boldsymbol{\varepsilon} f_c[t_{ON} + t_R])$ for the $\Delta p/p = 0$ case.

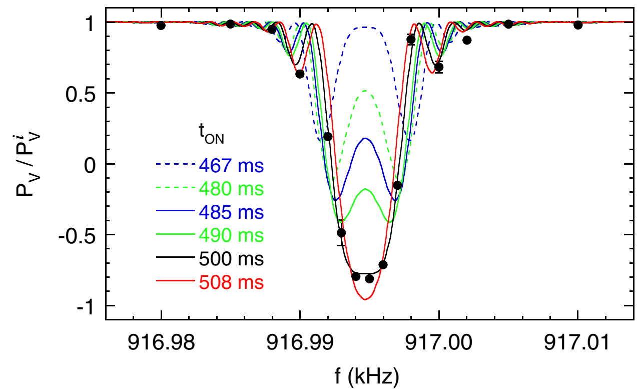

FIG. 4 (color online). Spin-averaged vertical vector polarization ratio P_V/P_V^i for ***bunched*** beam plotted vs the rf-solenoid's fixed f. The curves were calculated for the indicated t_{ON} values using the parameters in Fig. 2 caption; t_R was 200 ms.

P_V/P_V^i data are plotted in Fig. 2. The resonance was very narrow and had almost full spin flip at its center f_r^c. The data also suggest two symmetric smaller dips, each about 5 Hz from f_r^c. We fit these ***bunched*** P_V/P_V^i data to Eq. (11) with f_r^c, σ_A, and $\boldsymbol{\varepsilon}$ as fit parameters. The fit's minimum $\chi^2/(N - 3)$ of 42 occurred at f_r^c of $916\,994.7 \pm 0.5$ Hz, σ_A of 23 ± 4 Hz, and $\boldsymbol{\varepsilon}$ of $(1.052 \pm 0.015) \times 10^{-5}$. The errors include our χ^2-estimated systematic errors. The fit curve agrees qualitatively with the data; it reproduces the narrow width, the spin-flip at the center, and the side dips.

We also studied the ***bunched*** calculation's dependence on various rf-solenoid and beam parameters. By varying each parameter with the others fixed, we found that the ***bunched*** calculation was independent of the synchrotron phase φ_s in Eq. (9) and depended only weakly on f_s and σ_A. However, the P_V/P_V^i value depended strongly on the rf solenoid's on-time t_{ON}. The calculated P_V/P_V^i at the resonance center f_r^c is plotted vs t_{ON} in Fig. 3 for both ***unbunched*** and ***bunched*** beams; it oscillates as a function of t_{ON}. For an ***unbunched*** beam, the calculated oscillation's amplitude is small and decreases at large t_{ON}. However, for a ***bunched*** beam, P_V/P_V^i oscillates from almost +1 to −1 with only slight damping. The ***bunched*** calculation is compared with the formula [20] for the $\Delta p/p = 0$ case:

$$P_V/P_V^i = \cos(2\pi\boldsymbol{\varepsilon} f_c[t_{ON} + t_R]). \tag{12}$$

Our ***unbunched*** and ***bunched*** data points at the resonance centers in Figs. 1 and 2, respectively, are also plotted in Fig. 3 at $t_{ON} = 500$ ms; as expected from Figs. 1 and 2, both agree well with the calculations. Figure 3 indicates that, for a ***bunched*** beam, one can adjust P_V/P_V^i at an rf resonance's central frequency f_r^c anywhere from fully flipped to unpolarized to its initial value. Similar behavior was apparently seen at IUCF, but it was not fully understood [36,37].

To understand the narrow resonance's dependence upon t_{ON}, we calculated resonance maps for different t_{ON} values, as shown in Fig. 4 along with the spin-averaged ***bunched*** data from Fig. 2. Note that P_V/P_V^i varies from +1 to −1 at the resonance's center. Figures 3 and 4 suggest that a properly timed pulse at the resonance's center f_r^c might spin flip a ***bunched*** beam's polarization. However, this would require a far more precise knowledge of f_r^c than the frequency sweep technique [14–17].

We also calculated ***bunched*** resonance maps for different values of the synchrotron amplitude's spread σ_A; they are shown in Fig. 5 along with the spin-averaged data from Fig. 2. The resonance width depends very weakly on σ_A; its depth does depend on σ_A. The two sidebands are probably not due to the synchrotron motion since they also occur in the $\Delta p/p = 0$ calculation (the dashed curve). Thus, for ***bunched*** beams, the A_s spread σ_A increased the resonance's width very little from its $\Delta p/p = 0$ value.

In summary, we recently measured spin resonance maps of both ***bunched*** and ***unbunched*** beams of 1.85 GeV/c polarized deuterons stored in COSY, by operating an rf

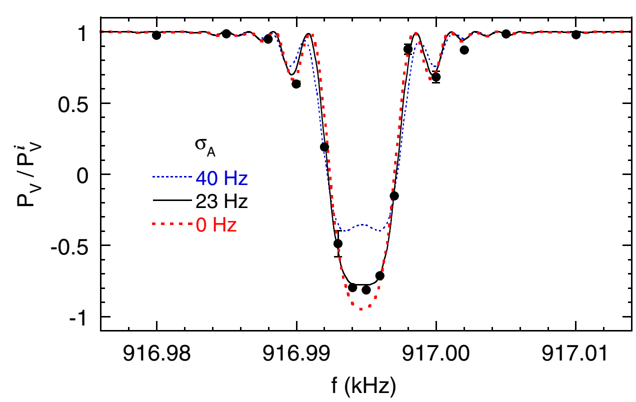

FIG. 5 (color online). Spin-averaged vertical vector polarization ratio P_V/P_V^i for ***bunched*** beam plotted vs the rf-solenoid's fixed f. The curves were calculated for the indicated σ_A values using the parameters in Fig. 2 caption.

solenoid at different fixed frequencies near a spin resonance. With an ***unbunched*** beam, we found a shallow and wide (full width at half maximum ~25 Hz) depolarization dip; we found earlier [34] that for ***unbunched*** deuterons, the dip's depth and width depend upon the beam's momentum spread $\Delta p/p$. Our new experiment with ***bunched*** deuterons found that the resonance was very narrow (fwhm ~5 Hz) and the polarization at the resonance's center was almost fully flipped. We used the theoretical approach adopted in Ref. [20] to explain this data and then to predict the dependence of P_V/P_V^i on various rf-solenoid and beam parameters. The predictions in Figs. 3 and 4 could be easily tested by varying the on-time t_{ON} and frequency f of an rf solenoid installed in a ring containing stored polarized deuterons. Our data and calculations both indicate that a ***bunched*** deuteron beam's polarization can behave as if the beam has zero momentum spread.

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