Quasi-Bloch Oscillations in Curved Coupled Optical Waveguides

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We report the observation of quasi-Bloch oscillations, a recently proposed, new type of dynamic localization in the spatial evolution of light in a curved coupled optical waveguide array. By spatially resolving the optical intensity at various propagation distances, we show the delocalization and final relocalization of the beam in the waveguide array. Through comparisons with other structures, we show that this dynamic localization is robust beyond the nearest-neighbor tight-binding approximation and exhibits a wavelength dependence different from conventional dynamic localization.

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The dynamics of an electron in a one-dimensional (1D) spatially periodic potential in the presence of a spatially uniform electric field has generated great interest over the past 70 years. When the electric field is static (dc), this yields the well-known phenomena of Bloch oscillations (BOs) wherein an initially localized electron wave packet delocalizes over several lattices and then relocalizes back to its initial position at times given by the Bloch oscillation period, τ_B [1,2]. More recently, it was shown that a similar phenomenon, dynamic localization (DL), can occur even for ac fields [3].

Two distinct types of DL exist: approximate dynamic localization (ADL) and exact dynamic localization (EDL). ADL occurs for a particular amplitude of sinusoidal electric fields if the electron band dispersion can be accurately described in the nearest-neighbor tight-binding (NNTB) approximation [3]. EDL occurs for arbitrary dispersion but only for ac fields that are discontinuous at every sign change [4]. In both ADL and EDL, the electron relocalizes at times given by the period, τ, of the ac field.

Quasi-Bloch oscillations (QBOs), a new type of DL, can occur when both ac and dc electric fields are present. QBOs differ from ADL and EDL in that they occur regardless of the shape and amplitude of the ac component of the field. Relocalization occurs with a period of $T = N\tau$ ($N > 1$) if the BO period associated with the dc component of the field is $\tau_B = N\tau$. Although QBOs do not occur for an arbitrary band dispersion, they are less restrictive than ADL because they can occur if the band dispersion is accurately described in the $(N-1)$th-order NNTB approximation [5].

Because of domain formation, electron-electron interactions, and electron-phonon interactions, BOs and DL are difficult to observe in solid-state systems. BOs have been observed in semiconductor superlattices [6], which make possible the completion of several BOs before decoherence dominates. DL also requires large-amplitude, THz-frequency ac fields with periods shorter than the electron decoherence times. To avoid these difficulties, researchers have investigated a number of alternative systems to observe BO and DL, such as periodic arrays of optically trapped atoms [7–9] and waveguide structures [10–19]. Nonetheless, until now, the observation of QBOs has not yet been reported in any system. In this Letter we present the first experimental demonstration of QBOs in curved coupled optical waveguides (CCOWs) and explicitly show that they are robust beyond the NNTB approximation. By resolving the spectral response of the QBOs and comparing it to other DL structures, we show that QBOs can have a larger relocalization wavelength bandwidth than other DL schemes.

To demonstrate QBOs, we measured the propagation of a continuous wave optical beam in a CCOW array and compared the results with EDL and ADL. Figure 1(a) shows a schematic of the CCOW. The transverse period of the waveguides is d, u (v) is the coordinate perpendicular (parallel) to the direction of beam propagation, $n(u)$ is the refractive index profile, λ is the free space wavelength, and c is the vacuum speed of light. The curvature of the central waveguide at position v is $R(v)$. If the width of the waveguide array is much less than $\min|2R(v)|$, the spatial evolution of the envelope of the out-of-plane component of the magnetic or electric field along v maps onto the temporal evolution of the wave function of an electron in a 1D periodic potential $V(u)$ with period d under the influence of an external electric field $E(t)$. The mapping from Maxwell

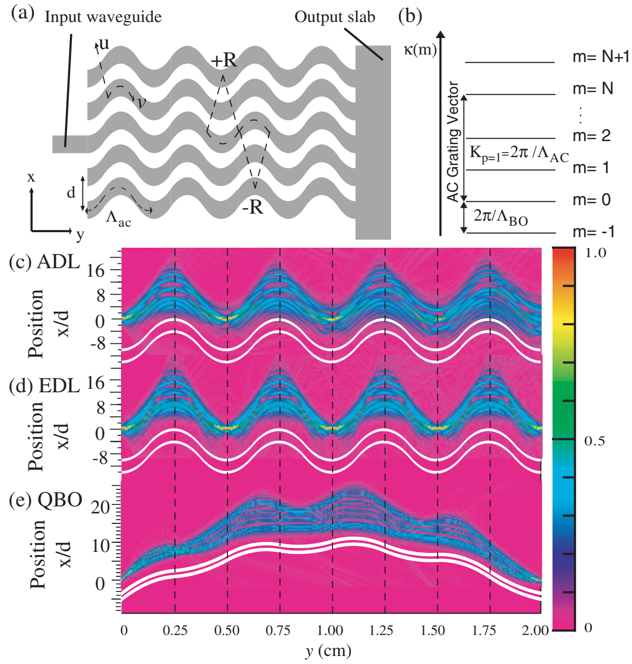

FIG. 1 (color online). (a) Schematic of a CCOW. The waveguides are terminated at the slab. (b) Illustration of QBO as the coupling between WSL modes with a grating vector due to the periodic ac curvature along v. The ac grating vector has net zero coupling strength. Scalar beam propagation simulation of light propagation in (c) ADL, (d) EDL, and (e) QBO structures. The CCOWs are not well described by the NNTB approximation. Some neighboring waveguides are marked white for visibility. The plots are in the x-y coordinate system.

equations to the continuous Schrodinger equation is [20]

$$t \to \frac{v}{c}, \qquad m^* \to \frac{\bar{n}h}{c\lambda}, \qquad V(u) \to -hc\frac{\delta n^2(u)}{2\bar{n}\lambda}, \qquad eE(t) \to \frac{hc\bar{n}}{\lambda R(v)}. \tag{1}$$

In the electron system, t is time and $-e$ and m^* are the charge and effective mass of the electron, respectively. In the CCOW system, $\bar{n}$ is the effective index of a single, straight waveguide and $\delta n^2(u) \equiv n^2(u) - \bar{n}^2$.

In a superlattice, the electric field periodically relocalizes the electron wave function in time, while in the optical domain, the waveguide curvature periodically relocalizes the optical beam in space. The radius of curvature in the optical domain plays the role of the electric field in the electronic system. If light is launched into the single input waveguide in the CCOW [Fig. 1(a)], then DL manifests as the periodic relocalization of the light back into a single waveguide along the direction of propagation. DL in the optical domain is controlled via the choice of the radius of curvature profile, $R(v)$, and the optical wavelength.

BOs and DL can be explained in the context of the CCOWs. We begin by examining the effects of curvature on the optical modes in the CCOW array. The optical modes of an infinite array of straight, coupled waveguides are Bloch waves that can be labeled by the Bloch wave vector, k, where $-\pi/d < k \leq \pi/d$. In analogy to the energy bands of electron states in solids, the infinite waveguide array supports modes with a band of propagation constants $\beta(k)$ [21]. The propagation constant of a Bloch mode, $H_k(u)e^{i\beta(k)v}$, is

$$\beta(k) = \sum_{p=-\infty}^{\infty} \tilde{\beta}_p e^{ikpd}, \tag{2}$$

where the $\tilde{\beta}_p$ are the Fourier expansion coefficients of the band. In general, the description of $\beta(k)$ requires knowledge of all $\tilde{\beta}_p$'s. However, if the propagation constants are well described using only the $\tilde{\beta}_p$'s up to $|p| = N$, the bands are said to be in the Nth-order NNTB approximation. The usual NNTB approximation is $N = 1$. The optical field in the straight waveguide array can be expressed as a linear superposition of the Bloch modes. Because each mode has a different $\beta(k)$, light initially localized in a single waveguide of the array spreads over many waveguides along v [21].

The addition of a constant curvature to the array results in BOs, which can be understood through the optical Wannier-Stark ladder (WSL) modes. The waveguide curvature can be modeled by adding $\bar{n}u/R_{dc}$ to the index of refraction profile. The individual waveguides become detuned so they no longer form delocalized Bloch modes. In a one-band model, the new modes are the WSL modes with propagation constants

$$\kappa(m) = \kappa(0) + \frac{2\pi m}{\Lambda_{BO}}, \tag{3}$$

where in the NNTB approximation, $\kappa(0)$ is the single waveguide propagation constant, $\Lambda_{BO} = \lambda R_{dc}/\bar{n}d$ is the BO period, m is an integer, and the mth mode is spatially centered at the mth waveguide [20]. An initially localized field can be expressed as a superposition of the WSL modes. After propagating a distance equal to a multiple of Λ_{BO}, the phases of the WSL modes are equal to multiples of 2π, resulting in the relocalization of the light.

Certain ac curvature profiles, which are periodic in v with period Λ_{ac}, can result in DL. An example that exhibits EDL is a "square-wave" profile, where $R(v)$ is piecewise constant, such that $R(v) = \Lambda_{ac}\bar{n}d/2\lambda \equiv R_{0E}$ for half of the period and $R(v) = -R_{0E}$ for the other half [22]. Light undergoes one BO over $R(v) = R_{0E}$ and a second BO over $R(v) = -R_{0E}$. For EDL, $R(v)$ must remain finite [4]. However, if the structure can be described in the $N = 1$ NNTB approximation, then DL can be approximated as ADL and can occur even when $R(v)$ diverges. An example is the curvature of the form $R_{ADL}(v) = R_{0A}\sin(2\pi v/\Lambda_{ac})^{-1}$, where R_{0A} satisfies $J_0(d\bar{n}\Lambda_{ac}/\lambda R_{0A}) = 0$ and $J_0(x)$ is the zeroth-order Bessel function of the first kind [10].

If the waveguide curvature consists of a dc component plus a periodic ac component, then QBO can occur. QBOs can be understood by considering the effect of the ac curvature on the WSL modes formed by the dc curvature. As illustrated in Fig. 1(b), the ac curvature couples the WSL modes and changes the amplitude and phase of the field in each waveguide. For QBOs to occur, the ac period should satisfy $\Lambda_{BO} = N\Lambda_{ac}$, where N is an integer. Since the ac curvature is periodic in v, it only has Fourier components at wave vectors (along v) of $K_p = 2\pi p/\Lambda_{ac} = 2\pi pN/\Lambda_{BO}$, where p is an integer. Combining this result with Eq. (3), the grating from the ac curvature can only couple WSL modes with a difference in their indices, Δm, that are multiples of N [Fig. 1(b)]. Moreover, the coupling strength due to the ac curvature, $n_{ac}(u) = \bar{n}u/R_{ac}$, between WSL modes with propagation constants that differ by $\Delta\kappa = 2\pi\Delta m/\Lambda_{BO}$ is proportional to $\tilde{\beta}_{\Delta m}$ [23], but in the $(N-1)$th-order NNTB approximation, $\tilde{\beta}_p = 0$ for $|p| > (N-1)$ in Eq. (2). Therefore, there is no net coupling between the WSL modes after propagating through Λ_{BO}, though each WSL mode accumulates a phase-shift determined by Eq. (3). At $v = \Lambda_{BO}$, the WSL-mode phase differences are multiples of 2π, so a beam localized at $v = 0$ exactly relocalizes at $v = \Lambda_{BO}$.

To demonstrate QBOs and show their robustness beyond the NNTB approximation, we fabricated three CCOWs that differ only in their curvature profiles: one to exhibit QBOs and one each to examine ADL and EDL. Each CCOW consisted of an array of 50 waveguides of length $\Lambda = 2.0$ cm and was designed to operate at a wavelength of $\lambda = 1546$ nm. The single-mode waveguides were lithographically defined on an AlGaAs wafer to have an effective index of $\bar{n} = 3.261$ [11,13]. The waveguide spacing of $d = 6.7$ μm yields a photonic band gap of 77.2 cm^{-1} between the band of interest and the next band, resulting in minimal coupling to other bands. The first band has Fourier expansion coefficients $\tilde{\beta}_1 = -12.3$ cm^{-1}, $\tilde{\beta}_2 = 1.82$ cm^{-1}, $\tilde{\beta}_3 = -0.45$ cm^{-1}, and $\tilde{\beta}_4 = 0.15$ cm^{-1}, which is well-described in the 3rd-order NNTB approximation.

In the three systems, $\Lambda_{ac} = 0.5$ cm. The ADL structure had a curvature profile of $R_{ADL}(v) = R_{0A}\sin(2\pi v/\Lambda_{ac})^{-1}$ with $R_{0A} = 29.4$ mm, which would yield ADL only if the structure could be described in the NNTB. The EDL structure had a square-wave curvature profile with an amplitude of $R_{0E} = 35.33$ mm, so that $R_{0E} = \Lambda_{ac}\bar{n}d/2\lambda$. The QBO structure had a curvature profile $1/R_{QBO} = 1/R_{dc} + 1/R_{ac}$, where

$$R_{ac} = \begin{cases} R_{0ac} & \text{for } (q+\frac{1}{4}) < v/\Lambda_{ac} < (q+\frac{3}{4}) \\ -R_{0ac} & \text{for } (q+\frac{3}{4}) < v/\Lambda_{ac} < (q+\frac{5}{4}), \end{cases}$$

$R_{dc} = 281.9$ mm, $R_{ac} = 58.7$ mm, q is an integer, and $0 < v < 4\Lambda_{ac}$. R_{dc} was chosen such that $\Lambda_{BO} = 4\Lambda_{ac} = 2.0$ cm. This ac component was chosen to not result in DL in the absence of R_{dc}. The radii of curvature of the three structures were much larger than the total array width of 0.335 mm, ensuring the validity of Eq. (1).

We calculated the optical intensity profiles in the three non-NNTB CCOWs using a 2D scalar beam propagation method [Figs. 1(c)–1(e)]. In the EDL structure, light completely relocalizes to the center waveguide after each ac period, but in the ADL structure, it only partially relocalizes at each ac period and is not well localized at the end of the structure. In the QBO structure, light relocalizes only after four ac periods at $v = 4\Lambda_{ac} = \Lambda_{BO}$.

To characterize the samples, TM polarized light from a tunable laser was coupled into an input waveguide to array via a tapered optical fiber. To observe the transverse profile of the optical power, we focused our imaging system through the slab waveguide [Fig. 1(a)] to the CCOW termination and imaged with an InGaAs camera. The details of the measurement technique are described in [11].

The solid lines in Figs. 2(a)–2(l) show measured power profiles at $v = \Lambda_{ac}/2$, Λ_{ac}, $2\Lambda_{ac}$, and $4\Lambda_{ac}$. Consistent with numerical simulations using the beam propagation method (dashed lines), the light in the EDL structure relocalized after each oscillation period, while the light in the ADL structure only partially relocalized. To our knowledge, this is the first experimental verification of the failure of ADL in non-NNTB structures. The light in the QBO structure relocalizes only after 4 ac periods, demonstrating that QBOs occur even for non-NNTB structures.

According to Eq. (1), DL depends on the wavelength. Figures. 3(a)–3(c) show the spatially resolved optical spectrum at $v = 4\Lambda_{ac}$ for the structures. For the EDL and QBO structures, localization is maximum at the design wave-

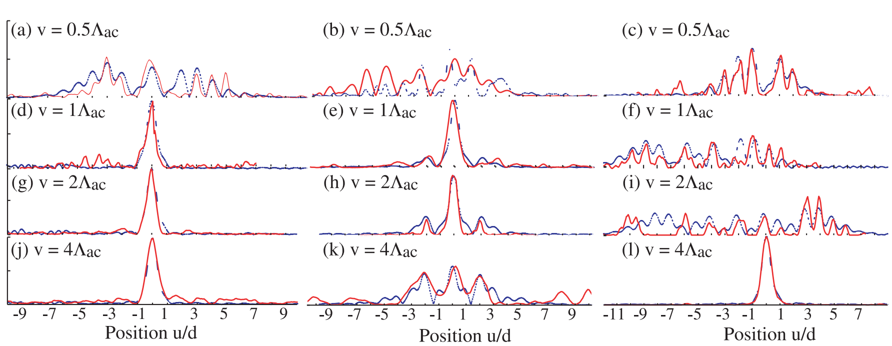

FIG. 2 (color online). Calculated (dashed lines) and measured (solid lines) optical power as a function of the transverse coordinate for EDL (leftmost column), ADL (center column), and QBO (rightmost column), respectively, at $v = 0.5\Lambda_{ac}$, $1\Lambda_{ac}$, $2\Lambda_{ac}$, and $4\Lambda_{ac}$.

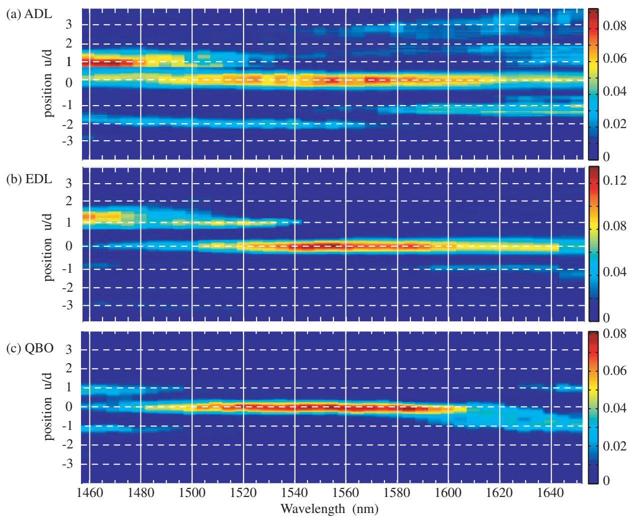

FIG. 3 (color online). Spatially resolved spectrum of (a) ADL, (b) EDL, (c) QBO.

length of 1546 nm. For the ADL structure, complete relocalization does not occur at any wavelength.

The full width at three-quarter maximum (FWTM) of the EDL structure was computed using scalar beam propagation to be 45 nm, and the ratio between the FWTM of the QBO and EDL structures was 1.7. The measured ratio and width were 1.6 and 55 nm, respectively. We chose the three-quarter reference instead of the traditional half reference, because the transmittance of the central waveguide does not drop below half its maximum in the experimental wavelength range.The discrepancy between the measured and numerical results can be due to the wavelength dependence of 2D refractive indices which was neglected in our numerical model.

The difference between the EDL and QBO transmission spectra can be more easily understood in the NNTB limit, where approximate analytical expressions can be obtained. In the NNTB approximation, the transmittance of the central waveguide is $T \simeq |J_0[2\tilde{\beta}_1|S(4\Lambda_{\mathrm{ac}}, \lambda)|]|^2$, where $S(v, \lambda) \equiv \int_0^v e^{i\gamma(z)}dz$, and $\gamma(z) \equiv \frac{2\pi\bar{n}d}{\lambda}\int_0^z R^{-1}(z')dz'$ [20]. If $\delta \equiv (\lambda - \lambda_c)/\lambda_c \ll 1$, for a square-wave EDL structure, $|S_{\mathrm{EDL}}(4\Lambda_{\mathrm{ac}}, \lambda)| \simeq 4\Lambda_{\mathrm{ac}}|\delta|$, while for an $N = 4$ QBO structure, $|S_{\mathrm{QBO}}(4\Lambda_{\mathrm{ac}}, \lambda)| \simeq \sqrt{2}\pi|\delta||S_{\mathrm{QBO}}(\Lambda_{\mathrm{ac}}, \lambda)|$ [24]. Thus, for small δ, the EDL transmission is independent of the square-wave radius of curvature (as long as an integer number of BOs fit in each half period for $\lambda = \lambda_c$), while the strong dependence of $|S(\Lambda_{\mathrm{ac}}, \lambda)|$ on the QBO ac field makes it possible to modify the QBO bandwidth by changing the amplitude and/or shape of the ac curvature. For example, neglecting material dispersion, the FWTM is an order of magnitude larger than the EDL bandwidth for a QBO structure with $R_{\mathrm{ac}} = 30$ mm.

Although we have discussed only TM modes, similar results were obtained for the TE modes. The propagation constants of the TE and TM modes differ by only 0.01%, which changes the localization wavelength by only 0.15 nm.

A reason for the good agreement between our experimental results and predictions is the low bend loss of the waveguides. Bend losses, which are necessarily present in curved waveguides, correspond to interband coupling. For our structures, the maximum expected bend losses are only 0.04 dB/cm, which leads to a power loss of less than 2%. The low power coupling to other propagation bands ensures that our single band model is valid.

In summary, we have demonstrated a new type of dynamic localization called quasi-Bloch oscillations. The observation and comparison of QBO with DL was made possible using the CCOW, in which we can precisely control the nearest-neighbor and interband coupling as well as the magnitude and form of the curvature. In electronic systems, QBOs are of interest because they can occur for ac fields with small amplitudes, thereby eliminating the requirement of large-amplitude THz fields to observe dynamic localization.

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