

All Optical Metamaterial Circuit Board at the Nanoscale

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Optical nanocircuits may pave the way to transformative advancements in nanoscale communications. We introduce here the concept of an optical nanocircuit board, constituted of a layered metamaterial structure with low effective permittivity, over which specific traces that channel the optical displacement current may be carved out, allowing the optical “local connection” among “nonlocal” distant nanocircuit elements. This may provide “printed” nanocircuits, realizing an all-optical nanocircuit board over which specific grooves may be nanoimprinted within the realms of current nanotechnology.

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In modern optical communications, it is of great importance to realize the bridge between silicon-based circuits and plasmonics [1], which may allow the interconnection between signal processing and optical communication technology in a wide variety of applications. An interesting alternative is represented by metamaterial-inspired optical nanocircuit concepts: nanoinductors, nanocapacitors, and nanoresistors may be realized and connected together by properly designing arrangements of nanoparticles with different permittivity [2]. A set of laws analogous to Kirchhoff’s current and voltage laws has been derived for such nanocircuit elements [2], and the simple impedance definitions used in regular electronic circuits may be straightforwardly applied to characterize light interaction with optical nanoparticles. This may allow processing the optical signal at the nanometer scale, without passing through low-frequency electronic circuits, with several potential advantages in terms of speed, bandwidth, and compactness. Optical nanofilters, as one such example, may be formed as combinations of nanoparticles placed within optical waveguides [3] or loading nanoantennas [4]. When the circuit scale becomes relatively large, however, these concepts may be affected by unwanted coupling among distant nanoparticles and strong leakage of displacement current in the background material. This effect may limit the operation and use of optical nanocircuits, as compared to classic low-frequency electronic circuits. A way to overcome these limitations for a single nanoelement is to make use of optical nanoinsulators [5,6], in the form of “shields” made of zero-permittivity materials confining the displacement current. Here, we generalize and extend these concepts by proposing the use of a layered metamaterial board with effective epsilon-near-zero (ENZ) property, over which nanocircuit traces may be carved out in order to channel the optical displacement current “flow” along specific paths. In this way, we introduce the concept of optical nanocircuit board. It is worth mentioning that the concept of circuit modeling of the light interaction with

plasmonic structures is well established in the literature [7]. However, one of the novel aspects of our paradigm consists in the possibility of tailoring and designing nanoparticles to act as “lumped” circuit elements, with desired relevant *quantitative* values in terms of lumped impedance at optical frequencies. This concept and its modularity, which may open doors to the design of more complex nanocircuits and nanosystems in the optical domain, is now brought to a new level with the introduction of the concept of optical nanocircuit board here.

Consider a metamaterial substrate that may be formed by alternating thin layers of epsilon-positive Si_3N_4 and epsilon-negative Ag, with equal thickness t , as in the lower inset of Fig. 1. Its effective permittivity for electric field in the plane of the layers (x - y plane) may be obtained using the effective medium theory by simply averaging the permittivity of silicon nitride and silver [8], obtaining a zero real part of effective permittivity around $f_0 = 725$ THz with sufficiently small imaginary part [9]. Around the frequency f_0 , the flux of displacement current $J_d = -i\omega \text{Re}(\epsilon_{\text{eff}})E$ (with ϵ_{eff} being the local effective permittivity, E the local electric field, and assuming an $e^{-i\omega t}$ time convention) that may flow through such substrate is strongly reduced and the displacement current may indeed be redirected and confined within channels with $\text{Re}(\epsilon_{\text{eff}}) \neq 0$, properly carved in such low-permittivity substrates.

Consider, for instance, the square loop groove with side L , shown in Fig. 1, carved out in this layered metamaterial with ENZ properties. Here, we consider realistic losses and material dispersion for silver and silicon nitride at optical frequencies. For simplicity the geometry is assumed to be two dimensional (2D), i.e., infinite in the direction of stratification (i.e., the z coordinate in Fig. 1). The carved loop, filled with air, is surrounded by an effective ENZ substrate that prevents leakage of displacement current. As a result, J_d is confined within the air groove, resembling the conduction current staying within a metallic wire in a conventional low-frequency electronic circuit board. The

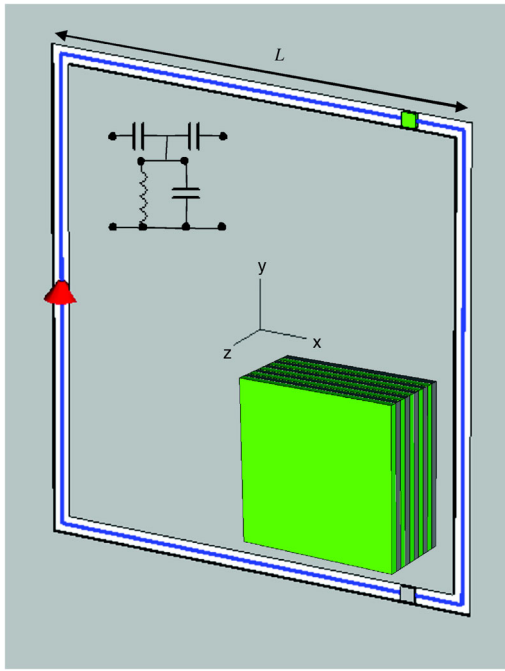


FIG. 1 (color online). A square loop nanocircuit channel carved in a low-permittivity (ENZ) metamaterial substrate. A layered metamaterial (bottom inset) constitutes the nanocircuit board over which specific grooves may be carved out. The channel may be excited by an optical source (left) and loaded by plasmonic (gray, darker) and nonplasmonic (green, lighter) nanorods. The transmission-line model for the field distribution along the channel is reported in the upper inset.

confinement of displacement current along the groove is in some sense analogous to an *optical nanowire* [10], for which we have theoretically shown very low phase variation along the wire, independent of its electrical length. In particular, using a similar approach as in [10], the guided wave number inside the groove here may be derived as $\beta \simeq k_{\text{in}} \sqrt{(\epsilon_{\text{out}}/\epsilon_{\text{in}}) + (4\epsilon_{\text{out}}^2/k_{\text{in}}^2 d^2 \epsilon_{\text{in}}^2)}$, where ϵ_{in} and ϵ_{out} are the effective permittivities of the groove and the background, respectively, d is the groove width, and k_{in} is the groove wave number. This expression is accurate in the limit for which $\epsilon_{\text{out}} \rightarrow 0$ and confirms the possibility of having a very low phase variation along the groove, independent of its electrical length, consistent with the following full-wave numerical results. In addition, this formula allows one to predict the bandwidth of operation of the nanocircuit board, which turns out to be as large as several THz at the visible frequencies of interest considered in this Letter.

Modeled as a transmission-line segment, as shown in the upper inset of Fig. 1, an infinitesimal portion of the groove may indeed be modeled as a series capacitor, provided by the insulator (air) filling the groove, and a parallel inductor-capacitor pair as shunt elements, which are at resonance at the frequency f_0 (in this model for simplicity we are neglecting the presence of losses in the substrate materials,

which are taken into account in our numerical simulations). Around f_0 , the shunt elements effectively constitute an open circuit, reducing the groove to a cascade of series capacitors. This implies that, independently of the electrical length of the groove, the displacement current remains in phase all along the groove, analogous to a wire in a regular circuit board. The groove may then be excited by an optical source (e.g., fluorescent molecule, quantum dot, laser spot, near-field scanning optical microscope) and loaded along its path by different nanoloads, as suggested in Fig. 1.

Consider, for instance, a relatively more complex nanocircuit board, as shown in Fig. 2(a). Here, the nanocircuit groove, carved out in the ENZ nanocircuit board, is excited in the left branch by an optical dipole source (e.g., a fluorescent molecule) emitting at frequency f_0 . At this frequency, the displacement current is evidently “channeled” along the carved groove, all in phase despite the groove length. The figure plots the electric field distribution everywhere, showing how the electric field is nonzero all over the background substrate. However, J_d remains

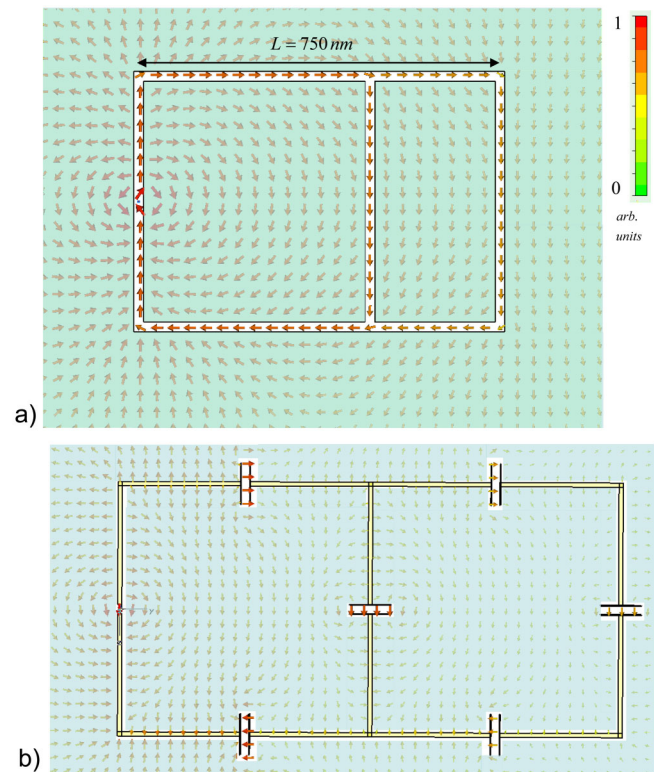


FIG. 2 (color online). Analogy between an optical nanocircuit board and a conventional low-frequency electronic circuit. Snapshots in time of the electric field distribution at frequency f_0 calculated using [14] for (a) a nanocircuit channel carved in the board of Fig. 1 and excited by an optical source (left branch) and (b) the equivalent low-frequency electronic circuit, excited by a current generator. Each branch is modeled as a series capacitor, consistent with the inset of Fig. 1. In this and the following simulations, we have assumed the thickness of each layer in the board to be $t = 2$ nm.

very small everywhere in the ENZ substrate, due to its low effective permittivity, except in the air groove. This results in channeling of the displacement current flow, with almost no phase variation along the groove, consistent with the previous analytical formula shown above and analogous to wires in the low-frequency electronic circuit depicted in Fig. 2(b), made of realistic copper wires that conduct the conduction currents across a series of capacitors in the rf regime. The model of Fig. 2(b) resembles the interconnections in the nanocircuit board of Fig. 2(a). Here in Fig. 2(b), the circuit is excited in the left branch with an ac current generator [analogous to the fluorescent molecule exciting the left branch of Fig. 2(a)]. Each branch is modeled as a lumped capacitor, taking into account the distributed capacitance (inset of Fig. 1) of each side of the circuit along the groove. It is evident how the electric field in this circuit of Fig. 2(b) is also distributed all over the background material, as it may be expected, in a similar fashion to the optical nanocircuit of Fig. 2(a). We usually neglect this effect, since the current (mainly conduction current in the copper and displacement in the capacitors) remains strongly confined along the circuit trace. In the same way, the displacement current remains strongly confined in the nanocircuit groove of Fig. 2(a), with limited leakage due to the low permittivity of the layered ENZ nanocircuit board.

It can be seen that the current prefers to flow in the central branch of the circuit of Fig. 2(b), due to the lower impedance of this branch, rather than in the right branch with higher impedance; a similar feature is verified in the nanocircuit board of Fig. 2(a). The overall dimension of the circuit of Fig. 2(b) is supposed to be much smaller than the operating wavelength, as required for conventional electronic circuit concepts at low frequencies. However, the “spatially staticlike” properties of the ENZ substrate (i.e., uniform phase in the ENZ) essentially relax this requirement for the optical nanocircuit board of Fig. 2(a), for which the total length may become several free-space wavelengths long (while it is electrically small compared to the very long wavelength in ENZ).

Analyzing the electric field distribution along the nanocircuit channels in Fig. 2(a), we have numerically verified that Kirchhoff’s current and voltage laws are indeed satisfied around the frequency f_0 . Despite the presence of realistic losses in the substrate, the displacement current is well confined along the channels and at each node it is properly divided between the two channels according to the different impedance associated with the length of each channel. Equally well, the net sum of the voltages along any closed loop is very close to zero (the residual voltage is provided by Faraday’s law due to the nonzero flux of magnetic field across the cross section of each loop, consistent with the results in [11]). Our numerical results for the nanocircuit of Fig. 2(a) show that these Kirchhoff laws are effectively satisfied over a fractional bandwidth of about 5% around the frequency f_0 for this design, which corresponds to an operational bandwidth of over 35 THz.

This is consistent with the analytical formula derived above.

As a further example, Fig. 2 in [12] shows the nanoloop of Fig. 1 with $L = 500$ nm loaded by two nanorods acting as a nanoinductor and a nanocapacitor (made, respectively, of silver and silicon nitride rectangular nanorods). Having the same size and opposite-signed permittivity at the frequency f_0 , the two nanocircuit elements are in “series” resonance, canceling out their impedances and effectively reducing the total capacitive load of the groove seen by the source, by an amount proportional to their lengths in the x - y plane. This is verified in [12], where, independent of the relative position of the two elements in the nanocircuit channel, the electric field is oppositely polarized in the two nanorods, ensuring continuous displacement current and zero voltage drop across the pair. It is evident from the two panels how this effect is independent of the relative position of the two elements, similar to what is expected in a conventional electronic circuit. In other words, due to the confinement of the displacement current within the carved-out grooves in the ENZ substrate, the two nanorods are being coupled “locally,” even though they may be arbitrarily located “nonlocally” [13]. In [11], we report the magnetic field distribution (oriented along z , snapshot in time) in the plane of the loop for this same geometry, showing how the spatially staticlike property of the ENZ substrate allows one to keep a spatially constant, but time-varying, magnetic field inside the loop, despite its relatively large size. At the two sides of the groove the magnetic field rapidly flips sign, analogously to what happens in a conventional electronic circuit board around the conducting wire, as depicted in the bottom panel of Fig 2 for the analogous rf circuit.

Figure 3 shows a more complex nanocircuit board with several grooves carved out in the ENZ substrate, with parallel and series connections for several nanoinductors (gray, darker for silver nanorod) and nanocapacitors (green, lighter for Si_3N_4 nanorod) displaced along the channel. Their impedance may be tailored by choosing

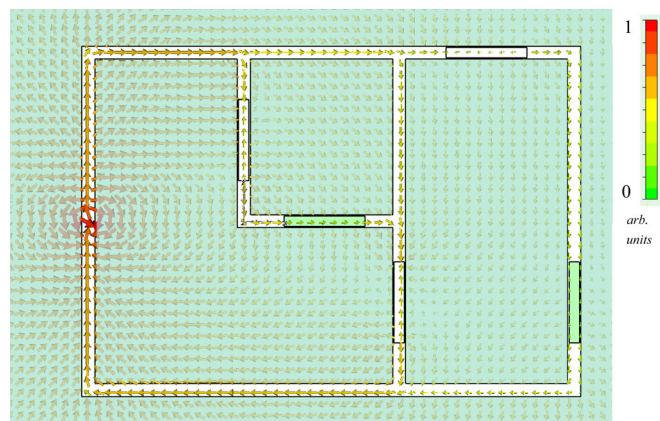


FIG. 3 (color online). A relatively more complex optical nanocircuit board: snapshot of the electric field distribution at frequency f_0 .

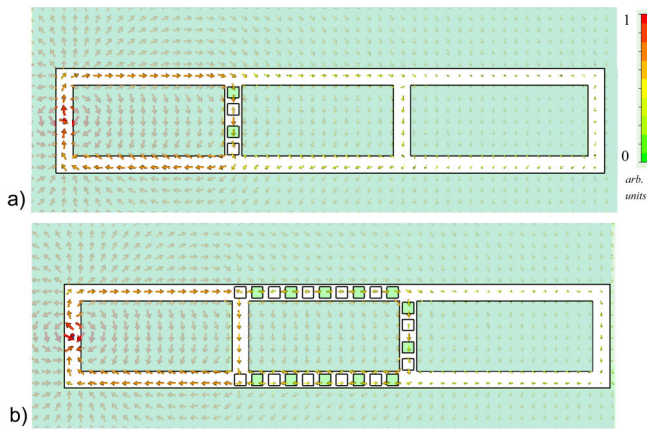


FIG. 4 (color online). The concept of an optical nanobusline: an optical dipole source (e.g., a fluorescent molecule) (left branch) excites the displacement current that may be distributed along the different branches of the nanobusline according to their loading, analogous to a busline in a low-frequency rf micro-electronic circuit.

the materials and their length (in this case, for simplicity we have used only silver and silicon nitride for both the loads and the possible realization of the layered substrate.) In general, due to the specific orientation of the electric field in the groove, which is parallel to the groove axis, the impedance (per unit thickness of substrate) of the nanorod is simply given by $Z = (-i\omega\epsilon d/l)^{-1}$ [2], with l being their cross-sectional length of the nanorod along the channel and d their cross-sectional transverse size. Even for this complex example it is evident that the displacement current is well confined by the circuit channels carved in the ENZ substrate and splits at each node, depending on the impedance of each branch loaded with different nanocircuit elements. In this example, it is clear that by properly loading each branch of the nanocircuit board it is possible to reroute and filter the optical displacement current at will, similarly to the functionalities of a conventional printed circuit in rf electronics. In this sense, Fig. 4 introduces the concept of an optical “nanobusline,” for which, depending on the loading of the different parallel grooves with silver and silicon nitride square nanorods, it is possible to reroute the circulation of displacement current into the desired branch, analogous to a rf microelectronic busline loaded with regular circuit elements. We believe that the concepts outlined in this Letter may pave the way to the realization of “printed” optical nanocircuits, transplanting the concepts and functionalities of conventional printed circuit boards and buslines onto optical frequencies, in a nanocircuit board over which specific grooves may be nano-imprinted within the current potentials of nanotechnology, and loaded with nanocircuit elements and nanofilters composed of proper combinations of nanoparticles. These findings may open various venues for possible practical applications related to the general paradigm of optical

nanocircuits introduced in [2–6]. Such relevant functionalities described here may lead to unprecedented possibilities for expanding the field of nano-optics with optical information processing and manipulation at the nanoscale.

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- [11] See EPAPS Document No. E-PRLTAO-103-021941. Figure 1 shows an analogy between the optical magnetic field distribution (snapshots in time) in a nanoscale loop of air groove carved in the nanocircuit board, analogous to Fig. 1, and in the corresponding low-frequency regular rf electronic circuit. For more information on EPAPS, see <http://www.aip.org/pubservs/epaps.html>.
- [12] See EPAPS Document No. E-PRLTAO-103-021941. Figure 2 is similar to Fig. 2, but showing how the confinement of displacement current along the carved channel ensures that the relative position of a pair of nanocircuit elements in a series circuit does not affect the circuit response in its entirety. In the plasmonic (gray) nanoparticle, with negative real part of permittivity, the electric field (snapshot in time) is reversed, ensuring continuity of the displacement current and zero voltage drop along the resonant pair, independent of their relative position. For more information on EPAPS, see <http://www.aip.org/pubservs/epaps.html>.
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