Instanton Glass Generated by Noise in a Josephson-Junction Array

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We compute the correlation function of a superconducting order parameter in a continuous model of a two-dimensional Josephson-junction array in the presence of a weak Gaussian noise. When the Josephson coupling is large compared to the charging energy, the correlations in the Euclidian space decay exponentially at low temperatures regardless of the strength of the noise. We interpret such a state as a collective Cooper-pair insulator and argue that it resembles properties of disordered superconducting films.

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Systems consisting of densely packed metallic grains have been studied for decades; see, for review, Ref. [1]. They exhibit peculiar electronic properties that stem from the quantum tunneling of electrons between the grains. Numerous models of Josephson-junction arrays have been employed to describe properties of granular superconductors [2]. The most recent theoretical research in this area has been inspired by the experimental evidence of a sharp low-temperature superconductor-insulator transition in disordered films [3–6] (see also earlier experimental works [7]). Various theoretical scenarios of the effect have been proposed: suppression of the superconducting order parameter by disorder in two dimensions [8], Bose condensation of vortices [9], trapping of Cooper pairs due to Coulomb blockade [10], collective superinsulator phase [11], insulating Josephson phase quenched by the magnetic field [12], overheating of electrons due to inefficient electron-phonon processes [13]. Weak localization of Cooper pairs and the possibility of Anderson localization in the Coulomb blockade regime have been recently considered by Syzranov, Efetov, and Altshuler [14]. In the model of Ref. [14] the Josephson-junction array is dominated by the charging energy which determines the activation gap for Cooper pairs. In this Letter we argue that the localization of Cooper pairs may occur regardless of the relation between the charging energy and the Josephson coupling energy. We compute the gap for a weakly disordered array dominated by strong Josephson coupling. We show that in this regime the dependence of the gap on disorder and charging energy is entirely different from the one in the Coulomb blockade regime.

We are concerned with the correlations of the order parameter in a two-dimensional array of strongly coupled superconducting grains in the presence of a weak Gaussian noise. Below we demonstrate that regardless of the strength of the Josephson coupling between the grains, the imaginary-time correlation function of the order parameter decays exponentially in $2+1$ dimensions. We call such a state an instanton glass and interpret it as a collective Cooper-pair insulator in which Cooper pairs are weakly localized within areas that include a large number of grains. We consider superconducting grains that are sufficiently large so that the fluctuations of the magnitude of the complex order parameter Δ can be ignored. This condition is satisfied when the distance between electron energy levels in the grain, δ, is small compared to $|\Delta|$. However, the phase θ_i of Δ on each grain is a dynamical variable described by the Hamiltonian

$$\mathcal{H} = \sum_i E_C^i n_i^2 + \sum_{\langle i<j\rangle} E_J^{ij}[1-\cos(\theta_i-\theta_j)] + \sum_i \epsilon_J^i[1-\cos(\theta_i-\phi_i)]. \tag{1}$$

The sum is over all grains, with $\langle i<j\rangle$ denoting the summation over the nearest-neighbor pairs of grains. The first term in Eq. (1) corresponds to the charging effect due to the Cooper-pair exchange between the grains, with n_i being the number operator for the excess Cooper pairs at the ith grain and E_C^i being the charging energy of the grain. The second term describes the Josephson coupling of strength E_J^{ij} between the grains. The form of the last term in Eq. (1) implies the existence of additional weak links that allow some leakage of the Cooper pairs in and out of the superconducting grains. In our model these weak links are external to the two-dimensional Josephson-junction array of the grains. They exist on top of the strong links between the grains. One possible origin of such external weak links could be, e.g., the presence of metallic islands in the substrate in the vicinity of the film. We assume random distribution of phases ϕ_J and call such a disorder "Josephson noise."

The prevailing view is that the ground state of a granular superconductor depends on the ratio of the Josephson coupling energy and the charging energy. If this ratio is large, the Cooper pairs move freely between the grains and the system is a superconductor. If the ratio is small, which should be expected for small grains with large $E_C = (2e)^2/(2C)$, then moving an excess Cooper pair into the grain costs too much energy. In such a case the Cooper pairs are localized on individual grains and the system is an insulator. In our model we require

$$E_J \gg E_C, \tag{2}$$

so that our granular film is electrically close to a homogeneous film. Quantitatively, this condition translates into a large dimensionless tunneling conductance, $g = 2\pi\hbar/(e^2 R) \ll 1$, with R being the normal resistance of the Josephson contact between the grains. In this case the charging energy becomes renormalized by the Coulomb screening of the excess charge on the grain, so that [1]

$$E_C \approx \frac{|\Delta|}{g} \ll |\Delta|. \tag{3}$$

Large E_J, provides small differences between θ_i and θ_j at the neighboring grains. On the contrary, we assume the Josephson noise to be weak,

$$\langle \epsilon_J^2 \rangle \ll \langle E_J^2 \rangle, \tag{4}$$

so that the corresponding small tunneling conductances allow an arbitrarily large difference between θ_i and ϕ_i. In what follows we treat ϕ_i as a dynamical random field. We show that contrary to previous theoretical findings, in the presence of the noise, the Cooper pairs are localized at $T = 0$ within areas of size $r \propto 1/\langle \epsilon_J^2 \rangle$ even under the condition (2).

Since n_i and θ_i are canonically conjugated variables,

$$n_i = -i\frac{d}{d\theta_i}, \tag{5}$$

$$i\hbar\frac{d\theta_i}{dt} = [\theta_i, \mathcal{H}] = 2iE_C^i n_i. \tag{6}$$

This allows one to replace n_i in Eq. (1) by $\hbar\dot{\theta}_i/(2E_C^i)$. Then the first term in Eq. (1) acquires the form of the "kinetic energy." The Euclidean action corresponding to the Hamiltonian (1) is

$$S_{\rm eff} = \int d\tau \bigg\{ \sum_i \frac{\hbar^2}{4E_C^i}\left(\frac{d\theta_i}{d\tau}\right)^2 + \sum_{\langle i<j \rangle} E_J^{ij}[1-\cos(\theta_i - \theta_j)] + \sum_i \epsilon_J^i[1-\cos(\theta_i-\phi_i)]\bigg\}, \tag{7}$$

where $\tau = it$. Without the kinetic term this action is equivalent to the XY spin model in a random field that has been intensively studied in the past [15]. Without the last term Eq. (7) has been also intensively studied (with and without dissipation) in the 1980s in connection with the possibility of the low-temperature reentrant superconductor—normal metal transition due to quantum fluctuations of the phase [16]. The superconductor-insulator transition at $E_C \sim 2E_J$ in a two-dimensional Josephson-junction array has been confirmed by Monte Carlo simulations [17]. We are not aware of any theoretical investigation of the ground state of the model described by Eq. (7) under the condition (2).

The essential features of the model can be studied by considering a square Josephson-junction array with a lattice spacing a, $E_C^i = E_C$, $E_J^{ij} = E_J$ at $T = 0$. Small difference of the phase for the neighboring grains allows one to write for the nearest neighbors

$$\cos[\theta(\mathbf{r_i}) - \theta(\mathbf{r}_j)] \approx 1 - \tfrac{1}{2}[\theta(\mathbf{r_i}) - \theta(\mathbf{r}_j)]^2, \tag{8}$$

$$\theta(\mathbf{r}_j) \approx \theta(\mathbf{r_i}) + (\mathbf{r}_j - \mathbf{r}_i)\cdot[\boldsymbol{\nabla}\theta(\mathbf{r})]_{\mathbf{r}=\mathbf{r}_i}. \tag{9}$$

Substitution of these equations into Eq. (7), summation over the four nearest neighbors in the square lattice, and replacement of the summation over i by the integration according to $\sum_i \to \int d^2r/a^2$ yields a continuous field model described by the action

$$S_{\rm eff} = \int d\tau \int d^2r \left\{ \frac{\hbar^2}{4a^2E_C}\left(\frac{d\theta}{d\tau}\right)^2 + E_J\left(\frac{d\theta}{d\mathbf{r}}\right)^2 \right\} + \int d\tau \int \frac{d^2r}{a^2}\epsilon_J(\mathbf{r})\{1-\cos[\theta(\mathbf{r},\tau) - \phi(\mathbf{r},\tau)]\}. \tag{10}$$

It is convenient to use dimensionless variables: $(\bar{x}, \bar{y}) = (x/a, y/a)$, $\bar{\tau} = \tau/\tau_0$, with

$$\tau_0 = \frac{\hbar}{2\sqrt{E_J E_C}}. \tag{11}$$

In terms of these variables Eq. (10) becomes

$$\frac{S_{\rm eff}}{\hbar} = \left(\frac{E_J}{E_C}\right)^{1/2} \int d^3\bar{r} \left\{ \frac{1}{2}(\bar{\boldsymbol{\nabla}}\theta)^2 + \frac{\epsilon_J}{2E_J}[1-\cos(\theta-\phi)] \right\}. \tag{12}$$

Here the integration is over dimensionless Euclidian coordinates ($d^3r = d\bar{x}d\bar{y}d\bar{\tau}$); $\bar{\boldsymbol{\nabla}}\theta$ is the $3d$ gradient of θ with respect to these coordinates.

We are interested in the limit of $E_J \gg E_C$ when $S_{\rm eff}$ is large compared to $\hbar$ and the phase $\theta(\bar{\mathbf{r}})$ is a well-defined semiclassical field. Quantum dynamics of such a field is dominated by the extremal trajectories of Eq. (12), satisfying

$$\bar{\boldsymbol{\nabla}}^2\theta = \frac{\epsilon_J}{2E_J}\sin(\theta - \phi). \tag{13}$$

At $\epsilon_J = 0$ this equation possesses a solution $\bar{\boldsymbol{\nabla}}\theta = \text{const}$ that describes a global superconducting current. In general, for such a current to exist, the phases at distant points must be correlated. We, therefore, want to compute the correlation function

$$C(\bar{\mathbf{r}}_1, \bar{\mathbf{r}}_2) \equiv \frac{\langle \Delta(\bar{\mathbf{r}}_1)\Delta^\dagger(\bar{\mathbf{r}}_2)\rangle}{|\Delta|^2} = \langle e^{i[\theta(\bar{\mathbf{r}}_1) - \theta(\bar{\mathbf{r}}_2)]}\rangle = \langle \cos[\theta(\bar{\mathbf{r}}_1) - \theta(\bar{\mathbf{r}}_2)]\rangle, \tag{14}$$

where $\mathbf{r} = (\bar{x}, \bar{y}, \bar{\tau})$. The average in Eq. (14) is over all pairs of points $(\bar{\mathbf{r}}_1, \bar{\mathbf{r}}_2)$ in $2+1$ dimensions that are separated by the same distance $|\bar{\mathbf{r}}_1 - \bar{\mathbf{r}}_2|$.

The nonlinear dynamics of the field expressed by Eq. (13) usually presents a problem for the computation of the correlation function in Eq. (14). Below we use a mathematical trick that under the condition (4) allows one to obtain $C(\bar{\mathbf{r}}_1, \bar{\mathbf{r}}_2)$ exactly with a conventional choice for the noise. We introduce a random two-component vector field,

$$\mathbf{f}(\bar{\mathbf{r}}) = [f_1, f_2] = [\epsilon_J(\bar{\mathbf{r}})\cos\phi(\bar{\mathbf{r}}), \epsilon_J(\bar{\mathbf{r}})\sin\phi(\bar{\mathbf{r}})], \quad (15)$$

and write Eq. (13) in the integral form:

$$\theta(\bar{\mathbf{r}}) = \frac{1}{2E_J}\int d^3\bar{r}' G(\bar{\mathbf{r}} - \bar{\mathbf{r}}')[f_1(\bar{\mathbf{r}}')\sin\theta(\bar{\mathbf{r}}') - f_2(\bar{\mathbf{r}}')\cos\theta(\bar{\mathbf{r}}')], \quad (16)$$

where $G(\bar{\mathbf{r}}) = -1/(4\pi|\bar{\mathbf{r}}|)$ is the Green function of the $3d$ Laplace equation, satisfying $\bar{\nabla}^2 G(\bar{\mathbf{r}}) = \delta(\bar{\mathbf{r}})$. Substituting Eq. (16) into Eq. (14) one obtains

$$C(\bar{\mathbf{r}}_1, \bar{\mathbf{r}}_2) = \left\langle \exp\left\{\frac{i}{2E_J}\int d^3\bar{r}[G(\bar{\mathbf{r}}_1 - \bar{\mathbf{r}}) - G(\bar{\mathbf{r}}_2 - \bar{\mathbf{r}})] \times [f_1(\bar{\mathbf{r}})\sin\theta(\bar{\mathbf{r}}) - f_2(\bar{\mathbf{r}})\cos\theta(\bar{\mathbf{r}})]\right\}\right\rangle. \quad (17)$$

To proceed with the calculation of the space average in Eq. (17) one needs to choose the model of the noise. The simplest choice corresponds to the Gaussian distribution for the probability, P, of any given realization $\mathbf{f}(\bar{\mathbf{r}})$:

$$P[\mathbf{f}(\bar{\mathbf{r}})] \propto \exp\left[-\frac{1}{2\langle\epsilon_J^2\rangle}\int d^3\bar{r}\mathbf{f}^2(\bar{\mathbf{r}})\right], \quad (18)$$

which also provides the definition of $\langle\epsilon_J^2\rangle$. With this assumption Eq. (17) becomes

$$C(\bar{\mathbf{r}}_1, \bar{\mathbf{r}}_2) = \left[\int D^2\{\mathbf{f}\}\exp\left\{-\frac{1}{2\langle\epsilon_J^2\rangle}\int d^3\bar{r}\mathbf{f}^2\right\}\right]^{-1}\int D^2\{\mathbf{f}\} \times \exp\left\{\int d^3\bar{r}\left(\frac{i}{2E_J}[G(\bar{\mathbf{r}}_1 - \bar{\mathbf{r}}) - G(\bar{\mathbf{r}}_2 - \bar{\mathbf{r}})] \times [f_1(\bar{\mathbf{r}})\sin\theta(\bar{\mathbf{r}}) - f_2(\bar{\mathbf{r}})\cos\theta(\bar{\mathbf{r}})] - \frac{\mathbf{f}^2}{2\langle\epsilon_J^2\rangle}\right)\right\}, \quad (19)$$

where $\int D^2\{\mathbf{f}(\bar{\mathbf{r}})\} = \int D\{f_1\}D\{f_2\}$ denotes functional integration over the realizations of disorder.

At first glance the evaluation of the above correlator may seem hopeless because it requires the explicit knowledge of $\theta(\bar{\mathbf{r}})$ created by $\mathbf{f}(\bar{\mathbf{r}})$. To see that $C(\bar{\mathbf{r}}_1, \bar{\mathbf{r}}_2)$ can be calculated exactly in the limit of weak noise, we first notice that according to Eqs. (4) and (13) the contribution of $\mathbf{f}(\bar{\mathbf{r}})$ to the spatial derivatives of $\theta(\bar{\mathbf{r}})$ is generally small. In fact, as is shown below, the significant change in $\theta(\bar{\mathbf{r}})$ occurs over the distances $\bar{r} \sim E_J^2/\langle\epsilon_J^2\rangle \gg 1$. This means that the value of θ at a certain point $\bar{\mathbf{r}}$ has very little correlation with $\mathbf{f}$ at that point. Consequently, to the lowest order on the noise, the dependence of $\theta(\bar{\mathbf{r}})$ on $\mathbf{f}(\bar{\mathbf{r}})$ in Eq. (19) can be neglected, and the remaining Gaussian integration over $\mathbf{f}$ can be easily performed. As a result, $\sin\theta(\bar{\mathbf{r}})$ and $\cos\theta(\bar{\mathbf{r}})$ in the exponent nicely combine into $\sin^2\theta(\bar{\mathbf{r}}) + \cos^2\theta(\bar{\mathbf{r}}) = 1$, yielding

$$C = \exp\left\{-\frac{\langle\epsilon_J^2\rangle}{8E_J^2}\int d^3\bar{r}[G(\bar{\mathbf{r}}_1 - \bar{\mathbf{r}}) - G(\bar{\mathbf{r}}_2 - \bar{\mathbf{r}})]^2\right\} = \exp\left\{-\frac{\langle\epsilon_J^2\rangle}{4E_J^2}\int\frac{d^3\bar{q}}{(2\pi)^3}\frac{1 - \cos[\bar{\mathbf{q}}\cdot(\bar{\mathbf{r}}_1 - \bar{\mathbf{r}}_2)]}{\bar{q}^4}\right\}. \quad (20)$$

Further integration gives

$$C(\bar{\mathbf{r}}_1, \bar{\mathbf{r}}_2) = \exp\left(-\frac{|\bar{\mathbf{r}}_1 - \bar{\mathbf{r}}_2|}{l}\right), \qquad l = \frac{32\pi E_J^2}{\langle\epsilon_J^2\rangle}. \quad (21)$$

Phase correlations that decay exponentially in both space and imaginary time indicate that the Josephson-junction array is in the insulating state [18]. Since the instanton solutions of Eq. (13) describe tunneling trajectories of Cooper pairs, we call such a state an "instanton glass." The presence of a finite correlation length in the (x, y) plane,

$$R_c = \frac{32\pi E_J^2}{\langle\epsilon_J^2\rangle}a, \quad (22)$$

implies that at any given moment of time the phase correlation is lost over spatial distances greater than R_c. At any point in space there is also a finite correlation length in the imaginary time,

$$\tau_c = \frac{32\pi E_J^2}{\langle\epsilon_J^2\rangle}\tau_0, \quad (23)$$

that implies the existence of the energy gap,

$$\Delta_{\mathrm{IG}} = \frac{\hbar}{\tau_c} = \frac{\langle\epsilon_J^2\rangle}{16\pi E_J}\left(\frac{E_C}{E_J}\right)^{1/2}, \quad (24)$$

characteristic of an insulator [18]. It represents the localization energy of Cooper pairs within overlapping areas of size R_c. Note the importance of the charging energy, E_C, in the formation of the gap.

Applied to a granular film, the above results mean that Cooper pairs are localized within regions of size R_c that are large compared to the average size of the grain a. If the dimensions of the film, L, are greater than R_c, the film should be a Cooper-pair insulator. At $T \ll \Delta_{\mathrm{IG}}$ the conductivity of such a film must be due to the thermal hopping of Cooper pairs between regions of size R_c, obeying the law $\exp(-\Delta_{\mathrm{IG}}/T)$. Note that in our model the insulating gap is small compared to the Kosterlitz-Thouless temperature, $T_{\mathrm{KT}} = \pi E_J$. So far we have not included the magnetic field into the problem. Its treatment within our model is much more involved, but a plausible speculation can be made about the expected effect of the field. It is known to suppress Josephson tunneling. The weaker the coupling the smaller is the critical field that destroys the tunneling. In our model the weakest links are the external ones characterized by the coupling strength ϵ_J. They will be destroyed by the field first, making the localization length (22) infinite and destroying the insulating gap (24). The nonmonotonic dependence of the Josephson coupling on the field [19] may lead to the nonmonotonic field dependence of the gap. The expected temperature and field behavior of the instanton glass is, therefore, in a qualitative agreement with experimental findings in disordered superconducting films [5].

Also in agreement with experiment is the fact that such a behavior is pertinent to a two-dimensional system and does not appear in a three-dimensional superconducting sample. Indeed, for a $3d$ sample, the calculation similar to the one performed above, but with the Green function $G_4(\bar{\mathbf{r}}) = -1/(4\pi^2|\bar{\mathbf{r}}|^2)$ instead of $G_3(\bar{\mathbf{r}}) = -1/(4\pi|\bar{\mathbf{r}}|)$, provides a power-law decay of the correlations of the phase in a $3+1$ Euclidian space. The corresponding $3+1$ correlation length is, therefore, infinite and the gap is zero. Such a dependence of the correlations on the dimensionality of space is typical for models with a continuous order parameter in the presence of quenched disorder [20,21]. Our approach is, in effect, an extension of the Larkin-Imry-Ma theorem to the Euclidean space-time for problems that involve tunneling in the presence of noise. Note in this connection that similar to problems with quenched disorder the exact form and origin of the noise may not be important for the onset of the insulating phase.

In conclusion, we have demonstrated that Josephson noise, no matter how weak, destroys long-range space-time correlations of the order parameter in a two-dimensional Josephson-junction array regardless of the tunneling conductance. We leave it to the experimentalists to analyze whether the noise necessary to generate the insulating phase was present in experiments that reported low-temperature superconductor-insulator transition in disordered films.

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