Electron Self-Injection and Trapping into an Evolving Plasma Bubble

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The blowout (or bubble) regime of laser wakefield acceleration is promising for generating monochromatic high-energy electron beams out of low-density plasmas. It is shown analytically and by particle-in-cell simulations that self-injection of the background plasma electrons into the quasistatic plasma bubble can be caused by slow temporal expansion of the bubble. Sufficient criteria for the electron trapping and bubble's expansion rate are derived using a semianalytic nonstationary Hamiltonian theory. It is further shown that the combination of bubble's expansion and contraction results in monoenergetic electron beams.

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An important sea change for the field of plasma-based laser wakefield accelerators (LWFA) occurred several years ago [1–4] when quasimonoenergetic high-energy electron beams were produced. Although it has been known for some time [5] that strong plasma wakes excited by a relativistic laser pulse propagating in the x direction through the plasma of density n_0 can accelerate electrons with an ultrahigh gradient $E_x \sim \sqrt{n_0[\mathrm{cm}^{-3}]}$ V/cm, it is the high quality of the self-injected electrons [1–4,6] that makes the LWFA scheme important for high-energy physics. These recent experiments were carried out in the so-called bubble (or blowout) regime [7,8] in which ambient plasma electrons are fully expelled radially by the transverse ponderomotive force of the laser pulse.

The resulting plasma bubble follows in the wake of the laser pulse propagating with the group velocity $v_0/c \approx 1 - 1/2\gamma_0^2$ ($\gamma_0 \approx \omega_0/\omega_p$), where $\omega_p = \sqrt{4\pi e^2 n_0/m}$ is the electron plasma frequency and $\omega_0 \gg \omega_p$ is the laser frequency. Strong longitudinal (E_x) and transverse electric fields inside the bubble are responsible for acceleration and focusing of the relativistic electrons which can, under the right circumstances, be injected into the bubble from the ambient plasma. The latter process of injection and trapping is the focus of this Letter. Although several analytic and semiempirical models of the plasma bubble have recently been put forward [9,10], the complex process of trapping ambient plasma electrons into the bubble is not well understood. Understanding electron trapping or injection from the background plasma has important practical implications because the alternatives (such as, for example, using a second laser pulse [11–13]) are technically demanding. Moreover, injection into a nonevolving bubble in the most interesting case of $\gamma_0 \gg 1$ (corresponding to tenuous plasmas) that could potentially result in single-stage multi-GeV acceleration [14] requires a very large bubble size [9].

In this Letter we demonstrate that robust electron trapping into the ultrarelativistic bubble can be achieved when the bubble's size is adiabatically evolving. The robustness of electron injection is ensured by the *sufficient* trapping criterion in Eq. (1). We also demonstrate that the expansion rate of the bubble's radius must exceed a certain threshold value to cause trapping. Moreover, we show that the combination of the bubble's expansion and subsequent contraction can result in a highly monoenergetic beam. Our simplified model [validated by particle-in-cell (PIC) simulations] assumes that (a) the majority of the background plasma electrons supporting the bubble structure are not trapped by the bubble, and (b) the small fraction of background electrons trapped by the expanding bubble does not affect the bubble's evolution.

We illustrate the concept of plasma electron trapping into an evolving bubble by adopting a simple phenomenological model [9] of a spherical bubble with radius R devoid of electrons moving with the velocity $\boldsymbol{v} = v_0 \boldsymbol{e}_x$. The motion of a plasma electron in the comoving reference frame is described [9] by the Hamiltonian $H(\boldsymbol{\rho}, t) = \sqrt{1 + (\mathbf{P} + \mathbf{A})^2} - v_0 P_x - \phi$, where $\boldsymbol{\rho} = (\xi, y, z)$, $\xi = x - v_0 t$, $\mathbf{P}$ is the canonical momentum, $\mathbf{A}$ and ϕ are the vector and scalar potentials. We normalize time to ω_p^{-1}, length to c/ω_p, and the potentials to $m_e c^2/|e|$. Electron equations of motion are $d\mathbf{P}/dt = -\partial H/\partial \boldsymbol{\rho}$ and $d\boldsymbol{\rho}/dt = \partial H/\partial \mathbf{P}$. For a spherically symmetric bubble moving with $v_0 \approx 1$, the only nonvanishing electromagnetic potentials are A_x and ϕ. Using the $A_x + \phi = 0$ gauge and introducing $\Phi = A_x - \phi$ [hence, $d\mathbf{P}/dt = -0.5(1 + \beta_x)\nabla\Phi$], we find $\Phi = (\rho^2 - R^2)/4$ inside and $\Phi = 0$ outside the bubble. A smooth transition layer of width $d \ll R$ introduced similarly to Ref. [9] was found to have a small effect.

For an ultrarelativistic nonevolving bubble ($R = \mathrm{const}$) we have found no evidence of trapping for a wide range of R and d. From energy conservation, $H = 1$ for all times for the initially quiescent electrons. The situation changes when the bubble is allowed to grow in time: $R(t) = R_0(1 + \epsilon t)$, where $\epsilon \ll 1$. Because the bubble's potential is changing, the Hamiltonian evolves according to $dH/dt = \partial H/\partial t$. We observe that for any untrapped particle far away from the bubble $A_x = \phi = 0$, and $H = \sqrt{1 + \mathbf{P}^2} - v_0 P_x > 0$. Thus, a particle with $H < 0$ must be trapped by the bubble. This constitutes a *sufficient* condition for electron trapping: if the bubble is expanding fast enough, the Hamiltonian of some electrons changes by

$$\Delta H \equiv \int dt \frac{\partial H}{\partial t} = \int \frac{d\xi}{v_0 - v_x} \frac{\partial H}{\partial t} < -1, \qquad (1)$$

and those electrons are trapped by the bubble.

To visualize the process of electron trapping and confirm the trapping condition, we ran a test particle simulation by solving equations of motion for the initially quiescent electrons ($\gamma = 1$) with different initial impact parameters y_0 assuming an expanding bubble with $R_0 = 5$, $d = 0.4$, $\varepsilon = 0.009$, and $\gamma_0 = 66.1$. The bubble expands from R_0 to $1.2R_0$, at which point the expansion stops. It is found that electrons within a small range of impact parameters are trapped and accelerated to high energies as shown in Figs. 1(a) and 1(b). The time evolution of H shown in Fig. 1(c) for the same particles indicates that electrons experiencing a change in Hamiltonian $\Delta H < -1$ are trapped and accelerated to high energies, while electrons with $\Delta H > -1$ are not. By defining the trapping time t_{trap} such that $H(t_{\text{trap}}) = 0$, we observe that $\gamma(t_{\text{trap}}) \ll \gamma_0$. Therefore, bubble expansion is crucial for accelerating particles that would normally slip back: electrons become trapped or accelerated by the expanding bubble. Electron

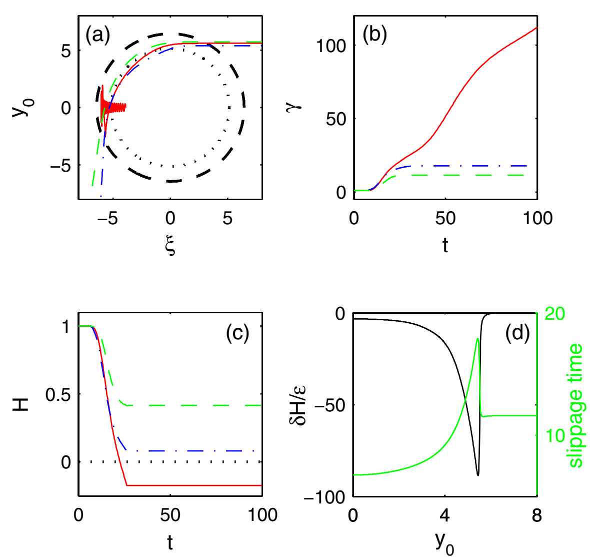

FIG. 1 (color online). Electron trapping by an expanding ultrarelativistic ($\gamma_0 = 66.1$) bubble. (a)–(c) Dynamics of particles with three impact parameters. Trapped particle with $k_p y_0 = 5.6$ (red solid line) satisfies $\Delta H < -1$. Untrapped particles: $k_p y_0 = 5.73$ (green dashed line) and $k_p y_0 = 5.4$ (blue dash-dotted line). (a) Particle orbits, (b) evolution of the particles' energies, (c) evolution of the Hamiltonian. (d) Variation rate of the $\delta H/\epsilon$ (black line) and the slippage time T_{slip} [green (light gray) line] for the unperturbed orbits.

trapping into the bubble is a fully three-dimensional process as evidenced by the particles' trajectories in Fig. 1(a). Earlier analytic and numerical work addressed the issue of separatrix crossing [15] and particle trapping or acceleration [16,17] for *one-dimensional* time-dependent accelerating potentials. A conceptually similar 3D acceleration mechanism (albeit in vacuum) by a diffracting laser pulse [18] has been considered.

In this example, the bubble's radius would double after propagating over $x = 22R_0$ in the plasma. Such a slow evolution implies that the majority of electrons are weakly perturbed by the bubble expansion ($\Delta H \approx 0$), and only the small group acquiring large $|\Delta H|$ can be trapped. To evaluate the minimal rate of bubble inflation sufficient for particle trapping, and to better understand which ambient plasma electrons are most likely to get trapped, we relate the change in Hamiltonian sufficient for trapping ($|\delta H_\epsilon| \sim 1$) to the relative expansion rate ϵ. Using Eq. (1) and carrying out the integration along the unperturbed trajectories (i.e., trajectories with a static bubble), $1 \sim |\delta H_\epsilon| = |dR/dt \int (\partial H/\partial R)(v_0 - v_x)^{-1} d\xi|$ where $dR/dt \equiv R_0 \epsilon$, we arrive at the threshold expansion rate $\epsilon_{\text{tr}} = \min_{y0} |\delta H_\epsilon/\epsilon|^{-1}$, where $\delta H_\epsilon/\epsilon = \int R_0 (\partial H/\partial R) \times (v_0 - v_x)^{-1} d\xi$. For $\epsilon \ll 1$ it is expected that $\delta H_\epsilon \approx \Delta H \propto \epsilon$, making $\delta H_\epsilon/\epsilon$ independent of ϵ. As evidenced by Fig. 1(d), only the electrons with $y_0 \approx R_0$ experience a large $|\delta H_\epsilon/\epsilon|$, which makes them candidates for trapping. The threshold expansion rate for particle trapping into the bubble can be estimated from the nonevolving bubble simulations: for the present example, $\epsilon_{\text{tr}} \approx 0.01$.

The interaction time between the electrons and the bubble estimated as $T_{\text{slip}} \equiv \int d\xi/(v_0 - v_x)$ is also shown in Fig. 1(d), indicating that the electrons with large δH are the ones that interact with the bubble the longest. The physical meaning of the trapping criterion thus becomes transparent: the bubble must be expanding rapidly enough to change its size by an appreciable fraction (about 20% in this example) during the slippage time T_{slip}. For the majority of initially quiescent plasma particles $\delta H/\epsilon \sim 1$; i.e., the adiabatic ($\epsilon \ll 1$) bubble expansion does not result in a massive electron trapping.

This single-particle model establishes the conceptual framework for understanding electron trapping into an evolving bubble. It does not account for a number of important effects such as the perturbation of electron orbits by the laser pulse preceding the bubble, realistic bubble evolution due to self-focusing or diffraction of the laser pulse, and beam loading by the trapped particles. Additionally, the exact potential structure of the relativistic bubble is quite different from the presented simplified model, especially near the back of the bubble ($\xi \approx -R_0$) where electron trajectories are most sensitive to the details of the accelerating field [9]. Nevertheless, satisfying the trapping condition (1) implies robust acceleration also when the above-mentioned effects are accounted for. We demonstrate this for the most challenging case of ultrarelativistic bubble of modest size ($\gamma_0 \sim 100$, $R \approx 5$). We perform a set of numerical experiments in realistic 3D geometry using two numerical approaches: the fully kinetic PIC modeling by the 3D Virtual Plasma Physics Lab (VLPL) [19] code and quasistatic PIC modeling via the axisymmetric time-averaged code WAKE [20] with test particles [21]. The latter accurately describes the dynamics of electron injection until the onset of beam loading [22] and can thus be used as a fast and noiseless diagnostic tool. Moreover, because wakefields are described by A_x and ϕ in WAKE, verifying the sufficient trapping condition (1) is straightforward.

The first set of simulations is done for a LWFA driven by a 100 fs, 2 PW Gaussian laser pulse propagating in a plasma of density $n_0 = 4 \times 10^{17}$ cm^{-3} [14]. Strong bubble expansion is initiated close to the entrance of the plasma by focusing the pulse to spot size $r_0 = 16.8$ μm (2.5 times smaller than that required for self-guiding). The results of the VLPL and WAKE simulations are shown in Fig. 2. Simulations show that the overfocused laser diffracts essentially as in vacuum, and the bubble diameter grows by 13.5% over roughly 10 bubble lengths. Both simulations indicate that self-injection begins at the moment when the bubble starts growing and proceeds continuously. At high energies, the longitudinal phase spaces of both VLPL macroparticles and WAKE test electrons overlap at all times over the entire simulation length. This can be seen by comparing the top and bottom panels in Fig. 2 as well as from Fig. 3(a).

To investigate the relation between the trapping condition (1) and electron acceleration, we map the Hamiltonian H against both the energy gain γ and the injection position x of the (continuously injected) test electrons [Fig. 3(b)]. The majority of self-injected WAKE test electrons have $H < 0$, and according to the sufficient condition (1) are trapped. We have checked that when the laser is initially focused to the self-guided spot size of $r_0 = 42$ μm there is very little bubble evolution and such trapping does not occur over the same propagation distance. Thus, rapid bubble expansion is responsible for trapping of the background plasma electrons.

Figure 3(b) shows that the electrons injected in the beginning of the bubble expansion ($x \approx 0.5$ mm) not only gain the highest energy ($\gamma \approx 250$) but also are trapped in a strict sense with $\Delta H \approx -4.5$. Because of the continuous electron injection into the bubble, the spectrum of accelerated electrons is not monoenergetic. The necessary condition for generating a monochromatic beam is that the electron injection into the bubble terminates at some point. Below we demonstrate that when the bubble *expansion* is followed by *contraction* it is possible to limit the number and energy spread of the accelerated electrons. We illustrate this effect by another 3D VLPL simulation where both expansion and contraction of the plasma bubble are observed for the parameters experimentally accessible for present-day lasers [23]: pulse power 70 TW, duration 30 fs, $r_0 = 13.6$ μm, and $n_0 = 6.5 \times 10^{18}$ cm^{-3}.

Simulations indicate that between $x = 0.68$ mm [Fig. 4(a)] and 1.046 mm [Fig. 4(b)] the bubble undergoes a steady expansion accompanied by continuous electron self-injection. Electron beam energy is broad at $x = 1.046$ mm, as shown in Fig. 5(a) (black markers). After this point, as shown in Fig. 4(d), the bubble starts contracting and the electron injection stops. Some of the recently injected electrons become detrapped from the bubble. By the time the laser pulse advances by $x = 1.32$ mm into the plasma [Fig. 4(c)], there is a clear evidence of the emerging monochromatic electron beam shown in Figs. 5(a) and 5(b). Termination of the electron injection is also observed in WAKE simulations (not shown) and is not, therefore, related to the moderate beam loading of the bubble. Further monochromatization of the beam occurs

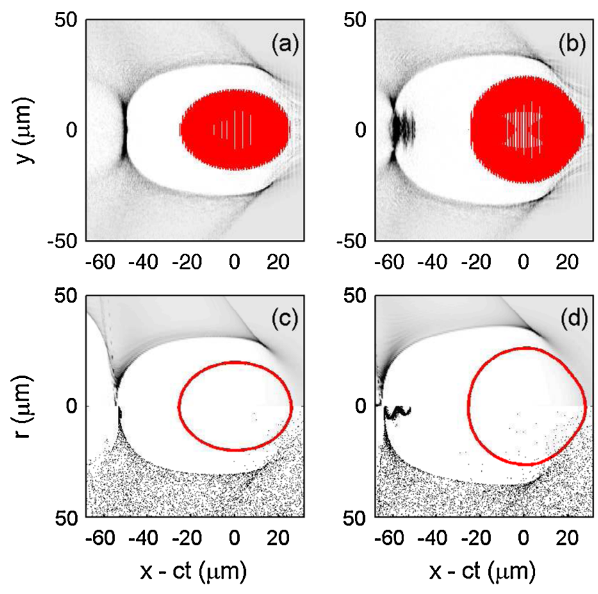

FIG. 2 (color online). Defocusing of a 2 PW, 100 fs laser pulse in the $n_0 = 4 \times 10^{17}$ cm^{-3} plasma causes expansion of the bubble and electron self-injection. Electron density (grayscale) and laser intensity [isocontour at e^{-2} of the peak, red (dark gray)] from the 3D VLPL (a),(b) and WAKE simulations (c),(d) at 0.7 mm (left) and 1.3 mm (right) from the plasma border. Dots in (c),(d): test electron radial positions.

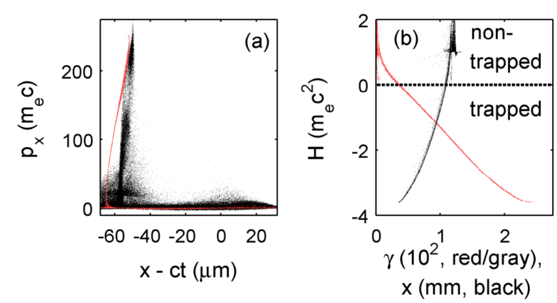

FIG. 3 (color online). (a) Longitudinal momentum of electrons after 1.3 mm propagation. Black: VLPL macroparticles from Fig. 2(b); red (gray): WAKE test electrons from Fig. 2(d). (b) Normalized Hamiltonian for WAKE test electrons as a function of their energy [red (gray)] and initial position (black).

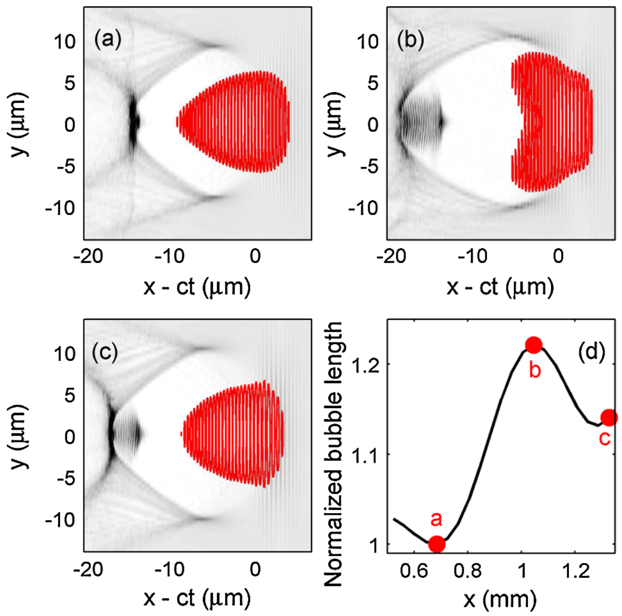

FIG. 4 (color online). Bubble size variation and shortening the electron bunch due to the bubble contraction. (a)–(c) Electron density in grayscale and isocontour of laser intensity in red (dark gray) at (a) $x = 0.68$ mm, (b) 1.046 mm, and (c) 1.32 mm. (d) Variation of the bubble length over the propagation distance; red (dark gray) dots mark the positions of snapshots.

because of the phase space rotation of the remaining trapped electrons indicated by the flattening of the longitudinal phase space shown in Fig. 5(a) [red (light gray) markers]. Energy spectra in Fig. 5(b) show that the resulting quasimonoenergetic electron bunch accelerates without quality degradation to 400 MeV (see the movie in the supplementary material [24]).

To conclude, we have proposed and supported with 3D PIC simulations a concept of robust electron self-injection into an evolving "electron bubble" generated behind an ultraintense laser pulse. Even in the very rarefied plasmas corresponding to the prospective single-stage GeV LWFA, self-injection can be accomplished owing to the temporal

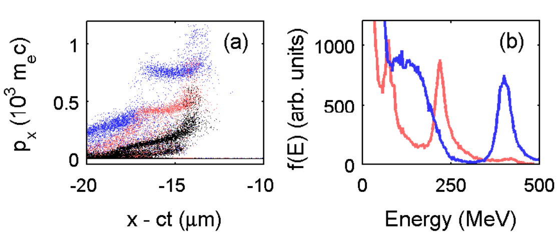

FIG. 5 (color online). Acceleration of the quasimonoenergetic electron bunch generated due to the bubble contraction in the simulation of Fig. 4. (a) Longitudinal phase space of electrons for (black) $x = 1.046$ mm, [red (light gray)] 1.32 mm, and blue (dark gray)] 1.77 mm. (b) Electron energy spectrum for $x =$ 1.32 mm [red (light gray)] and 1.77 mm [blue (dark gray)].

expansion of the bubble. Moreover, the structure evolution guarantees that the self-injected electrons become trapped in the bubble. We arrive at this conclusion using the sufficient condition (1) from the nonstationary Hamiltonian theory. The expansion rate of the bubble sufficient for the electron trapping is derived. It is also shown that if the bubble expansion is followed by its contraction, then further self-injection of the background plasma electrons is terminated, and a monoenergetic beam of trapped electrons emerges.

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[24] See EPAPS Document No. E-PRLTAO-103-029939 for a supplementary movie. For more information on EPAPS, see http://www.aip.org/pubservs/epaps.html.