Independent Control of Scattering Lengths in Multicomponent Quantum Gases

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We develop a method of simultaneous and independent control of different scattering lengths in ultracold atomic gases, such as ^{40}K or a ^{40}K-^{6}Li mixture. Our method can be used to engineer multicomponent quantum phases and Efimov trimer states.

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Introduction.—Recently, multicomponent gases of degenerate fermions [1–3] or boson-fermion mixtures [4] have attracted broad interest both theoretically and experimentally. Novel quantum phases [1] and Efimov states [3,5] have been predicted in three-component Fermi gases. If interspecies scattering lengths can be altered independently, one can engineer Efimov states and the quantum phases of a three-component Fermi gas, and control an effective interaction between two-component fermions immersed in a Bose gas [4]. The magnetic Feshbach resonance (MFR) technique [6] can control only one scattering length, or a few scattering lengths but not independently. The optical Feshbach resonance technique [7] can be directly generalized to the independent control of more than one scattering length [8]. However, it would significantly shorten the lifetime of the system unless the atom has a long-lived electronic excited state (e.g., the $^3P_0$ state of the ^{171}Yb or ^{173}Yb atom [9]).

In this Letter, we combine the two ideas of MFR and rf-field-induced Feshbach resonance [10] to propose a method for independently controlling two scattering lengths in three-component atomic gases. In our scheme, atoms are dressed via couplings between different hyperfine states. With a proper magnetic field, the independent tuning of scattering lengths of atoms in different dressed states can be achieved by control of the Rabi frequencies and detunings for different couplings.

Control of a single scattering length.—We first consider atoms with three ground hyperfine levels [Fig. 1(a)] $|f_1\rangle$, $|f_2\rangle$, and $|g\rangle$. In this Letter, we use the Dirac bracket $|\rangle$ to denote the hyperfine levels of one or two atoms; $|)$ to indicate the spatial states of the relative motion between two atoms, and $|\rangle\rangle$ for the total two-atom state, which includes both spatial motion and hyperfine state. We assume that $|f_1\rangle$ is coupled to $|f_2\rangle$ through Rabi frequency Ω and detuning Δ. Such a coupling can be realized with a two-color stimulated Raman process (TCSRP), i.e., coupling the hyperfine states $|f_{1,2}\rangle$ via a common excited state $|e\rangle$ (not shown in the figure) by two laser beams [11]. The atomic loss caused by the spontaneous decay of $|e\rangle$ can be suppressed when the laser frequencies are far detuned from the resonance. We assume the Rabi frequencies to be at least 1 order of magnitude smaller than the typical depth (∼ a few 100 MHz) of optical traps (see, e.g., [12]), so that the loss due to the decay of the state $|e\rangle$ may be ignored. A stable coupling can also be realized via an rf field that induces a direct transition between two hyperfine states. The Rabi frequency caused by the rf field can be 0.1 –1 MHz with the oscillating amplitude of magnetic field $\lesssim$0.1 G.

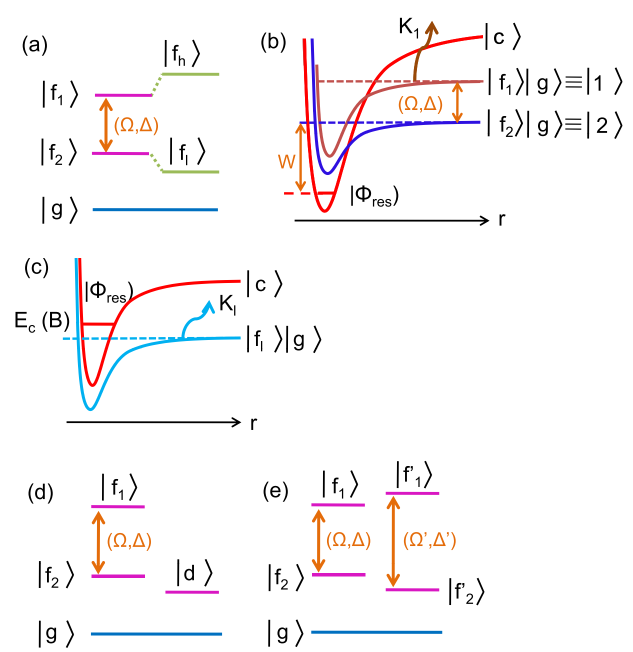

FIG. 1 (color online). (a) Schematic hyperfine levels used for the control of a single scattering length. The states $|f_{1,2}\rangle$ are coupled to form two dressed states $|f_{h,l}\rangle$. (b) Bare channels with hyperfine states $|1\rangle\equiv|f_1\rangle|g\rangle$ and $|2\rangle\equiv|f_2\rangle|g\rangle$ are coupled with parameters (Ω,Δ), and $|2\rangle$ is coupled to the bound state $|\phi_{res})$ in the closed channel with hyperfine state $|c\rangle$ via interaction W. The state $|1\rangle$ can decay via a hyperfine relaxation process with two-body loss rate K_1. (c) Dressed channels. a_{lg} is resonantly enhanced when the threshold energy of the channel with $|f_l\rangle|g\rangle$ crosses the bound state $|\Phi_{res})$ in $|c\rangle$. (d),(e) Hyperfine states used for the control of two scattering lengths with our first (d) and second (e) methods. All the levels are plotted in the rotating frame of reference. In (b) and (c), r indicates the interatomic distance.

Without loss of generality, we assume that the scattering channel with respect to hyperfine state $|2\rangle \equiv |f_2\rangle|g\rangle$ is stable, while the channel with $|1\rangle \equiv |f_1\rangle|g\rangle$ can decay to another channel with hyperfine state $|a\rangle$ through a hyperfine relaxation (HFR) process. We further assume that a static magnetic field is tuned to the region of MFR between the diatomic channel with $|2\rangle$ and a bound state $|\phi_{\rm res})$ in a closed diatomic channel with hyperfine state $|c\rangle$ [Fig. 1(b)]. The binding energy $E_c(B)$ of $|\phi_{\rm res})$ can be controlled by magnetic field B. Therefore, in the rotating frame of reference, the Hamiltonian for the relative motion of two atoms is given by

$$\hat{H} = \hat{H}^{(\rm bg)} + E_c(B)|\phi_{\rm res})(\phi_{\rm res}| \otimes |c\rangle\langle c| + \hat{W} + \hat{W}^\dagger, \quad (1)$$

where

$$\hat{H}^{(\rm bg)} = -\nabla^2 \sum_{i=1,2,a} |i\rangle\langle i| + \hat{V}^{(\rm bg)};$$
$$\hat{V}^{(\rm bg)} = \sum_{i=1,2,a} [V_i^{(\rm bg)}(r) + E_i]|i\rangle\langle i| + [\Omega|1\rangle\langle 2| + V_{1a}(r)|1\rangle\langle a| + {\rm H.c.}];$$
$$\hat{W} = W(r)|\phi_{\rm res})(\phi_{\rm res}| \otimes |2\rangle\langle c|,$$

where we set $\hbar = 2m_* = 1$ with m_* being the reduced mass. In Eq. (1), $V_i^{(\rm bg)}(r)$ $(i = 1, 2, a)$ is the background scattering potential in the channel $|i\rangle$ with r the distance between the two atoms; $W(r)$ is the coupling between $|2\rangle$ and $|c\rangle$; $V_{1a}(r)$ is the coupling between $|a\rangle$ and $|1\rangle$; E_i is the asymptotic energy of the channel $|i\rangle$ in the rotating frame. Here we choose $E_2 = 0$, which implies $E_1 = \Delta$ and $E_a = \Delta - \delta$ with δ being the energy gap between $|1\rangle$ and $|a\rangle$.

Hamiltonian (1) shows that scattering channels $|1\rangle$ and $|2\rangle$ are coupled via the Rabi frequency Ω. Since this coupling is given by the single-atom TCSRP, it does not vanish in the limit $r \to \infty$. The scattering length is therefore not well defined for the bare channels $|1\rangle$, $|2\rangle$. To overcome this problem, we diagonalize the Hamiltonian by introducing dressed states $|f_l\rangle = \alpha|f_1\rangle + \beta|f_2\rangle$ and $|f_h\rangle = \beta|f_1\rangle - \alpha|f_2\rangle$ with eigenvalues $\mathcal{E}_{h/l} = \Delta/2 \pm (\Omega^2 + \Delta^2/4)^{1/2}$ and coefficients $\alpha = \Omega[\Omega^2 + (\mathcal{E}_l - \Delta)^2]^{-1/2}$ and $\beta = (\mathcal{E}_l - \Delta)[\Omega^2 + (\mathcal{E}_l - \Delta)^2]^{-1/2}$ which can be controlled via Δ and Ω. Since the effective coupling between the dressed scattering channels $|f_l\rangle|g\rangle$ and $|f_h\rangle|g\rangle$ vanishes in the limit $r \to \infty$, the scattering length a_{lg} between $|f_l\rangle$ and $|g\rangle$ is well defined [Fig. 1(c)].

In presence of the interchannel coupling $\hat{W}$, both $|f_l\rangle|g\rangle$ and $|f_h\rangle|g\rangle$ are coupled with the bound state $|\phi_{\rm res})$ in the closed channel $|c\rangle$. When the threshold E_l of the channel $|f_l\rangle|g\rangle$ crosses the energy E_c of $|\phi_{\rm res})$, the Feshbach resonance between $|f_l\rangle|g\rangle$ and $|\phi_{\rm res})$ strongly alters the scattering length a_{lg} between $|f_l\rangle$ and $|g\rangle$. Therefore, for a given magnetic field, one can control a_{lg} by tuning E_l through the coupling parameters (Ω, Δ). By a straightforward generalization of the method in Ref. [13], we obtain

$$a_{lg} = a_{lg}^{(\rm bg)} - 2\pi^2 \frac{\Lambda_{ll} - \kappa e^{2i\eta}\Lambda_{al}}{\mathcal{D} - i(2\pi^2)\chi^{1/2}\Lambda_{aa}} \quad (2)$$

with the (Ω, Δ, B)-dependent parameters

$$\mathcal{D} = E_c + \langle c|(\phi_{\rm res}|\hat{W}^\dagger G_{\rm bg}^{(P)}\hat{W}|\phi_{\rm res})|c\rangle - \mathcal{E}_l;$$
$$\Lambda_{\alpha\beta} = \langle\langle\Phi_{\rm bg}^{(\alpha)}[0]|\hat{W}\hat{W}^\dagger|\Phi_{\rm bg}^{(\beta)}[0]\rangle\rangle \qquad (\alpha, \beta = l, a);$$
$$\Gamma = e^{2i\eta}\langle\langle\Phi_{\rm bg}^{(a)}[0]|\hat{W}\hat{W}^\dagger|\Phi_{\rm bg}^{(l)}[0]\rangle\rangle;$$
$$\kappa = 2\sqrt{{\rm Im}[a_{lg}^{(\rm bg)}]\chi}.$$

Here $a_{lg}^{(\rm bg)}$ is the background scattering length between $|f_l\rangle$ and $|g\rangle$ in the absence of MFR with $|\phi_{\rm res})$, and $\chi \equiv \sqrt{\mathcal{E}_l - E_a}$; $|\Phi_{\rm bg}^{(l,a)}[0]\rangle\rangle$ are the background zero-energy scattering states with incident particles in the channels $|f_l\rangle|g\rangle$ and $|a\rangle$, that is, we have $|\Phi_{\rm bg}^{(l)}[0]\rangle\rangle = (1 + G_{\rm bg}^{(+)}\hat{V}^{(\rm bg)}) \times (2\pi)^{-3/2}|f_l\rangle|g\rangle$ and $|\Phi_{\rm bg}^{(a)}[0]\rangle\rangle = (1 + G_{\rm bg}^{(+)}\hat{V}^{(\rm bg)})|\chi\hat{e}_z)|a\rangle$ with the zero-energy background Green's functions $G_{\rm bg}^{(\pm)} = (i0^\pm - \hat{H}^{(\rm bg)})^{-1}$ and $G_{\rm bg}^{(P)} = (G_{\rm bg}^{(+)} + G_{\rm bg}^{(-)})/2$; $|\chi\hat{e}_z)$ satisfying $(\vec{r}|\chi\hat{e}_z) = (2\pi)^{-3/2}e^{i\chi z}$ is the eigenstate of the relative momentum with eigenvalue $\chi\hat{e}_z$, and η is determined by $\hat{V}^{(\rm bg)}$.

Equation (2) shows that one can control the effective interaction between atoms in the states $|f_l\rangle$ and $|g\rangle$, which is determined by the real part ${\rm Re}[a_{lg}]$ of a_{lg}. Under a given magnetic field, one can tune ${\rm Re}[a_{lg}]$ by changing the coupling parameters (Ω, Δ) around the resonance point where $\mathcal{D} = 0$.

Because of the coupling term V_{1a} in the Hamiltonian (1), the HFR process also occurs from the dressed channel $|f_l\rangle|g\rangle$ to $|a\rangle$. The two-body loss rate K_l due to this HFR is proportional to the imaginary part ${\rm Im}[a_{lg}]$ of a_{lg}: $K_l = -16\pi\,{\rm Im}[a_{lg}]$ [14]. In Eq. (2), the change of ${\rm Im}[a_{lg}]$ due to (Ω, Δ) is described by $i(2\pi^2)\delta^{1/2}\Lambda_{aa}$ and $i\,{\rm Im}[\kappa e^{2i\eta}\Lambda_{al}]$. Because of these two terms, the two-body loss is enhanced in the resonance region. To avoid this difficulty, one should either use the scattering channels without hyperfine relaxation for both $|f_1\rangle|g\rangle$ and $|f_2\rangle|g\rangle$, or choose the proper atomic species for which the parameters $(|\Lambda_{al}|, |\Lambda_{aa}|, \kappa)$ are sufficiently small, so that the peak of ${\rm Im}[a_{lg}]$ is much narrower than the resonance of ${\rm Re}[a_{lg}]$.

The scattering and resonance between atoms in the dressed ground states have been discussed in the literatures [10,15]. A new point in our scheme is that we not only couple the two hyperfine states $|f_{1,2}\rangle$, but also employ the MFR between $|f_2\rangle|g\rangle$ and $|c\rangle$. As shown above, the enhancement of a_{lg} usually occurs when the dressed channel $|f_l\rangle|g\rangle$ is in the resonance region of $|\phi_{\rm res})$. Therefore with the help of the MFR, we can control the location of the resonance of a_{lg} by varying both the dressing parameters and the magnetic field.

Independent control of two scattering lengths.—There are two methods to realize the independent control of two scattering lengths in a three-component system with our approach. The first method, illustrated in Fig. 1(d) is to use the states $|g\rangle$, $|f_{1,2}\rangle$, and an additional hyperfine state $|d\rangle$. As shown above, the two states $|f_{1,2}\rangle$ are coupled and form two dressed states $|f_{h,l}\rangle$. We further assume the scattering length a_{dg} between $|g\rangle$ and $|d\rangle$ can be tuned via a MFR which is close to the one between $|2\rangle$ and $|c\rangle$. Therefore, in the three-component system with fermionic atoms in the states $(|g\rangle, |d\rangle, |f_l\rangle)$ we can control a_{dg} by changing the B field. Once the magnetic field is tuned to an appropriate value, we can control the scattering length a_{lg} by changing the coupling parameters (Ω, Δ) with the approach described above.

In the second method, we make use of five hyperfine states as shown in Fig. 1(e). We assume the static magnetic field is tuned to an appropriate value so that the bare channel $|2\rangle$ is near MFR with a bound state in a closed channel $|c\rangle$, while $|2'\rangle \equiv |f_2'\rangle|g\rangle$ is also in the region of MFR with another closed channel $|c'\rangle$. We assume that two states $|1\rangle$ and $|2\rangle$ are coupled with Rabi frequency Ω and detuning Δ, and form two dressed states $|f_{h,l}\rangle$. Similarly, the states $|f_{1,2}'\rangle$ are coupled with Rabi frequency Ω' and detuning Δ', and form dressed states $|f_{h,l}'\rangle$. Then according to the above discussion, the scattering length a_{lg} between $|f_l\rangle$ and $|g\rangle$ can be resonantly controlled by (Ω, Δ), while $a_{l'g}$ between $|f_l'\rangle$ and $|g\rangle$ can be resonantly controlled by (Ω', Δ'). Therefore in the three-component system with fermionic atoms in the states $(|g\rangle, |f_l\rangle, |f_l'\rangle)$, one can independently control a_{lg} and $a_{l'g}$ by changing the four parameters $(\Omega, \Delta, \Omega', \Delta')$.

In the above two methods, two conditions are required to make the independent resonance controls of a_{lg} and a_{dg} $(a_{l'g})$ practical. First, the two MFRs for the bare channels $|2\rangle$ and $|d\rangle|g\rangle$ $(|2'\rangle|g\rangle)$ should be close to each other (e.g., the distance between the two resonance points $\lesssim 10$ G). Second, Rabi frequencies Ω and Ω' should be large enough (on the order of 10 MHz). Fortunately we can find such resonances in many systems [12,16–18]. In principle, the second method can be generalized to the independent control of $n-1$ scattering lengths in an n-component system. To this end, one should make use of $n-1$ MFRs which are close together.

Possible experimental realizations.—Now we discuss possible implementations of our method. In a mixture of ^{40}K and ^{6}Li [12], we can realize the three-level system with the first method in the above section. To this end, we use the hyperfine state $|1/2, 1/2\rangle$ of ^{6}Li to be $|g\rangle$ and consider the state $|9/2, -9/2\rangle$ of ^{40}K to be $|d\rangle$. We also take the states $|9/2, -7/2\rangle$ and $|9/2, -5/2\rangle$ of ^{40}K as $|f_2\rangle$ and $|f_1\rangle$, and form the dressed states $|f_l\rangle$. With the help of the MFR in the channel $|d\rangle|g\rangle$ with $B_0 = 157.6$ G, $\Delta B = 0.15$ G [12] and the one of $|2\rangle$ with $B_0 = 159.5$ G, $\Delta B = 0.45$ G [12], one can tune a_{dg} by changing the magnetic field, and then tune a_{lg} by changing the coupling parameters (Ω, Δ). We note that, since both of the two bare channels $|1\rangle$ and $|2\rangle$ are stable, there is no hyperfine relaxation in the collisions between atoms in the states $|g\rangle$ and $|f_l\rangle$. The unequal masses of ^{40}K and ^{6}Li can lead to many new phenomena in such a three-component heteronuclear system (^{40}K atoms in $|d\rangle$, $|f_l\rangle$ and ^{6}Li atoms in $|g\rangle$), e.g., the appearance of an Efimov state formed by two heavy atoms and one light atom when both a_{dg} and a_{lg} are large enough [5].

With a multichannel calculation based on the realistic ^{40}K-^{6}Li interaction potential, one can determine the scattering length a_{lg} as a function of Ω and Δ. For simplicity, we use a square-well model [19] for the ^{40}K-^{6}Li interaction: $V_i^{(\mathrm{bg})}(r) = -v_i^{(\mathrm{bg})}\theta(\bar{a} - r)$, $W(r) = w\theta(\bar{a} - r)$ and $V_{1a}(r) = 0$, where $\theta(x)$ is the unit step function and $\bar{a}$ is defined as $\bar{a} = 4\pi\Gamma(1/4)^{-2}R_{\mathrm{vdW}}$ with $\Gamma(x)$ being the Gamma function and R_{vdW} the van der Waals length [19]. $v_i^{(\mathrm{bg})}$, w, v_{1a} and δ are determined by the experimental scattering parameters. In the absence of the coupling between $|f_1\rangle$ and $|g\rangle$, our model gives the background scattering lengths in the channels $|f_1\rangle|g\rangle$ and $|f_2\rangle|g\rangle$, and the correct resonance point and width for the MFR between $|f_2\rangle|g\rangle$ and $|c\rangle$. Although the square-well model cannot provide a quantitatively accurate result for a_{lg}, it can give a qualitative and intuitive illustration of our method. The results of our calculation are shown in Fig. 2(a). It is clear that, for every given value of the magnetic field (or the scattering length a_{dg}), a_{lg} can be resonantly controlled via the laser induced coupling (Ω, Δ).

It is also possible to implement our method in ultracold ^{40}K atoms with the second method described above. We can utilize five hyperfine states to construct the five-level structure as schematically illustrated in Fig. 1(e). The ground hyperfine state $|9/2, -9/2\rangle$ is used for $|g\rangle$, while four other states $|7/2, -5/2\rangle$, $|9/2, -7/2\rangle$, $|9/2, -3/2\rangle$ and $|9/2, -5/2\rangle$, respectively, are taken as $|f_1\rangle$, $|f_2\rangle$, $|f_1'\rangle$ and $|f_2'\rangle$. To utilize the MFR in the channel $|2\rangle$ with $B_0 = 202.1$ G, $\Delta B = 8$ G [16] and the one in the channel $|2'\rangle$ with $B_0 = 224.2$ G, $\Delta B = 10$ G [17], we choose $B = 200$ G. As shown above, dressed state $|f_l(f_{l'})\rangle$ can be formed by the couplings between $|f_1(f_1')\rangle$ and $|f_2(f_2')\rangle$, and one can independently tune a_{lg} and $a_{l'g}$ by choosing different dressing parameters in the three-component gas of atoms in the states $|g\rangle$ and $|f_l\rangle$, $|f_{l'}\rangle$.

In this scheme, the upper hyperfine states $|f_1(f_1')\rangle$ can induce the HFR processes of both bare channels $|1\rangle$, $|1'\rangle \equiv |f_1'\rangle$ and dressed channels $|f_l(f_{l'})\rangle|g\rangle$. The values of the loss rates of the bare channels $|1(1')\rangle|g\rangle$ of ^{40}K are not available. However, the loss rate of the process $|9/2, 5/2\rangle|9/2, 7/2\rangle \rightarrow |9/2, 9/2\rangle|9/2, 3/2\rangle$ has been measured to be as low as 10^{-14} cm^3/s [20]. We take this value to be the loss rate of $|1(1')\rangle$ in our calculation with the square-well model, where $V_{1a(1'a')}(r)$ is assumed to be

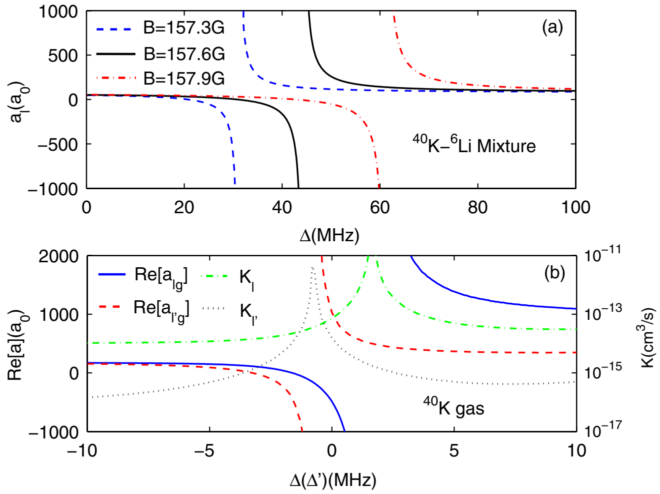

FIG. 2 (color online). (a) Independent control of two scattering lengths in the three-component gases of a ^{40}K-^{6}Li mixture. The scattering length a_{lg} is plotted in units of the Bohr radius a_0 as a function of Δ with $\Omega = 40$ MHz and $B = 157.3$ (blue dashed curve), 157.6 (black solid curve), and 157.9 G (red dash-dotted curve). (b) Independent control of two scattering lengths in the three-component gases of ^{40}K with $B = 200$ G and $\Omega = 2$ MHz, $\Omega' = 9$ MHz. Re$[a_{lg}]$ (blue solid curve) and K_l (green dash-dotted curve) are plotted as functions of Δ; Re$[a_{l'g}]$ (red dashed curve) and $K_{l'}$ (black dotted curve) are plotted as functions of Δ'. Since the scattering lengths between $|f_1(f_1')\rangle$ and $|g\rangle$ are not available, we take the values to be the same as those between $|f_2(f_2')\rangle$ and $|g\rangle$, which are given in Refs. [12,16,17].

$v_{1a(1'a')}\theta(\bar{a} - r)$. Both intrachannel scattering lengths (Re$[a_{lg}]$, Re$[a_{l'g}]$) and the two-body loss rate (K_l, K_l') of $|f_l(f_l')\rangle|g\rangle$ are shown in Fig. 2(b). We find that, around the resonance point where the loss rates (K_l, K_l') peak, there are broad regions with large absolute values of (Re$[a_{lg}]$, Re$[a_{l'g}]$) and small (K_l, K_l'). Although we do not perform the calculation with realistic interaction potential between ^{40}K atoms, our result based on the square-well model also shows the feasibility of our scheme in a gas of ^{40}K.

Conclusion and discussion.—In this Letter we propose a method for the independent control of (at least) two scattering lengths in three-component gases by preparing atoms in dressed states and via independent tuning of the couplings among the hyperfine states. Under suitable conditions, our method can be generalized to the control of $n - 1$ ($n > 3$) scattering lengths in an n-component system. This would be a powerful technique for engineering different types of homonuclear or heteronuclear Efimov states and for control of quantum phases. We have shown that our scheme can be implemented in cold gases of ^{40}K or a ^{40}K-^{6}Li mixture. It is also possible to apply our method to bosonic systems, such as the ^{40}K-^{87}Rb mixture, where two close Feshbach resonances at $B = 464$ G and 467.8 G [18] are available.

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