Finite Range Effects in $(\alpha, 2\alpha)$ Reactions

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Finite range calculations for the $(\alpha, 2\alpha)$ reactions are performed for the first time to remove huge inconsistencies obtained earlier in conventional zero range analyses. Vagaries of the energy dependent experimental observations up to 200 MeV are understood using the well-established nuclear radii and distorting optical potentials. The results are found to be sensitive to the short distance behavior of the α-α interaction, indicating the utility of the knockout reactions as a probe of the knockout vertex at short distances. Our approach paves the way to include finite range effects in atomic and molecular physics as also in neutron multiplication calculations.

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Conventional analysis of the knockout of α cluster by proton and α projectiles with the distorted wave impulse approximation (DWIA) using zero range (ZR) interaction has resulted in large inconsistencies [1–6]. While the absolute cross section predictions for the $(p, p\alpha)$ reactions are close to the experimental data [1–3,7,8] the corresponding comparison for the $(\alpha, 2\alpha)$ reactions lead to almost 2 orders of magnitude lower predictions [1]. Exceptions to these observations, however, were seen for the $(\alpha, 2\alpha)$ reactions on ^{9}Be [9] and ^{12}C [10] at 197 and 200 MeV, respectively. Similar inconsistencies were detected in the case of the knockout of d, t, and ^{3}He clusters [11]. The small predictions of absolute cross sections and hence large α-cluster spectroscopic factors from $(\alpha, 2\alpha)$ reactions up to 140 MeV were ascribed to induced α clustering, simulated by using the large bound state potential radius [4], or in terms of reduced optical distortion effects [12]. These *ad hoc* prescriptions cannot, however, account for the 197–200 MeV data [9,10].

In the conventional ZR-DWIA treatment of the knockout transition matrix element, the factorization of the knockout vertex contribution is built in [2,13,14]. The factorization can arise either from the zero range nature of the knockout vertex transition operator or from the optical distortion free scattering states. While the ZR-DWIA calculations exhibit large optical distortion characteristics [5,6] the cluster knockout data indicate little influence from it [6].

We have examined the nature of the α-α knockout vertex transition operator $t_{\alpha\alpha}(\vec{r})$ at various energies and found it to be of fairly long range [15]. The α-α t-matrix effective interaction was also found to be strongly dependent on the nature of the α-α realistic optical potentials. These optical potentials are not unique and equally good fits to the elastic scattering data can be obtained by very different optical potentials [15]. Two drastically different types of α-α nuclear optical potentials in common use are (i) with short range repulsive core [16] and (ii) a potential which is purely attractive [17,18]. These two types of α-α optical potentials, with their phase shifts matched, fit the α-α elastic scattering data. In a detailed study [15] it has been demonstrated that different α-α optical potentials yield different α-α t-matrix effective interactions, $t_{\alpha\alpha}(\vec{r})$'s. For example with a repulsive core [16] the t-matrix effective interaction peak is shifted away from $r = 0$ by about 1.5 fm, Fig. 1(a). On the other hand for a purely attractive α-α optical potential [18] the t-matrix effective interaction, although fairly long ranged peaks close to $r = 0$, Fig. 1(b). Although these $t_{\alpha\alpha}(\vec{r})$'s are seen to be shorter ranged as compared to the α-bound state wave function in the target nucleus, they are fairly long ranged enough to cause a failure of the zero range approximation [13,14]. This finding suggests the need for finite range (FR)-DWIA calculations for the cluster knockout reactions. In this Letter we present these much needed FR-DWIA analyses and put the results in perspective as a resolution for the inconsistencies mentioned above. Our formalism has immediate application in knockout reactions in atomic, molecular, and intermediate energy nuclear physics.

The transition amplitude T_{fi} for the knockout reaction $A(\alpha, 2\alpha)B$ in the FR-DWIA formalism from the initial state i to the final state f can be written [2,13,14],

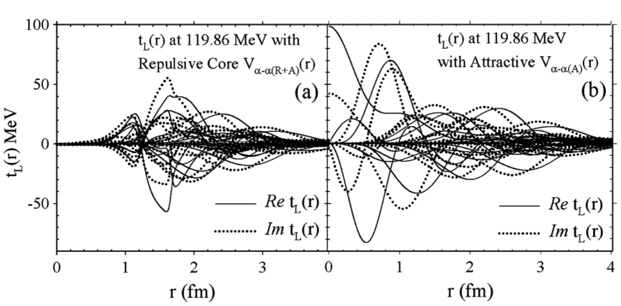

FIG. 1. Effective α-α t-matrix interaction $t_L(r)$ vs r at 119.86 MeV for many L values, (a) using $V_{\ell,\alpha-\alpha}(r)$ with repulsive core and a longer range attraction, (b) using a purely attractive $V_{\alpha-\alpha}(r)$.

$$\frac{d^3\sigma^{L,J}}{d\Omega_1 d\Omega_2 dE_1} = F_{\rm kin} S_\alpha^{LJ} \sum_{\wedge} |T_{fi}^{\alpha L\wedge}(\vec{k}_f, \vec{k}_i)|^2, \quad (1)$$

where J and L ($\wedge$) are the total and orbital (its azimuthal component) angular momenta of the bound α particle in the target nucleus, $F_{\rm kin}$ is a kinematic factor, and S_α^{LJ} is the cluster spectroscopic factor. The conventional transition matrix element for the knockout reaction $T_{fi}^{\alpha L\wedge}(\vec{k}_f, \vec{k}_i)$ using the finite range α-α t-matrix effective interaction $t_{12}(\vec{r}_{12})$ is given by [2,13,14]

$$T_{fi}^{\alpha L\wedge}(\vec{k}_f, \vec{k}_i) = \int \chi_1^{(-)*}(\vec{k}_{1B}, \vec{r}_{1B}) \chi_2^{(-)*}(\vec{k}_{2B}, \vec{R}_{2B}) t_{12}(\vec{r}_{12}) \times \chi_0^{(+)}(\vec{k}_{1A}, \vec{r}_{1A}) \varphi_{L\wedge}(\vec{R}_{2B}) d\vec{r}_{12} d\vec{R}_{2B}. \quad (2)$$

Here the $t_{12}(\vec{r}_{12})$, evaluated at the final state relative energy E_f, is given by [15,19]:

$$t_{12}^+(E, \vec{r}) = e^{-ikz} V(\vec{r}) \Psi_{12}^+(\vec{r}) \equiv \sum_{L=0,1,2,\ldots} t_L(E, r) P_L(\hat{r}), \quad (3)$$

$$\Psi_{12}^+(\vec{r}) = \sum_{\ell=0,2,4,\ldots} i^\ell (2\ell+1) \frac{u_\ell(kr)}{kr} e^{i\sigma_\ell} P_\ell(\hat{r}). \quad (4)$$

As discussed in Ref. [15], the Lth multipole of the $t_{\alpha\alpha}^+(E, \vec{r})$ can be written

$$t_L(E, r) = \frac{2L+1}{2} \sum_{\ell,n} V_\ell(r) i^\ell (2\ell+1) \frac{u_\ell(kr)}{kr} j_n(kr)(-i)^n \times (2n+1) e^{i\sigma_\ell} \int_{-1}^{+1} P_L^*(\cos\theta) P_\ell(\cos\theta) \times P_n(\cos\theta) d(\cos\theta). \quad (5)$$

The distorted waves χ_0, χ_1, and χ_2 of Eq. (2) are evaluated using the optical potentials for the α_1-A, α_1-B, and α_2-B. Finally all the relative coordinates are expressed in terms of $\vec{r}_{12}$ ($\equiv \vec{r}$) and $\vec{R}_{2B}$ ($\equiv \vec{R}$). While using the ZR-DWIA the transition matrix element T_{fi} of Eq. (2) was factorized into integrals over $\vec{r}$ and $\vec{R}$ separately. The same is not possible when one uses the full finite range $t_{12}(\vec{r}_{12})$ due to the presence of optical distortions. This is because in the FR-DWIA formalism the chosen relative coordinates $\vec{r}$ and $\vec{R}$ get coupled through the distorted waves $\chi_0^{(+)}(\vec{k}_{1A}, \vec{r}_{1A})$ and $\chi_1^{(-)*}(\vec{k}_{1B}, \vec{r}_{1B})$.

For the evaluation of $T_{fi}^{\alpha,L,\wedge}$ of Eq. (2) the distorted waves $\chi(\vec{k}, \vec{r})$ were expanded in terms of partial waves and then on the mesh of the spherical polar coordinates, r, θ, ϕ and R, Θ, Φ the values of χ_0, χ_1, χ_2, $\varphi_{L\wedge}(\vec{R})$ and $t_{12}(\vec{r})$ were evaluated. The final result of T_{fi} is obtained by doing a 6-dimensional integration over the mesh of $\vec{r}$ and $\vec{R}$ coordinates. The computer code was checked by performing FR-plane wave impulse approximation (PWIA) calculations using the present six-dimensional integration approach as well as through the three-dimensional integrations approach [because in the plane wave case the six-dimensional integral of Eq. (2) factorizes into two three-dimensional integrals, one over $\vec{r}$ and the other over $\vec{R}$].

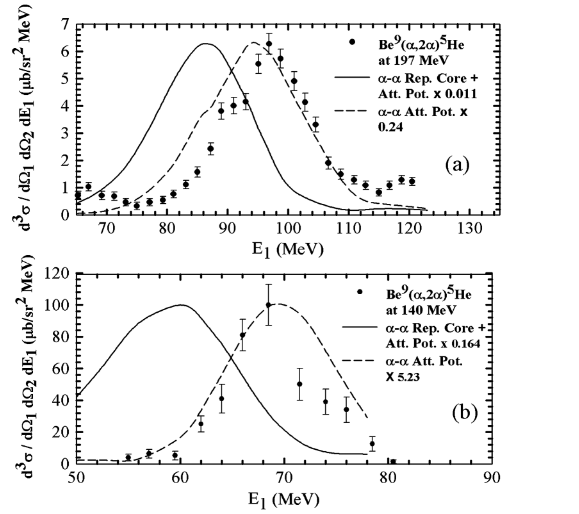

FIG. 2. Comparison of ^{9}Be$(\alpha, 2\alpha)$ data with the FR-DWIA calculations using α-α interaction which is purely attractive (A) and having a repulsive core ($R + A$), (a) for 197 MeV and (b) for 140 MeV.

We performed the FR-DWIA analyses of the typical (α, 2α) data on ^{9}Be at 197 [9] and 140 MeV [4], on ^{12}C at 200 [10] and 140 MeV [4] and on ^{16}O at 140 [4] and 90 MeV [20]. For these analyses, out of the many possible $t_{12}(\vec{r})$'s, we used the α-α t-matrix effective interactions obtained from the two types of α-α optical potentials, attractive with a repulsive core ($R + A$) on the one hand and purely attractive (A) on the other hand. These $t_{12}(\vec{r})$'s are evaluated at the final state relative energy, E_f.

In the calculations we employed the improved prescription for the entrance channel potentials [21] where the

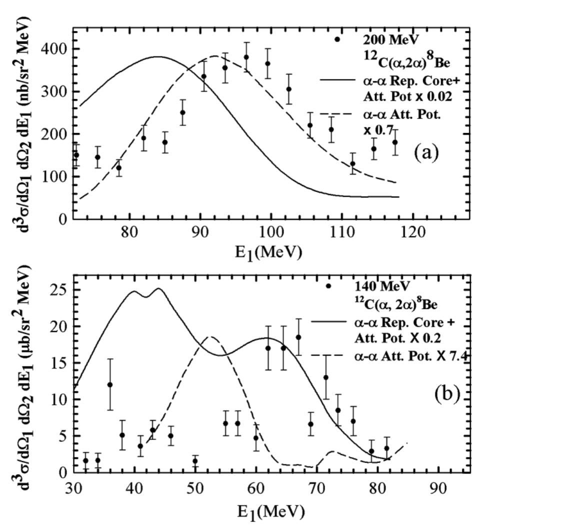

FIG. 3. Same as Fig. 2 but for ^{12}C$(\alpha, 2\alpha)$ reaction, (a) for 200 MeV and (b) for 140 MeV.

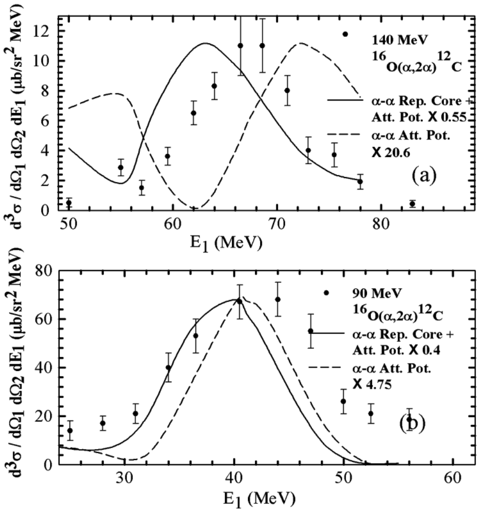

FIG. 4. Same as Fig. 2 but for $^{16}O(\alpha, 2\alpha)$ reaction, (a) for 140 MeV and (b) for 90 MeV.

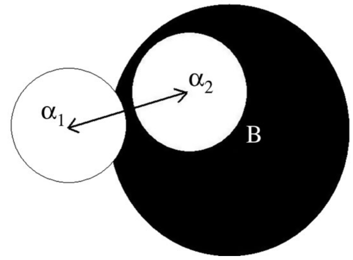

FIG. 5. Schematics of the knockout of α cluster by incident α projectile by a repulsive core α-α interaction.

folding model replaces the conventional ($\frac{B}{A}$) prescription [1]. The α_2-B bound wave function $\varphi_{L\wedge}(\vec{R})$ is generated as usual from the conventional nuclear potential of the Woods-Saxon form with $R_{\text{bound}} = 1.09B^{1/3}$ fm.

The results of the FR-DWIA computations, normalized to the data peak values, are presented in Figs. 2–4. Although the shapes of the energy sharing distributions [$\sigma_{(\alpha,2\alpha)}(E_1)$ vs E_1] are not very satisfactory the curves obtained from the attractive, $t_{\alpha\alpha(A)}(\vec{r})$ are much closer to the data. This arises because the $t_{\alpha\alpha(A)}(\vec{r})$'s peak close to $r = 0$, which simulates the zero range behavior and hence the results are similar to the ZR-DWIA results [1]. The repulsive core ($R + A$) results are seen to be at much variance. This could arise due to the uncertainty in the choice of the repulsive core α-α potential parameters. The most important conclusion, however, can be drawn by comparison (bold face entries) of the absolute peak cross section values from the FR-DWIA calculations with the data and the derived S_α values with the expectations from the structure theory [7,8] in Table I.

In Table I it is seen that the absolute cross sections and S_α values for the ~197–200 MeV $(\alpha, 2\alpha)$ reactions on ^{9}Be and ^{12}C using the purely attractive $t_{\alpha\alpha(A)}(\vec{r})$ are in better agreement with data in comparison to that using $t_{\alpha\alpha(R+A)}(\vec{r})$ where the absolute cross sections are 20 to 35 times larger. For energies at and below ~140 MeV, both the $t_{\alpha\alpha(A)}(\vec{r})$ and $t_{\alpha\alpha(R+A)}(\vec{r})$ yield somewhat distorted shapes. Yet the peaks close to the zero recoil momentum position (normalized to the data peak values) yield S_α values, seen in Table I, much closer to the theoretical values when $t_{\alpha\alpha(R+A)}(\vec{r})$'s are employed. On the other hand, the S_α values obtained from the $t_{\alpha\alpha(A)}(\vec{r})$'s are 10 to 90 times too large as compared to the theoretical estimates [7,8].

Differences of almost 2 orders of magnitude are seen between the FR-DWIA predictions of the $(\alpha, 2\alpha)$ reaction cross sections using the repulsive core ($R + A$) and purely attractive (A) α-α potentials. An obvious conclusion is that use of the conventional ZR-DWIA formalism and hence the factorization approximation for the analysis of $(\alpha, 2\alpha)$ reactions below ~197 MeV was incorrect.

While, due to factorization, the FR-PWIA $(\alpha, 2\alpha)$ results for the ($R + A$) and (A) α-α potentials match, the enhancement of the FR-DWIA results for the ($R + A$) case over that of the (A) case can be understood qualitatively from Fig. 5. Here it is seen that due to the α-α repulsion the incident α particle can knock out the bound α cluster while remaining outside the absorbing (shaded) region. On the other hand when the α-α interaction is purely attractive the $t_{\alpha\alpha(A)}(\vec{r})$ peaks at $r = 0$ and hence the α_1 has to enter the absorbing (shaded) region to knock the bound α_2 out. Thus the enhancement in the ($R + A$) case arises due to the

TABLE I. Comparison of $(\alpha, 2\alpha)$ cross sections from FR-DWIA calculations and experimental data on ^{9}Be, ^{12}C, and ^{16}O at various energies and spectroscopic factors (S_α) derived from the FR-DWIA calculations and theory. Comparison of boldface entries is emphasized.

Reaction	E_α (MeV)	$\sigma_{\alpha,2\alpha}$(peak)/Sr^2 MeV ($R + A$)	(A)	Expt.	Ref.	S_α ($R + A$)	(A)	Theory	Ref.
$^9Be(\alpha, 2\alpha)^5He$	197	575 μb	**26.4** μb	**6.3** μb	[9]	0.011	**0.24**	**0.57**	[7]
	140	**609** μb	19.1 μb	**100** μb	[4]	**0.164**	5.23		
$^{12}C(\alpha, 2\alpha)^8Be$	200	19.9 μb	**552** nb	**380** nb	[10]	0.02	**0.7**	**0.55**, **0.29**	[7,8]
	140	**92** μb	2.5 μb	**18.5** μb	[4]	**0.2**	7.4		
$^{16}O(\alpha, 2\alpha)^{12}C$	140	**19.1** μb	0.51 μb	**10.5** μb	[4]	**0.55**	20.6	**0.23**, **0.3**	[7,8]
	90	**171** μb	14.3 μb	**68** μb	[20]	**0.4**	4.75		

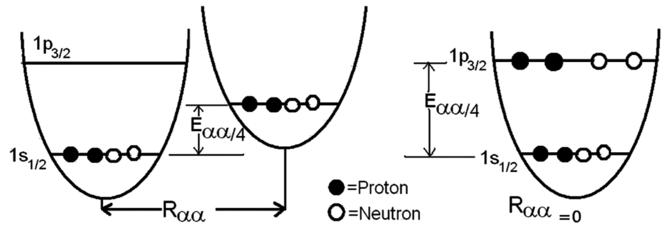

FIG. 6. Shell model (RGM) scheme of two α particles when separated or overlapping at relative energy of $E_{\alpha-\alpha}$.

reduction in the optical absorption as a result of the α-α repulsion.

From these FR-DWIA results it is obvious that the α-α potential character changes drastically at α energies, E_α somewhere between 140 and 200 MeV, corresponding to the center of mass energy $E_{\alpha-\alpha}$ of 70 to 100 MeV. Again this can be qualitatively understood in the resonating group method (RGM)-shell model picture (taking care of Pauli's exclusion principle), see Fig. 6. Here the four neutrons (n) and four protons (p) of the two α particles can exist in an overlapping position if the two n's and two p's of one α particle are in the lowest $1s_{1/2}$ shell model state and the other two n's and two p's of the other α in the next shell model state ($1p_{3/2}$, which is situated around 21 MeV above the ground state of α particle). The total energy of this overlapping system $E_{\alpha-\alpha}$ will thus be $\sim 4\times 21 =$ 84 MeV (corresponding to $E_\alpha \sim 2\times 84 = 168$ MeV). Thus below this energy, $E_\alpha \sim 168$ MeV, the two α's would find it energetically more favorable to avoid their overlap with a repulsive core in their interaction. Above this energy, however, the two α's have no such restriction and are free to have the usual attractive force between them. This understanding of the change in the nature of the α-α interaction is clearly validated by the present FR-DWIA analyses of the (α, 2α) data.

As there are continuous ambiguities in the optical model parameters it is premature to choose one particular parameter set in comparison to the other. Hence the present FR-DWIA results are only indicative of the general behavior of the influence of the α-α interaction. In order to get improved FR-DWIA fits to the (α, 2α) reaction data one may need to search various parameters more exhaustively to fit both the α-α elastic scattering as well as the (α, 2α) reaction data. While the RGM may be used to guide the choice of the repulsive core ($R + A$) α-α potential parameters, the folding model potentials can restrain the choice of purely attractive (A) α-α potential.

Similar arguments with repulsive core, ($R + A$) interaction between α-d, α-t, and α-^{3}He are expected to remove the inconsistencies [11] in the (α, αd), (α, αt), and (α, α^3He) reactions in comparison to the corresponding knockout reactions using the proton projectiles.

Extreme sensitivity of the cluster knockout reactions to the short range behavior of the colliding partners opens up the possibility of probing this aspect of the particles involved at the knockout vertex. In (p, $2p$) reactions, for example, one should be able to see the behavior of the nucleon-nucleon interaction at short distances or otherwise, if there is a possibility of dibaryon formation at some energy then one should be able to decipher it from the FR-DWIA analyses of the (p, $2p$) reactions [22]. Similarly one can visualize observing the Δ resonance in (π, πp) reaction [23] and the pentaquark, Θ^+ [24] in (K^+, $K^+ n$) reaction due to enhanced distortion effects. In heavy ion knockout reactions also one can investigate the short range behavior of the heavy ions involved at the knockout vertex which is rather difficult to observe in the elastic scattering. The present results and conclusions may be very instructive in studies involving (e, $2e$) reactions on atoms, knockout of atoms from molecules, (n, $2n$) reactions for neutron multiplication, and in many other disciplines involving direct knockout reactions.

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