Proposal for Pulsed On-Demand Sources of Photonic Cluster State Strings

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We present a method to convert certain single photon sources into devices capable of emitting large strings of photonic cluster state in a controlled and pulsed "on-demand" manner. Such sources would greatly reduce the resources required to achieve linear optical quantum computation. Standard spin errors, such as dephasing, are shown to affect only 1 or 2 of the emitted photons at a time. This allows for the use of standard fault tolerance techniques, and shows that the photonic machine gun can be fired for arbitrarily long times. Using realistic parameters for current quantum dot sources, we conclude high entangled-photon emission rates are achievable, with Pauli-error rates per photon of less than 0.2%. For quantum dot sources, the method has the added advantage of alleviating the problematic issues of obtaining identical photons from independent, nonidentical quantum dots, and of exciton dephasing.

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The primary challenge facing optical quantum computation is that of building suitable photon sources. The majority of effort has been directed at single photon sources. Four single photons can be used in an interferometer to produce a maximally entangled Bell pair of photons [1], and given a source of Bell pairs, it is in principle possible to fuse them [2] into larger so-called *cluster states* [3]. These somewhat magical quantum states can be used for performing quantum computation via the simple procedure of making individual (single-qubit) measurements on the photons involved. Recently, a promising new approach has been to produce Bell pairs directly [4,5] via a radiative cascade in quantum dots. However, even an ideal such source would only reduce the overall resources required for a full optical quantum computation by a small factor.

We will show that with current technology, it is possible to manipulate certain single photon sources, in particular, quantum dots, so as to generate a continuous stream of photons entangled in long strings of (various varieties of) 1-dimensional cluster states. Using these strings, cluster states capable of running arbitrary quantum algorithms can be very efficiently generated by fusion. We analyze all error mechanisms and show that the error rates can be very low—close to fault tolerant thresholds for quantum computing—even if the source is operated for times much longer than the typical decoherence time scales.

We begin with a highly idealized description of the proposal. Consider a source with a degenerate spin-1/2 ground state manifold. The basis $|\uparrow\rangle$, $|\downarrow\rangle$ denotes the spin projection along the z axis. Furthermore, imagine that optical transitions at frequency ω_0 are possible *only* to a doubly degenerate excited state manifold. The excited states $|\Uparrow\rangle$, $|\Downarrow\rangle$ have $J_z = \pm 3/2\hbar$, thus only the (single photon) transitions $|\uparrow\rangle \leftrightarrow |\Uparrow\rangle$ and $|\downarrow\rangle \leftrightarrow |\Downarrow\rangle$ are allowed. Such transitions are well known to occur, for example, in quantum dots which emit single photons via charged-exciton decay [6]. We only consider the emitted photons propagating along the z axis. Therefore, if the initial state of the source is $|\uparrow\rangle$ ($|\downarrow\rangle$), an excitation to the state $|\Uparrow\rangle$ ($|\Downarrow\rangle$) followed by radiative decay, results in the emission of a single right (left)-circularly polarized photon $|R\rangle$ ($|L\rangle$) and leaves the source in the state $|\uparrow\rangle$ ($|\downarrow\rangle$). Now, consider the initial state $|\uparrow\rangle + |\downarrow\rangle$, and a coherent excitation pulse with a linear polarization along the x direction. (The exciting pulse itself need not necessarily propagate along the z direction, which is useful for separation of the coherent and emitted light). Such a pulse couples equally to both transitions. Therefore, the processes described above happen in superposition, and the emitted photon will be entangled with the electron: the joint state of both systems would be the Bell pair $|\uparrow, R\rangle + |\downarrow, L\rangle$. Repeating such a procedure would produce GHZ-type entangled states, which are not useful for quantum computing, and for which disentangling the photons from the electron spin is difficult. Moreover, the GHZ state is highly vulnerable to decoherence. By contrast, the cluster states suffer none of these problems.

To see how to create cluster states, we now imagine that before the second excitation of the system, when the state of the spin and the first photon is $|\uparrow\rangle|R_1\rangle + |\downarrow\rangle|L_1\rangle$, the spin undergoes a $\pi/2$-rotation about the y axis. Under this operation, described by $\exp(-iY\pi/4)$, the state evolves to $(|\uparrow\rangle + |\downarrow\rangle)|R_1\rangle + (-|\uparrow\rangle + |\downarrow\rangle)|L_1\rangle$. A second pulse excitation, accompanied by a second photon emission, will now result in the two photons and the electron spin being in the state $(|\uparrow\rangle|R_2\rangle + |\downarrow\rangle|L_2\rangle)|R_1\rangle + (-|\uparrow\rangle|R_2\rangle + |\downarrow\rangle|L_2\rangle)|L_1\rangle$. In terms of abstract (logical) qubit encodings, we will take $|R\rangle \equiv |0\rangle$, $|L\rangle \equiv -|1\rangle$. It can be readily verified that rotating the spin with another $\pi/2$ rotation now leaves the spin and two photons in the state: $|000\rangle + |001\rangle + |010\rangle - |011\rangle + |100\rangle + |101\rangle - |110\rangle + |111\rangle$, which is exactly the 3-qubit linear cluster state. Repeating the process of excitation followed by $\pi/2$ rotation will produce a third photon such that the electron and three photons are in a 4-qubit linear cluster state. The procedure can, in principle, be repeated indefinitely, producing a continuous chain of photons in an entangled linear cluster state. Note that one advantage of producing a cluster state is that the electron can be readily disentangled from the string of entangled photons, for example, by making a computational ($|R\rangle$, $|L\rangle$) basis measurement on the most recently created photon. In fact, since in general the initial state of the spin will be mixed, such a detection of a photon in state $|R\rangle(|L\rangle)$ polarization can also be used to project the spin to the $|\uparrow\rangle$ ($|\downarrow\rangle$) state, and initializes the cluster state (either outcome is ok). It can be readily verified that the whole idealized procedure just described is equivalent to the qubit quantum circuit depicted in Fig. 1.

A general analysis of how cluster states are generated by evolution of atoms in cavities undergoing general pumping and decay can be found in [7], and interesting cavity QED proposals can be found in [8]. We will primarily focus on a specific implementation of our proposal, namely, photon emission from a quantum dot, via the process of creation and subsequent decay of a charged exciton (trion). In practice, the expressions we derive, such as the structure of the emitted photon wave packets, can be easily applied to any systems which obey similar selection rules, and the imperfections we discuss are, for the most part, generic. The importance of the selection rules arises as follows. In semiconductor quantum dots, the $J = 3/2$, $J_z = \pm 1/2$ states are naturally split off from the $J = 3/2$, $J_z = \pm 3/2$ ones primarily due to confinement. They correspond to trions containing two electrons in the singlet state and a *light* hole or *heavy* hole, respectively. We can consider only the *heavy* trions and neglect the mixing between them. In other systems, while the transitions to $J_z = \pm 1/2$ may be energetically split off by an external field, or may simply have different couplings, generically they will still lead to imperfections equivalent to nonorthogonality of the emitted photons. Moreover, processes in other systems tend to be slower, and temporally longer pulses may well also be required because of nearby energy levels. Although these problems can be remedied somewhat by applying proper filtration protocols to the output cluster state (at the expense of larger loss rates), we focus on quantum dots for which the suppression is essentially perfect, the processes are fast, and the energy levels well separated.

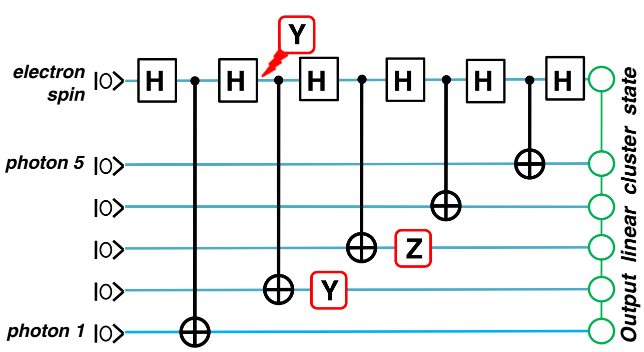

FIG. 1 (color online). A quantum circuit readily verified to output linear cluster state. For mapping to the cluster state machine gun, the top qubit line is the electron spin, the Hadamard gates are replaced by single-qubit unitaries $\exp(-i\pi Y/4)$ (requiring the careful tracking of certain phases), and the physical process of creating a photon with left/right circular polarization conditioned on the state of the electron spin becomes the controlled NOT gate which leaves the qubit (photon) in state $|0\rangle$ (i.e., $|R\rangle$) if the electron spin is in state $|0\rangle$ (i.e., $|\uparrow\rangle$), but otherwise flips it. Crucially, as depicted, a Pauli Y error on the spin localizes; i.e, it is equivalent to Y and Z errors on the next two photons produced.

Although other options exist, we will consider from now on the situation where the $\pi/2$ rotations on the spin are performed by placing the quantum dot in a constant magnetic field of strength B which is directed along the y direction (i.e., in the plane of the dot). The spin precession at frequency $\omega_B = g_e \mu B/\hbar$ in the z-x plane therefore implements the desired rotation every $T_{\text{cycle}} = \pi/2\omega_B$. Suitably timed strobing of the dot by the excitation pulse, followed by the rapid exciton decay, will therefore enable the machine-gun-like generation of $1d$ cluster state described above.

The potential imperfections to be considered are as follows: (i) The nonzero lifetime of the trion τ_{decay} means that the magnetic field causes precession of the electrons during the emission process. This leads to errors induced on the quantum circuit of Fig. 1; however, we shall find that they can be understood as implementing an error model on the final output cluster state which takes the form of Pauli errors occurring with some independent probability on pairs of (photonic) qubits. (ii) Interaction of the electron spin with its environment results in a nonunitary evolution of the spin. This evolution consists of two parts: decoherence (in which we include both dephasing and spin flips) and spin relaxation. Decoherence is characterized by a T_2 time. Fortunately, we will see that both these processes also lead only to errors occurring independently on two (photonic) qubits at a time. Efficient cluster state quantum computation can proceed even if every qubit has a finite (though small) probability of undergoing some random error [9]. This implies that the protocol's running time is not limited by T_2, while the errors are amenable to standard quantum error correction techniques for cluster states. Spin relaxation is characterized by a T_1 time, and is a process which projects the spin to the ground state. In semiconductor quantum dots, T_1 times are extremely long $T_1 \gg T_2 \gg \tau_{\text{decay}}$ [10]. Therefore, we shall not discuss the effects of this process further here. We point out, however, that it can be shown this process also leads to errors of a localized form, and so in principle is no obstacle to the continuous operation of the device even for times much longer than T_1. (iii) The last source of error is related to the issue of ensuring the photons are emitted into well-controlled spatial modes. In practice, this technological issue of mode matching (say by placing the dot in a microcavity) results in some amount of photon loss error in the final state. Significant progress on this issue is being made for a variety of quantum dots [11,12], although we emphasize that for our proposal, strong coupling to the cavity is *not* required. Fortunately, photonic cluster state computation can proceed even in the presence of very high (up to 50%) loss [13], and we will not consider this source of error further.

We now turn to detailed calculations of the error rate inflicted by imperfections (i) and (ii) discussed above. We first calculate the effect of a finite ratio of the trion decay time $\tau_{\rm decay}$ to the spin precession time. We denote by $\rho_n(\tau + t_n)$ the state of the system (the quantum dot and photons) at time τ after the nth excitation pulse, $t_n = nT_{\rm cycle}$. By $\rho_n(t_n^-)$, we mean the state of the system *just before* the nth excitation pulse (we assume the excitation is instantaneous). Following the excitation, the trion state decays, emitting a photon and leaving an electron in the quantum dot, the spin of which then precesses in the magnetic field. These lead to an evolution of the quantum state described by the following map (see [14] for details):

$$\rho(t_n + \tau) = U^\dagger(\tau)(G + F)^\dagger \rho(t_n^-)(G + F)U(\tau). \qquad (1)$$

The unitary operator $U = \exp(iY\omega_B\tau)\exp(iH_0\tau)$ describes the precession of the electron spin and the free propagation of the photons. The generalized creation operators $G^\dagger = G_R^\dagger|\uparrow\rangle\langle\uparrow| + G_L^\dagger|\downarrow\rangle\langle\downarrow|$, $F^\dagger = F_R^\dagger|\downarrow\rangle\langle\uparrow| - F_L^\dagger|\uparrow\rangle\langle\downarrow|$, describe the excitation and decay process, adding a photon to the state. The trion states decay exponentially with τ; therefore, we have omitted them from Eq. (1) (which describes the state of the system at times greater than the trion decay time, i.e., $\tau \gg \tau_{\rm decay}$). Note that the photons created in each cycle are well separated from the ones created in the previous cycles (formally, this is taken into account by the free propagation of the photons).

Equation (1) describes a circuit isomorphic to the one in Fig. 1. The operator $G^\dagger$ corresponds to a correct application of a controlled-NOT (CNOT) gate. This happens with an amplitude $g(k)$, which depends on the photon's energy k: $\langle k\varepsilon|G_\varepsilon^\dagger|0\rangle \equiv g(k) = \frac{\sqrt{\Gamma/(2\pi)}(k-Z)}{(k-Z)^2-(g_e\mu B)^2/4}$. Here, $|k\varepsilon\rangle = a_{k,\varepsilon}^\dagger|0\rangle$ and $\varepsilon = L, R$. The complex energy of the trion states is denoted by $Z = \omega_0 - i\Gamma/2$, where $\tau_{\rm decay} = 1/\Gamma$ is their lifetime. The operator $F^\dagger$ corresponds to a CNOT gate followed by a Y error on the spin qubit. This errored gate is applied with amplitude $f(k)$, where $\langle k\varepsilon|F_\varepsilon^\dagger|0\rangle \equiv f(k) = \frac{i\sqrt{\Gamma/(2\pi)}g_e\mu B/2}{(k-Z)^2-(g_e\mu B)^2/4}$.

Let us for the moment treat the processes described by $G^\dagger$ and $F^\dagger$ as incoherent with each other. Then, the resulting state is described by the circuit of Fig. 1 with a probability $p_B = \|f\|^2 = \frac{(g_e\mu B)^2}{2(g_e\mu B)^2+2\Gamma^2}$, that each CNOT gate is followed by Y error on the spin qubit. As noted in Fig. 1, a state with a Y error on the spin after generation of the nth photon (i.e., after the nth CNOT gate), is equivalent to a state Y and a Z error on the $n^{\rm th}+1$ and $n^{\rm th}+2$ photons, with no error on the spin. Note that the error probability increases with magnetic field strength because the spin can precess more during the lifetime of the trion [15]. Therefore, it is advantageous to consider relatively low magnetic fields, for which $g_e\mu_B B \ll \Gamma$. Taking the coherence between $G^\dagger$ and $F^\dagger$ into account, it can be seen that a unitary correction $e^{iY\phi}$ with $\tan\phi = (\langle g|f\rangle/\langle g|g\rangle)$ yields a further improvement of the error rate. We also point out that as $f(k)$ is more localized around ω_0 than $g(k)$ (inset of Fig. 2), selection of photons with energy $|k - \omega_0| > \Delta$ would yield a lower error rate at the expense of (heralded) loss.

The calculation above ignores the possibility of the exciton dephasing [16] during the decay process. Pure dephasing, in which both excited levels evolve the same (random) phase, will have no effect on the entanglement in polarization with which we are concerned. Cross dephasing (experimentally seen to be very small [16]) will lead to Z errors on the qubits, which also localize (see [14] for a detailed discussion).

We now turn to the issue of decoherence of the spin as a result of its interaction with the nuclei in the quantum dot. Assuming Markovian dynamics (discussed further in [14]), it is well known [17] that the resulting dephasing and spin flip dynamics are equivalent to the action of random Pauli operations X, Y, Z with some probabilities p_x, p_y, p_z. The

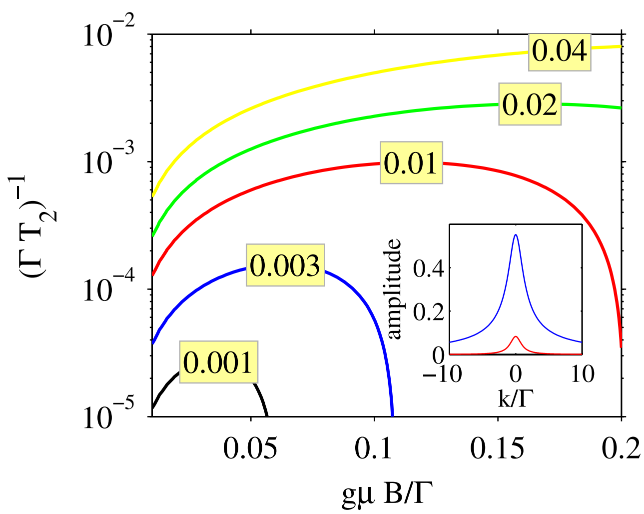

FIG. 2 (color online). Contour plots showing the probability of Pauli error on any given photon, as a function of $g_e\mu B/\Gamma$ and $(\Gamma T_2)^{-1}$. A stronger field causes faster precession which increases a chance of the error during decay, but reduces the standard dephasing error due to a finite T_2. The inset is a plot of the good mode function $|g(k)|$ (blue) vs the smaller error mode function $|f(k)|$ (red) at $g_e\mu B/\Gamma = 0.15$, from which it can be deduced that spectral filtering can reduce the error rates further.

probabilities p_x, p_z are suppressed due to the presence of the magnetic field, while p_y is characterized by T_2, the dephasing time. This can readily be shown to give $p_y = \frac{1}{2}(1 - e^{-T_{\rm cycle}/T_2})$ as the probability of a given spin error in 1 cycle. We already noted that a Y error on the spin becomes a Pauli error on the next 2 photons. Similarly, a Z error at the end of the nth cycle is equivalent to a Z error on the $n^{\rm th} + 1$ photon, as can be seen from Fig. 1 and the fact that the operators $C_{\rm NOT}(Z_{\rm spin} \otimes I_{\rm photon})$ and $(I_{\rm spin} \otimes Z_{\rm photon})C_{\rm NOT}$ have similar actions on the states $|00\rangle$ and $|10\rangle$. As $X = iZY$, an X error again affects only the next two photons to be generated.

In Fig. 2, we plot the total probability of error on any given qubit, $1 - (1 - p_B)(1 - p_y)$, as a function of the two dimensionless parameters $g_e\mu B/\Gamma$ and $(\Gamma T_2)^{-1}$. We include the aforementioned easily implemented unitary correction. Although free induction decay T_2 may be relatively short in low magnetic fields, using a spin echo pulse at half way along the cycle time can extend T_2 considerably and remove the dephasing caused by a wide distribution of nuclear (Overhauser) magnetic fields (often termed inhomogeneous broadening and characterized by a T_2^* time). To estimate an achievable error rate, we consider a decay time of $1/\Gamma = 100$ ps, and a dephasing time of $T_2 = 1$ μs with the addition of the spin echo pulses (a lower bound of 3 μs have been measured in high magnetic fields [18]). This gives $(\Gamma T_2)^{-1} = 10^{-4}$. From Fig. 2, one can deduce that a probability of error less then 0.2% can be achieved by applying a magnetic field of 15 mT (we take $g_e = 0.5$). We note that even without the spin echo pulses, error rates of about 1% are achievable, which enables the production of considerable longer and higher quality optical cluster states than those produced by current methods.

So far, we have considered pulse excitations that are timed to coincide with (integer multiples of) $\pi/2$ rotations of the spin. In fact, it can be advantageous to sometimes wait for a full π rotation to occur. This has the effect of emitting subsequent photons which are redundantly encoded [2]. Fusing together such qubits gives a highly efficient method for producing higher-dimensional cluster states which are universal for quantum computing. Photons which undergo fusion can be spectrally filtered (via a suitable prism), such that if they fail to pass the filter, they can still be measured and removed from the cluster state. This filtering does not lead to an increase in loss error rates, but simply decreases the overall success probability of the fusion gates.

Current experiments produce photonic cluster states via spontaneous parametric downconversion [19] and would seem to be limited to producing 6 to 8 photon cluster states. Our proposal in principle can produce strings of thousands of photons; however, initial experiments will be limited by collection efficiency. With the parameters above, a simple analysis shows that we would need a collection + photodetection efficiency of about 18% for a demonstration of on-demand 12-photon cluster states, where the full 12 qubits are expected to be detected about once every 10 seconds.

Finally, our proposal is suggestive of an efficient mechanism for entangling matter qubits [8,20], and we feel this is a topic worthy of further investigation.

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[1] Q. Zhang *et al.*, Phys. Rev. A **77**, 062316 (2008).
[2] D. E. Browne and T. Rudolph, Phys. Rev. Lett. **95**, 010501 (2005).
[3] R. Raussendorf and H. J. Briegel, Phys. Rev. Lett. **86**, 5188 (2001).
[4] N. Akopian *et al.*, Phys. Rev. Lett. **96**, 130501 (2006); M. Stevenson *et al.*, Nature (London) **439**, 179 (2006); A. Gilchrist *et al.*, Nature (London) **445**, E4 (2007).
[5] E. Meirom *et al.*, Phys. Rev. A **77**, 062310 (2008); J. E. Avron *et al.*, Phys. Rev. Lett. **100**, 120501 (2008).
[6] M. Bayer *et al.*, Phys. Rev. B **65**, 195315 (2002).
[7] C. Schoen *et al.*, Phys. Rev. Lett. **95**, 110503 (2005).
[8] J. Metz *et al.*, Phys. Rev. A **76**, 052307 (2007); C. Schoen *et al.*, Phys. Rev. A **75**, 032311 (2007); A. Beige *et al.*, J. Mod. Opt. **54**, 397 (2007).
[9] R. Raussendorf *et al.*, Ann. Phys. (N.Y.) **321**, 2242 (2006); C. M. Dawson *et al.*, Phys. Rev. Lett. **96**, 020501 (2006); M. A. Nielsen and C. M. Dawson, arXiv:quant-ph/0601066; Panos Aliferis and Debbie W. Leung, Phys. Rev. A **73**, 032308 (2006).
[10] M. Kroutvar *et al.*, Nature (London) **432**, 81 (2004).
[11] K. Hennessy *et al.*, Nature (London) **445**, 896 (2007); I. Fushman *et al.*, Phys. Status Solidi (c) **5**, 2808 (2008).
[12] S. Strauf *et al.*, Nat. Photon. **1**, 704 (2007); M. Rakher *et al.*, Phys. Rev. Lett. **102**, 097403 (2009).
[13] M. Varnava *et al.*, Phys. Rev. Lett. **97**, 120501 (2006); Phys. Rev. Lett. **100**, 060502 (2008).
[14] See EPAPS Document No. E-PRLTAO-103-037938 at http://www.aip.org/pubservs/epaps.html for detailed calculations of the emitted wave packets and the effects of exciton dephasing. For more information on EPAPS, see http://www.aip.org/pubservs/epaps.html.
[15] S. E. Economou *et al.*, Phys. Rev. B **71**, 195327 (2005).
[16] A. J. Hudson *et al.*, Phys. Rev. Lett. **99**, 266802 (2007).
[17] M. A. Nielsen and I. L. Chuang, *Quantum Computation and Quantum Information* (Cambridge University Press, Cambridge, England, 2000).
[18] A. Greilich *et al.*, Science **313**, 341 (2006).
[19] P. Walther *et al.*, Nature (London) **434**, 169 (2005); A.-N. Zhang *et al.*, Phys. Rev. A **73**, 022330 (2006).
[20] Sean D. Barrett and Pieter Kok, Phys. Rev. A **71**, 060310 (2005).