Acoustic Diode: Rectification of Acoustic Energy Flux in One-Dimensional Systems

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We numerically demonstrate a simple one-dimensional model of an acoustic diode formed by coupling a superlattice with a strongly nonlinear medium. The first numerical observation is presented of a significant rectifying effect on the acoustic energy flux within particular ranges of frequencies. By studying the underlying rectifying mechanism and the parameter dependence of the rectifying efficiency, the effectiveness of the acoustic diode is proved despite its simplicity. We also briefly discuss possible schemes of the experimental realization of this model as well as devising more efficient models.

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The origin of electrical diodes goes back more than one century, and they are the first devices to enable the rectifying of the current flux and have eventually led to substantial scientific revolutions around the world in many aspects. Motivated by the significant rectifying phenomenon of electrical diodes, considerable effort has been and continues to be dedicated to research in an attempt to control other kinds of energy in a similar manner [1–5]. Li *et al.* have numerically studied the heat conduction in one-dimensional (1D) nonlinear lattices and presented the fundamental model of the so-called thermal diode that has a rectifier effect on thermal energy [1,2]. It should be intriguing to devise an efficient model to rectify the acoustic energy flux, in the respect that the acoustic wave is an important form of classical wave with much longer research history than electricity. The investigation of such a model has potential practical applications, such as a unidirectional sonic barrier and controlled destruction of kidney stones via ultrasonic lithotripsy. As a matter of fact, the concepts of the "acoustic diode" (AD) have already been proposed for decades [3–5]. It is noteworthy, however, that most of the related works involve no more than the simplest possible mechanical devices designed to reduce the negative sound pressure, lacking in the crucial nonlinearity effect for observing the rectifying phenomenon on the energy flux [3,4]. Nesterenko *et al.* [5] have revealed the abnormal reflectivity at the interface of a strongly nonlinear system formed by granular materials which may be reasonably identified as an AD, but the principal information carrier in their system is solitary waves. To our knowledge, an effective model of an AD is still to be proposed that has the rectifying effect on the energy flux of a compressional wave that is the most frequently encountered mode of acoustic waves.

In this Letter, we demonstrate the possibility of building the model of an AD by numerically inspecting the propagation of a longitudinal wave in a simple nonlinear system. The first numerical observation of a significant rectifying effect on acoustic flux is presented that allows the acoustic waves of particular frequencies to propagate in one direction but causes the system to act like an insulator as the incident direction is reversed. Based on analysis of the underlying rectifying mechanism and detailed inspection of the parameter dependence of the rectifying efficiency, the present system may be reasonably identified as an AD model that works effectively in a wide range of structural parameters despite its simplicity. This model is efficient and simple enough to encourage practical efforts of experimental realization of an AD. A brief discussion has also been given on the potential schemes for observing the AD effect experimentally as well as devising more efficient models with complicated configurations.

Consider the propagation of a longitudinal wave of frequency ω in a 1D system fabricated by coupling a superlattice (SL) formed by laminating two linear media I and II periodically with the other medium, III, with particularly strong nonlinearity, as shown in Fig. 1(a). The total number of the periods is denoted as N. For the medium p ($p = \mathrm{I}, \mathrm{II}, \mathrm{III}$), the parameters of d_p, ρ_p, c_p, and Γ_p are employed to represent the thickness, the mass density, the longitudinal wave velocity, and the equivalent nonlinearity parameter, respectively. The nonlinearities of media I and II are assumed negligible as compared with the extremely large value of Γ_{III}, i.e., $\Gamma_{\mathrm{I}} = \Gamma_{\mathrm{II}} = 0$. The abbreviations LB and RB refer to the leftmost and the rightmost boundaries, respectively. There exist a few advantages in employing such a 1D system. First, this

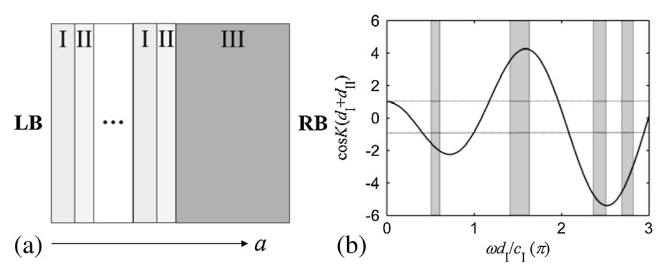

FIG. 1. (a) The schematic illustration of the 1D model of the AD. (b) The dispersion relationship of linear band structure of the SL.

system is sufficiently simple to warrant accurate extraction of many significant results. Second, this model is complicated enough to yield the rectifying effects. Third, such a simple system serves as the toy model of an AD that may be readily extended to be more complex and efficient in practice. Last but not the least, the experimental observation of the rectifying effects may be expected due to the simplicity as well as the efficiency of this system.

The media I and II chosen are water and glass, respectively, for which the material parameters are $\rho_{\rm I} = 998$ kg/m^3, $c_{\rm I} = 1483$ m/s and $\rho_{\rm II} = 2767$ kg/m^3, $c_{\rm II} = 5784$ m/s. The material parameters of medium III are assumed to be identical to medium I, except that the value of $\Gamma_{\rm III}$ may be manually adjusted in the numerical simulations to study the sensitivity of the results to nonlinearity. We assume the whole system is immersed in an infinite matrix of water. Unless otherwise stated, the system parameters are $d_{\rm II} = 0.7d_{\rm I}$, $d_{\rm III} = 15d_{\rm I}$, $N = 5$, and $\Gamma_{\rm III} = 10^4$. For this SL with simple 1D structure, the transmission property of the fundamental wave (FW) can be easily predicted by the dispersion relationship of linear band structure, as follows: [6]

$$\cos K(d_{\rm I} + d_{\rm II}) = \cos(\omega d_{\rm I}/c_{\rm I})\cos(\omega d_{\rm II}/c_{\rm II}) - \cosh\eta \sin(\omega d_{\rm I}/c_{\rm I})\sin(\omega d_{\rm II}/c_{\rm II}),$$

where $\eta = \ln(\rho_{\rm I} c_{\rm I}/\rho_{\rm II} c_{\rm II})$ and K is the usual Bloch wave number. Figure 1(b) displays the dispersion relationship for the SL. It is seen that pass bands and band gaps are produced in the frequency spectrum in an alternate manner, represented by the segments of curves inside and outside the region bounded by $\cos K(d_{\rm I} + d_{\rm II}) = \pm 1$ (two thin horizontal lines), respectively. In the SL, acoustic waves can propagate within the pass bands, whereas those within the band gaps undergo exponential attenuations as they propagate. This is the basic characteristic of the SL that plays a crucial role in the AD model.

The fundamental equation that governs the propagation of the acoustic wave in such a nonlinear system is given in Lagrangian coordinates as follows, with the effects of mode conversion and viscosity neglected:

$$\frac{\partial^2 \xi}{\partial a^2} - \frac{1}{c_p^2}\frac{\partial^2 \xi}{\partial t^2} = \Gamma_p \frac{\partial \xi}{\partial a}\frac{\partial^2 \xi}{\partial a^2},$$

where ξ and a refer to the displacements of Lagrangian particles and the Lagrangian coordinates, respectively. Boundary conditions are invoked to keep the continuity of the pressure and the particle velocity at every interface for both the FW and the second harmonic wave (SHW), and the discontinuity due to the accumulation of nonlinear distortion and the inherent nonlinearity of interfaces have not been taken into account [6].

The propagation of acoustic waves in this nonlinear system is investigated by using an extended transfer-matrix method combined with a perturbation technique, as described in Ref. [6], where the significant conclusion may be achieved that the amplitude of the incident wave affects the transmission of the nonlinear wave. It is expected that, therefore, such a remarkable pressure dependence of the transmission spectrum in the present nonlinear system could lead to an asymmetric manner in the propagation of acoustic flux; i.e., the amount of acoustic energy passing through the system is different as its amplitude or direction varies. This definitely serves as the necessary condition of the potential occurrence of the rectifying effect.

For the realization of an efficient AD model, the driving frequency must be appropriately chosen such that ω locates in the band gap of the SL while 2ω falls within the pass band. In other words, the AD effect can only be observed in a series of narrow frequency ranges discretely distributed in the frequency domain which are represented by the gray regions in Fig. 1(b) and denoted as effective rectifying bands (ERBs) here. We choose a particular value of normalized frequency $\omega d_{\rm I}/c_{\rm I} = 0.5$. The amplitude of the incident wave at the LB or RB is $v = 5 \times 10^{-6} c_{\rm I}$ with v being the particle velocity.

Figures 2(a) and 2(b) illustrate the spatial distribution of the time-averaged energy density as the acoustic wave incidents from the LB and the RB of the system, respectively. The total energy density of the acoustic wave E may be expressed, up to the second order approximation, as $E(a) \simeq E_1(a) + E_2(a)$. Here the subscripts 1 and 2 refer to the FW and SHW, respectively. Notice that in Fig. 2 the normalized energies may exceed unity due to the interference between incident and reflected waves in a standing wave field, which results in redistribution of acoustic energy in space. It is also noteworthy that the time-averaged energy of FW in the 1D system is piecewise constant, as expected [7]. Within the SL, the SHW only results from medium III due to the linear nature of media I and II, and the energy density is piecewise constant as a consequence. In medium III, however, the energy of the SHW is also generated from the FW and hence accumulates with respect to propagating distance, as observed in Fig. 2. As the acoustic wave incidents from the LB, the FW is evanescent in the SL, which leads to the result that the amount of

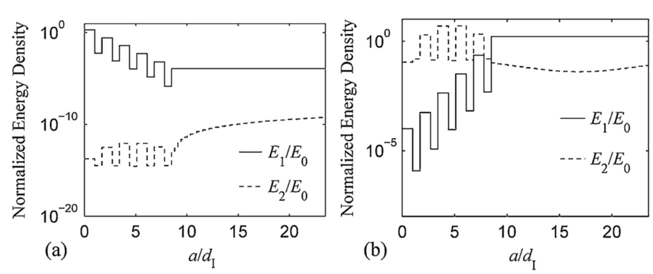

FIG. 2. The spatial distributions of the time-averaged normalized energy densities in the system as acoustic wave incidents from the LB (a) and the RB (b), respectively. E_1/E_0 and E_2/E_0 are ratios of the energy densities of the FW and SHW to the incident wave, respectively.

acoustic energy entering medium III is very small. Therefore the whole system virtually degenerates to a linear system in which the transmission spectrum of the incident wave is identical to the linear band structure of SL. This results in an exponential attenuation of the energy density for the FW and a periodically modulated distribution of energy density with extremely small amplitude for the SHW, as shown in Fig. 2(a). The total transmissions of FW and SHW can be calculated to be $T_1 = E_1(D)/E_0 \simeq 1.2 \times 10^{-4}$ and $T_2 = E_2(D)/E_0 \simeq 1.7 \times 10^{-9}$, respectively, with $D = N(d_{\mathrm{I}} + d_{\mathrm{II}}) + d_{\mathrm{III}}$ being the thickness of the whole system. Consequently the entire system behaves almost like a perfect insulator that prevents the acoustic energy flux from passing.

On the other hand, as the incident direction is reversed, the propagation of the incident wave may be expected to be substantially different as a result of the destruction of the system symmetry by the nonlinearity. Compared with the previous case, the transmission of the FW should be unaffected owing to the reciprocal theorem, but the SHW generated within the system is no longer negligible. Rather, the present system has been partially converted to a nonlinear system due to the strong nonlinearity of medium III. Observation of Fig. 2(b) shows that the reversal of the incident direction remarkably enhances the amount of the energy transferred into SHW within medium III, and the acoustic energy can partially pass through the 1D system in a periodically modulated manner. It is obtained from the numerical results that the transmission of SHW is enhanced to be $T_2 = E_2(0)/E_0 \simeq 0.1$.

Figure 2 demonstrates that a rectifying effect of the acoustic wave has been identified in the present system, which restricts the energy flux in one particular direction. For building an effective AD, however, it is necessary to analyze the pressure dependence of the transmission of the acoustic wave. The total transmission of energy flux versus p_0 is plotted in Fig. 3(a). Here p_0 and P_0 refer to the amplitudes of the pressures of the incident wave and the atmosphere, respectively, i.e., $P_0 = 1.01 \times 10^5$ Pa; the positive (negative) value of the pressure and the transmission indicates that the wave incidents from the RB (LB). It is apparent that the insulating property of the system always maintains under the action of a negative pressure, while the system allows more energy to pass as we gradually increase the strength of positive pressure. It is clearly observed that the intrinsic relation between pressure and energy flux illustrated in Fig. 3(a) exhibits a strong similarity with the relation between voltage and current flow of an electrical diode. The present system should therefore be reasonably identified as an effective AD model. It is worth pointing out that in Fig. 3(a) the amplitude of positive pressure has been cautiously controlled to guarantee the validity of the perturbation method. An additional enhancement of transmission may be expected, nevertheless, if the amplitude is further increased.

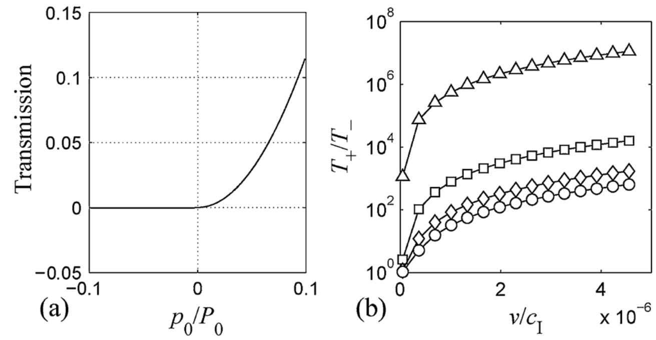

FIG. 3. (a) Total transmissions of energy flux versus the normalized amplitude of the incident wave. (b) Rectifying efficiency T_+/T_- versus the strength of the incident wave for the parameters: $N = 5$, $\Gamma_{\mathrm{III}} = 10^4$, $d_{\mathrm{III}} = 15d_{\mathrm{I}}$ (circles), $N = 5$, $\Gamma_{\mathrm{III}} = 10^4$, $d_{\mathrm{III}} = 25d_{\mathrm{I}}$ (diamonds), $N = 5$, $\Gamma_{\mathrm{III}} = 5 \times 10^4$, $d_{\mathrm{III}} = 15d_{\mathrm{I}}$ (squares), and $N = 15$, $\Gamma_{\mathrm{III}} = 10^4$, $d_{\mathrm{III}} = 15d_{\mathrm{I}}$ (triangles).

Notice that the abnormal transmission of acoustic wave in this system in fact results from the difference between the transmission spectra of FW and SHW. Therefore, as we vary the amplitude or the direction of the incident wave, the whole system may become a totally linear system forbidding the propagation of energy flux or a partially nonlinear one allowing the energy flux to pass. This should be interpreted as the underlying mechanism in this AD model, inherently similar to the model of the thermal diode [2]. It is noteworthy that, however, any composite structure (e.g., the present SL) only prohibits the sound propagation for some particular frequency ranges discretely distributed in the frequency domain. This is different from the case of the thermal diode where there exists one phonon band for the heat current to pass through [2]. Hence it is evident that the AD effect could only be observed within a series of ERBs, under the condition that the FW is forbidden while the SHW is allowed. It is understandable that the rectifying efficiency of the AD should depend on the extent to which the system is converted to a nonlinear one. It is thus particularly significant to study the parameter dependence of the rectifying efficiency. In an attempt to describe quantitatively the rectifying efficiency, we introduce the ratio T_+/T_- with T_+ and T_- being the transmission when the wave incidents from the RB and LB, respectively. Figure 3(b) plots T_+/T_- versus the strength of the incident wave by changing the system parameters of N, Γ_{III}, and d_{III}. The augment of the nonlinearity or the thickness of medium III increases the energy of SHW that is allowed to pass through the system, and the value of T_+ becomes larger. On the other hand, the increase of the number of periods in the SL apparently reduces the transmission of FW, and the value of T_- decreases. Consequently, an enhancement of rectifying efficiency is observed in Fig. 3(b) as we increase any of the parameters of N, Γ_{III}, and d_{III}, as expected.

Apparently, the attenuation in band gaps determines the extent to which the system becomes an insulator (i.e., the value of T_-) and should be large enough to guarantee the rectifying efficiency of the AD. For a finite SL, the attenuations are negligible at the edge of gaps, while they reach the maxima at the center. It is obvious however, that the ERBs locate generally at or near the centers of gaps, sufficiently far from the edges. Therefore the attenuations are large enough for the AD to work effectively, but their values may notably vary versus the frequency for ERBs not locating at the center, e.g., the first ERB. This is the reason why we choose the particular frequency $\omega d_{\mathrm{I}}/c_{\mathrm{I}} = 0.5$ in the study, which locates very near the lower limit of the first ERB. As proved by the numerical results, the rectifying efficiency of the AD is considerably high for this frequency at which the attenuation almost reaches the minimum ($\simeq$40 dB). Consequently it is expected that the AD will be more efficient as one chooses higher driving frequencies, which has been verified numerically.

The present model is sufficiently simple and efficient to encourage practical studies of experimental realization of an AD. It is of great significance to seek an appropriate medium with particularly strong nonlinearity and negligible viscosity, in the respect that the increase of N or d_{III} inevitably enlarges the sample size. It may be an effective scheme to employ a bubbly soft medium for which the natural frequency of bubble ω_0 is much greater than ω and the volume fraction β has been adjusted in an optimal manner. In the quasistatic case, according to Ostrovsky [8], the resonant effect vanishes and the viscosity is negligible, while the equivalent nonlinear parameter Γ_{III} depends on β only and reaches the maximum value $11\lambda/64\mu$ at the optimal volume fraction $4\mu/3\lambda$ with λ and μ being the Lame coefficients. Hence one needs to choose a bubbly soft medium with a substantially high ratio of λ/μ, e.g., plastisol for which $\lambda/\mu = 2 \times 10^5$ [9]; then an extremely large Γ_{III} may be obtained ($>10^4$) while the equivalent linear parameters are nearly unaffected due to the trivial magnitude of β ($\sim 10^{-6}$) [10]. Besides, β is small enough to break the multiple scattering effects between bubbles that may lead to other phenomena such as localization [11]. Therefore the assumption employed in the numerical simulations with regard to the physical parameters of medium III is reasonable owing to the fact the linear mechanical parameters of any soft medium approximate those of water. On the other hand, it should be possible to observe the AD effects in any nonlinear lattice with appropriately chosen structural parameters. The SL may also be replaced by any other composite with its own band structure. Particularly, it should be a promising way to employ metamaterials for building a more complex and effective AD system, which may largely reduce the sample size and broaden the ERBs due to the effect of subwavelength resonance [12,13]. This will be the goal of our future research.

In summary, we have presented a simple model of an AD formed by coupling a SL with a nonlinear medium and revealed a significant phenomenon of the rectifying effect of acoustic flux. We also have investigated the parameter dependence of rectifying efficiency as well as the necessary condition under which the AD works. The scheme of practical experimental realization of the AD model is briefly discussed.

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