Gravitational Fixed Points from Perturbation Theory

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The fixed point structure of the renormalization flow in higher derivative gravity is investigated in terms of the background covariant effective action using an operator cutoff that keeps track of powerlike divergences. Spectral positivity of the gauge fixed Hessian can be satisfied upon expansion in the asymptotically free higher derivative coupling. At one-loop order in this coupling strictly positive fixed points are found for the dimensionless Newton constant g_N and the cosmological constant λ, which are determined solely by the coefficients of the powerlike divergences. The renormalization flow is asymptotically safe with respect to this fixed point and settles on a $\lambda(g_N)$ trajectory after $O(10)$ units of the renormalization mass scale to accuracy 10^{-7}.

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Higher derivative gravity in four dimensions comes close to realizing a renormalizable quantum theory of gravity. Compared to the Einstein-Hilbert action two additional interaction monomials are added containing the independent curvature invariants with four derivatives of the metric tensor. The resulting gravity theory is perturbatively renormalizable to all loop orders [1], which is related to the strong, $1/p^4$ type, falloff of the free propagator at large momenta. With Euclidean signature the action reads

$$S = \int d^4x\sqrt{q}\Big[\tilde{\Lambda} - \frac{1}{\kappa^2}R + \frac{1}{2s}C^2 - \frac{\omega}{3s}R^2\Big]. \qquad (1)$$

Here $q_{\alpha\beta}$ is the metric entering the functional integral, C^2 is the square of its Weyl tensor, and total derivative terms like $\nabla^2 R$ and the integrand of the Gauss-Bonnet term have been omitted. In terms of the cosmological constant Λ one has $\tilde{\Lambda} = 2\Lambda/\kappa^2$ and the parameterization of the other coefficients by couplings s, ω is chosen for later convenience.

Two main issues need to be addressed in order to promote (1) to a viable theory of quantum gravity. First, since perturbation theory (PT) presumably captures only a small part of the physics content of the theory a formulation that is renormalizable in the Kadanoff-Wilson sense needs to be found. Second, taken at face value the free propagator is problematic from the viewpoint of positivity and one needs to make sure that physical quantities obey the relevant notion of unitarity. Schematically the $1/p^4$ decay arises from $1/p^2 - 1/(p^2 + s/\kappa^2)$ and the second term has negative norm in the conventional Fock space. The asymptotic safety scenario [2–5] outlines a route to achieving both goals. Central to it is the existence of a nontrivial fixed point for the dimensionless Newton constant, $g_N^* > 0$, and potentially also for the dimensionless cosmological constant, $\lambda_* \neq 0$. Here $g_N = \mu^2\kappa^2$ and $\lambda = \mu^{-4} g_N \tilde{\Lambda}/2 = \mu^{-2}\Lambda$, where μ is the renormalization scale. In addition the flow of all four dimensionless couplings g_N, λ, s, ω must be asymptotically safe, that is, bounded for all μ with finite limiting values for $\mu \to \infty$. Clearly both properties of the flow are parameterization dependent and as stressed by S. Weinberg [5] one should ultimately define "the coupling constants as coefficients in a power series expansion of the reaction rates themselves."

Related recent investigations used the flow equation for the average effective action Γ [6] and upon truncation of the (highly nonlocal) Γ to originally two later three [7] and four [8,9] terms in (1) obtained "hierarchical" approximations to the flow which revealed a nontrivial fixed point [10] for g_N and λ. This result is commonly attributed to the not perturbative nature of the technique, which, however, rests on a number of working hypotheses [2] whose validity in this context is hard to assess. In particular only *after* the ultraviolet renormalization problem is solved does the functional renormalization flow match PT by construction [11]. The recently reported nontrivial fixed point for s [9] lacks such a basis and may be an artifact.

We now want to argue that if (1) indeed has a nontrivial fixed point for g_N it should be visible already in PT [2]: any Wilsonian action of the form $S_\mu = \int d^4x\sqrt{q}\sum_{i\geq 1} u_i(\mu)\times P_i(q)$ with asymptotically safe couplings and scalar interaction monomials $P_i(q)$ of mass dimension $-d_i$ will for $\mu \to \infty$ depend only on $\mu^2 q_{\alpha\beta}$, as $u_i(\mu) \sim \mu^{d_i} u_i^*$. For the coefficient of the Ricci scalar in S_μ this is "as if" Newton's constant has picked up an integer anomalous dimension -2 along the trajectory connecting infrared to ultraviolet properties. A typical propagator would thus scale at low energies like $1/p^2$ and at high energies like $1/p^4$. But the latter is precisely the behavior which is in the realm of PT for (1). On the other hand an anomalous dimension -2 goes hand in hand with a nontrivial fixed point for g_N. This can be seen by taking into account g_N's double role as an inessential parameter ("wave function renormalization constant") and as a coupling. The yet-to-be-determined flow equation will thus naturally be parameterized by the anomalous dimension $\eta = \mu\frac{d}{d\mu}\ln\kappa^2$ [6], in which case $\mu\frac{d}{d\mu}g_N = (2 + \eta)g_N$, and $g_N^* \neq 0$ if and only if $\eta = -2$ [2]. Finally, we note that s and g_N are of degree

1 in the loop counting parameter $\hbar$ while the other couplings are of degree zero. Since s turns out to be asymptotically free in PT [12,13] one can regard the perturbative expansion as an expansion in powers of s. Then g_N may occur in degree zero ratios s/g_N (and in fact it does) and a putative nonzero fixed point value for g_N is well within the realm of the expansion.

The goal of the present Letter is to report the results of a computation augmenting the above heuristic argument. The nontrivial fixed points turn out to be related to the coefficients of the powerlike divergences in the one-loop effective action. To define these coefficients a background covariant operator regularization will be used [14]. Unlike dimensional regularization (which sees only logarithmic divergences) such a regulator in principle also allows one to make contact to nonperturbative results. We write $\Lambda_{\rm UV}$ for the UV cutoff and parameterize the divergent part of the one-loop background covariant [15] effective action as

$$\Gamma_1^{\rm div} = -\frac{1}{(4\pi)^2}\int d^4x\sqrt{g}[\Lambda_{\rm UV}^4\Upsilon_1 + \Lambda_{\rm UV}^2(\Upsilon_2 R + \mu^2\Upsilon_3) + \ln(\Lambda_{\rm UV}/\mu)(\zeta_1 C^2 + \zeta_2 R^2 + \mu^2\zeta_4 R + \mu^4\zeta_5)]. \quad (2)$$

Here $g_{\alpha\beta}$ is the background metric around which $q_{\alpha\beta}$ is expanded, $q_{\alpha\beta} = g_{\alpha\beta} + f_{\alpha\beta}$, and the functional integral over $f_{\alpha\beta}$ has been performed. Keeping track of the grading by the loop counting parameter ζ_1, ζ_2, ζ_4, ζ_5 and Υ_1, Υ_2, Υ_3 must be real valued functions of s/g_N, λ, ω. Using the field equations of (1) one sees that ζ_1, ζ_2, $\zeta_5/\lambda^2 + 4\zeta_4/\lambda$ contain only on-shell information and thus must be independent of the choice of gauge and field reparameterization constants. The coefficient Υ_1 and the combination $4\Upsilon_2 + \Upsilon_3/\lambda$ have a special status on which we comment later. To absorb the divergences we use the *nonminimal* subtraction ansatz

$$\begin{aligned}\tilde\Lambda_0 &= \mu^4\frac{2\lambda}{g_N}\Big\{1 + \frac{\hbar}{(4\pi)^2}\Big[a_{10} + a_{11}\ln(\Lambda_{\rm UV}/\mu) + a_{12}\Big(\frac{\Lambda_{\rm UV}}{\mu}\Big)^2 + a_{13}\Big(\frac{\Lambda_{\rm UV}}{\mu}\Big)^4\Big] + O(\hbar^2)\Big\},\\ \kappa_0^2 &= \mu^{-2}g_N\Big\{1 + \frac{\hbar}{(4\pi)^2}\Big[b_{10} + b_{11}\ln(\Lambda_{\rm UV}/\mu) + b_{12}\Big(\frac{\Lambda_{\rm UV}}{\mu}\Big)^2\Big] + O(\hbar^2)\Big\},\end{aligned} \quad (3)$$

and minimal subtraction with only log terms for s and ω with coefficients c_{11} and d_{11}. In principle a_{1i}, b_{1i}, c_{11}, d_{11} can be a power series of unit $\hbar$ degree of all renormalized couplings. The field renormalization we parameterize as

$$q^0_{\alpha\beta} = q_{\alpha\beta} + \frac{\hbar}{(4\pi)^2}\ln(\Lambda_{\rm UV}/\mu)g_N\xi q_{\alpha\beta} + O(\hbar^2), \quad (4)$$

where ξ can be a function of s/g_N, λ, ω. Inserting (3) and (4) into the bare action S_0, expanding, and requiring that the divergent terms equals $-\Gamma_1^{\rm div}$ yields the cancellation conditions which fix a_{1i}, b_{1i}, $i \neq 0$, and c_{11}, d_{11} in terms of the ζ_j, Υ_j, but leave a_{10}, b_{10} unconstrained.

So far the bare couplings κ_0^2, $\tilde\Lambda_0$ were only assumed to be μ independent. In a Wilsonian interpretation they should coincide with the running ("renormalized") couplings at scale $\mu = \Lambda_{\rm UV}$. This additional requirement fixes the subtraction point (3) in uniquely:

$$\begin{aligned}\kappa_0^2 &\overset{!}{=} \Lambda_{\rm UV}^{-2}g_N(\mu = \Lambda_{\rm UV}) \quad \text{iff } b_{10} + b_{12} = 0,\\ \tilde\Lambda_0 &\overset{!}{=} \Lambda_{\rm UV}^4\Big(\frac{2\lambda}{g_N}\Big)(\mu = \Lambda_{\rm UV}) \quad \text{iff } a_{10} + a_{12} + a_{13} = 0.\end{aligned} \quad (5)$$

Using (5) and the solution of the cancellation condition the flow equations are uniquely determined by the coefficients ζ_j, Υ_j and ξ. One recovers the universal s, ω flow equations of [12,13,16] and finds for g_N, λ:

$$\mu\frac{d}{d\mu}g_N = 2g_N + \frac{\hbar}{(4\pi)^2}g_N^2[\zeta_4 + \xi + 2\Upsilon_2], \quad (6)$$

$$\mu\frac{d}{d\mu}\lambda = -2\lambda + \frac{\hbar}{(4\pi)^2}\frac{g_N}{2}[\zeta_5 + 4\lambda\zeta_4 + \Upsilon_3 + 4\lambda\Upsilon_2 + 4\Upsilon_1 - (2\lambda\xi + 2\lambda\zeta_4 - \Upsilon_3)].$$

The above discussion was framed so as to stress that the system of flow equations for s, ω, g_N, λ can be derived without knowing the coefficients ζ, Υ explicitly. As a byproduct of the derivation one finds that

$$\Upsilon_1, \quad \Upsilon_2, \quad \frac{\Upsilon_3}{\lambda}, \quad \frac{\zeta_4}{\lambda}, \quad \frac{\zeta_5}{\lambda^2}, \quad \frac{\xi}{\lambda}, \quad (7)$$

are polynomials in $s/(g_N\lambda)$ and that the last three quantities cannot have constant terms. Anticipating that s is an asymptotically free coupling [12,13] it follows that $\zeta_4 = \zeta_5 = \xi = 0$ at the UV fixed point $s_* = 0$ so that the fixed points of (6) are determined by the Υ coefficients only. This is the main result of the Letter.

While the ζ coefficients are known in several gauges [12,13,16–18] the Υ coefficients have not previously been computed. To set up the computational framework a choice of gauge fixing and of regularization is needed. For the gauge fixing we use a three parameter harmonic gauge

$$\begin{aligned}S_{\rm gf} &= \frac{1}{2s}\int d^4x\sqrt{g}\chi_\mu Y^{\mu\nu}\chi_\nu, \quad \chi_\mu = \nabla^\nu f_{\mu\nu} + b_1\nabla_\mu f,\\ Y^{\mu\nu} &= -\frac{1}{a}[g^{\mu\nu}\nabla^2 + (b_2 - 1)\nabla^\mu\nabla^\nu - R^{\mu\nu}],\\ b_1 &= -\frac{1}{4c_1}\frac{1+4\omega}{1+\omega}, \quad b_2 = \frac{2c_2}{3}(1+\omega),\end{aligned} \quad (8)$$

where the gauge condition $\delta(\chi_\mu - \theta_\mu)$ has been averaged with a normalized Gaussian of covariance $Y^{\mu\nu}$. The reparameterization of b_1, b_2 in terms of c_1, c_2 is such that $a = c_1 = c_2 = 1$ corresponds to the so-called minimal gauge where in the gauge fixed Hessian all terms quartic in ∇_μ except $(\nabla^2)^2$ drop out. The ghost action associated with (8) has kernel $\Delta^{\mu\nu} := -g^{\mu\nu}\nabla^2 - (1 + 2b_1)\nabla^\mu\nabla_\nu - R^{\mu\nu}$.

In contrast to earlier PT computations we use a background covariant operator regularization [14] in combination with the heat kernel. For a (formally self-adjoint) differential operator $\mathbf{A}$ of order $2r$ our basic prescription is to replace $\ln\mathbf{A}$ with $F_{k^r,\Lambda_{\rm UV}^r}(A)(x,y)$ acting as an integral operator such that its trace is given by (9). Here $z\mapsto F_{k,\Lambda_{\rm UV}}(z)$ is a function that depends parametrically on an infrared cutoff k and an ultraviolet cutoff $\Lambda_{\rm UV}$. Specifically we take $F_{k,\Lambda_{\rm UV}}(z)=f(z/\Lambda_{\rm UV}^2)-f(z/k^2)$, for suitable f. Although conceptually distinct from μ we can for the purposes here take $0<rk=\mu\le\Lambda_{\rm UV}$ for some $r>0$. The normalization $\partial_z F_{0,\infty}(z)=1/z$ in combination with some additional properties ensures that functional traces are properly regularized. In particular $\mathrm{Tr}\ln\mathbf{A}$ is replaced with

$$\mathrm{Tr}\,F_{k^r,\Lambda_{\rm UV}^r}(A)=\int_0^\infty dt\tilde{F}_{k^r,\Lambda_{\rm UV}^r}(t)\int dx A(x,x;t), \qquad (9)$$

where $A(x,y;t)$ is the heat kernel of $\mathbf{A}$ and $\tilde{F}_{k,\Lambda_{\rm UV}}$ is the inverse Laplace transform of $F_{k,\Lambda_{\rm UV}}$. On a flat background one can also evaluate the operator trace directly in momentum space

$$\mathrm{Tr}\,F_{k^r,\Lambda_{\rm UV}^r}(A)=\sum_j m_j\int\frac{d^4p}{(2\pi)^4}F_{k^r,\Lambda_{\rm UV}^r}(\lambda_j(p)), \qquad (10)$$

where λ_j are the spectral values of $\mathbf{A}$ and m_j their multiplicities. Eventually, only certain moments of the cutoff function enter the results for the ζ and Υ coefficients. One has

$$\int_0^\infty dt t^{-n}\tilde{F}_{k,\Lambda_{\rm UV}}(t)=\frac{1}{\Gamma(n)}\int_0^\infty dz z^{n-1}F_{k,\Lambda_{\rm UV}}(z) =\begin{cases}-2\ln\Lambda_{\rm UV}/k, & n=0,\\ -q_n(\Lambda_{\rm UV}^{2n}-k^{2n}) & 0<n\le 2,\end{cases} \qquad (11)$$

where the q_n are positive constants of $O(1)$.

We define the one-loop effective action by

$$\Gamma_1=\tfrac{1}{2}\mathrm{Tr}F_{k^2,\Lambda_{\rm UV}^2}(\mathcal{H})-\tfrac{1}{2}\mathrm{Tr}F_{k,\Lambda_{\rm UV}}(Y)-\mathrm{Tr}F_{k,\Lambda_{\rm UV}}(\Delta), \qquad (12)$$

where $\mathcal{H}$ is the Hessian of $2s(S+S_{\rm gf})$. In a nongravitational context one usually subtracts from (12) a corresponding contribution from a reference operator. The reference operator is chosen so as to represent the noninteracting system and, in particular, removes quartic divergences. In gravity such a reference system bears on a definition of self-energy and it is unlikely that a preferred choice exists. The Gaussian normalization condition conventionally adopted for the kinematical measure over "metrics modulo diffeomorphisms" [19] amounts to having no subtractions in (12). Using this here for the time being one finds that neither Υ_1 nor $4\Upsilon_2+\Upsilon_3/\lambda$ are gauge independent. A more refined definition of the measure should render $4\Upsilon_2+\Upsilon_3/\lambda$ gauge independent and we shall formulate the flow equations in terms of this quantity.

Another modification of (12) would be to add to $\mathcal{H}$ its Vilkovisky–de Witt (VdW) correction [20]. We verified that the setting used here correctly reproduces the VdW form of ζ_5 [17] upon adding the correction, but that it leaves Υ_1, Υ_3 unaffected.

The evaluation of the divergent part of (12) now amounts to the determination of the short time asymptotics for the heat kernels of the operators $\mathcal{H}$, Y, and Δ. Both Y and Δ are second order operators with trivial principal part, for which tabulated heat kernel coefficients are available [21]. In a curved background and in a generic gauge (8) $\mathcal{H}$ is a very complicated operator for which no tabulated results are available; moreover there is no choice of gauge parameters for which its principal part is trivial. We thus resorted to an evaluation on a flat background in a generic gauge which allows one to determine Υ_1, Υ_3 (and as a check ζ_5) in a generic gauge. Finally, Υ_2 can be obtained by transversal-traceless decomposition of the Hessian on maximally symmetric backgrounds. As a check we also evaluated Υ_2 directly on a generic background in minimal gauge, where the principal part is a nontrivial but constant matrix.

The evaluation of (12) on a flat background reveals that—in contrast to the common wisdom about the system and in contrast to the situation in Einstein gravity—there is *no* problem with positivity. The Hessian on a flat background can be diagonalized exactly and the positivity of the spectrum can be investigated. There are four spectral values $\lambda_1(p)$, $\lambda_2(p)$, $\lambda_3(p)$, $\lambda_4(p)$, with multiplicities 5, 3, 1, 1, respectively. The last two are nonrational functions of the momenta with a large p expansion of the form $p^{-4}\lambda_i(p)=\mu_i+O(sp^{-2})$ (which also applies to λ_1, λ_2, where the expansion terminates). Spectral positivity is decided by the signs of the μ_i and one can show

$$\mu_i>0 \quad \text{for} \ -1<\omega<0, \qquad c_1>1/4, \qquad c_2/a>0. \qquad (13)$$

The interval $-1<\omega<0$ is invariant under the renormalization flow (6) and contains the known UV fixed point $\omega_*=(7\sqrt{6049}-549)/200\approx-0.0228$ [12,13,18]. Hence, for μ sufficiently large no problem with positivity of the propagator (i.e., the inverse Hessian) ever arises.

As noted before, the nontrivial fixed point of (6) is determined by the Υ coefficients only. Anticipating that also Υ_3 is linear in $s/(g_N\lambda)$ one sees that (6) has a nontrivial fixed point at

$$\frac{g_N^*}{(4\pi)^2}=-\frac{1}{\Upsilon_2^*}, \qquad \lambda_*=-\frac{\Upsilon_1^*}{2\Upsilon_2^*}, \qquad (14)$$

where $\Upsilon_2^*:=\Upsilon_2|_{\omega_*,s=0}$, $\Upsilon_1^*:=\Upsilon_1|_{\omega_*,s=0}$. The scheme dependence enters only through the q_n of Eq. (11). Importantly, these fixed points come out as robustly positive.

The results for Υ_1, Υ_2, Υ_3 in a generic gauge are too bulky to be reported here. For simplicity we present them here in minimal gauge. First, the fixed point values

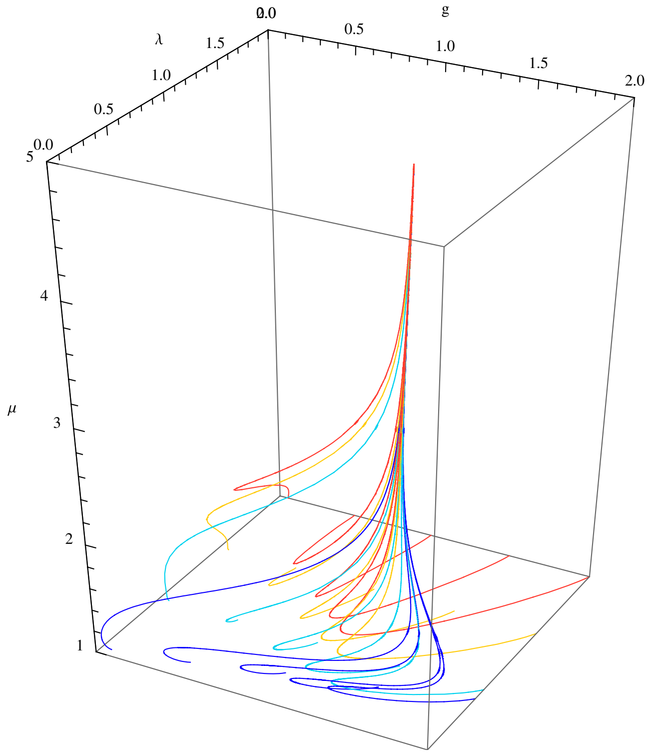

FIG. 1 (color online). Wilsonian one-loop flow (5), (6), and (17).

$$\Upsilon_2^* = -1.9867q_1 - 0.09836q_{1/2},\quad \Upsilon_1^* = 5.8114q_1 - 6.1026q_2, \tag{15}$$

where for all cutoffs usually considered $q_1/q_2 \geq 2$.

To study the flow (6) itself we now make the choice

$$\xi = -\zeta_4 + \Upsilon_3/(2\lambda), \tag{16}$$

which gives rise to (g_N, λ) flow equations depending only on the gauge independent ζ combination, $\Upsilon_2 + \Upsilon_3/(4\lambda)$ and Υ_1 without affecting the fixed point (14). One has $\Upsilon_1 = u_1(\omega)$, $\Upsilon_2 + \Upsilon_3/(4\lambda) = u_2(\omega) - \frac{s}{g_N\lambda}u_3(\omega)$, with

$$u_1(\omega) = q_1\frac{26\omega - 1}{12\omega} - q_2\left[\frac{9}{2} + \frac{9}{8}\frac{1}{(1+\omega)^2} + \frac{4}{9}(1+\omega)^2\right],$$

$$u_2(\omega) = -\frac{\sqrt{\pi}}{8}q_{1/2}\left[3(1+\omega) - \frac{\omega+2}{3}\sqrt{-\frac{1+\omega}{3\omega}}\right] - q_1\frac{87 + 118\omega + 56\omega^2 + 16\omega^3}{72(1+\omega)},$$

$$u_3(\omega) = \frac{3\sqrt{\pi}}{64}q_{1/2}\left[3 - \left(-\frac{1+\omega}{3\omega}\right)^{3/2}\right]. \tag{17}$$

In combination with the known ζ_1, ζ_2, $\zeta_5 + 4\lambda\zeta_4$ [12,13,16–18] this defines the flow (6).

Figure 1 shows the result of a numerical integration after rescaling $g_N \mapsto (4\pi)^2 g_N$, $s \mapsto (4\pi)^2 s$, with $s(1) = 1$, $\omega(1) = -1/2$, and cutoff $f(y) = -\ln(1 + 1/y) + (1 + y)^{-1} + 2^{-1}(1 + y)^{-2}$ [14] satisfying $q_{1/2} = 3\sqrt{\pi}/4$, $q_1 = 1/2$, $q_2 = 1/4$. The initial data for g_N, λ were varied in the range [0, 2]. One sees that g_N, λ are initially nonmonotonic functions of μ, monotonic behavior sets in quickly but nonuniformly in the initial data. At $\mu = 10$ the memory of the initial data is erased to accuracy 10^{-7} and the merged trajectory eventually hits the fixed point located at $g_N^* \approx 1.3697$, $\lambda_* \approx 0.9451$, however with 1% deviations even at $\mu = 10^9$.

In summary, higher derivative gravity has nontrivial fixed points for g_N and λ with respect to which the renormalization flow is asymptotically safe. By means of the improved perturbative framework (3), (5), and (9) they can be identified already in one-loop PT. A complete diagonalization of the Hessian for flat backgrounds reveals subject to (13) a strictly positive spectrum, rendering fourth order gravity a viable candidate for a fundamental field theory of gravity.

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