**Raghavan Replies:** The Comments by Potzel and Wagner [1] and Schiffer [2] attempt to contest the central conclusions of my Letter [3] on hypersharp neutrino emission from the perspective of the classical Mössbauer effect (ME) drawn exclusively from short-lived states. They miss the main role of the long nuclear lifetime in the tritium ($T$) neutrino ( $\tilde{\nu}_e$) resonance in which fluctuations *modulate* instead of *detune* the resonance. The averaging effect of modulation is ubiquitous in physics (indeed it forms the physics basis of the ME itself [4]). The long nuclear lifetime, encountered here for the first time, highlights a unique effect that relegates all line broadening to sidebands resolved from the central line instead of detuning it. In this case a significant fraction of the $T$ $\tilde{\nu}_e$ has *no option* but to be emitted with the natural linewidth. Ignoring this key feature leads to the misleading conclusions of [1,2].

*Periodic perturbations.*—We begin with a major common ground with [1] which agrees that *periodic* vibrations of the lattice *do* indeed result in motional averaging leading to the natural line width for tritium ($T$) $\tilde{\nu}_e$. This concession is crucial in recognizing the difference due to long and short lifetimes. For long lifetimes the $\tilde{\nu}_e$ line is modulated (not just detuned) by fluctuations with two parameters $\eta = (\Omega_o/\Omega)$, the ratio of the energy spread to the fluctuation frequency that affects the central line *intensity* and $\xi = (\Omega/\Gamma)$ the separation in linewidths $\Gamma$ of the sidebands created by the modulation [3]. Since in the long lived $T$ case $\xi$ is always $\gg 1$ for conceivable fluctuations in solids or from other sources, the sidebands are always many orders of magnitudes of linewidths removed from the central line. Thus, a significant fraction of the emitted $\tilde{\nu}_e$ line is *necessarily of natural width* with a usually acceptable loss of intensity to the sidebands given in terms of the Bessel coefficient $[J_o(\eta)]^4$ for the complete resonance $^3\mathrm{H} \leftrightarrow {}^3\mathrm{He}$ process. As the lifetime becomes shorter ($\sim \mu$s as in usual ME cases), $\Gamma$ is many orders of magnitude larger, $\xi$ decreases to $<1$, the sideband resolution is lost and the central line is broadened for the same fluctuation parameters. The line narrowing effect due to lattice (and in general any periodic) vibrational motion is pervasive for all coordinate $r$-dependent energy fluctuations.

*Stochastic perturbations.*— Comment [1] now asserts that line narrowing as above does not result from *stochastic* fluctuation. This directly contradicts well-known works such as that of Dicke [5] who showed that even the Doppler profile of spectral lines due to stochastic collisions in a *gas* manifests a sharp unshifted line, anticipating ME type line narrowing by many years. The fluctuation spectrum due to random relaxation process of [1] was shown by Kubo [6] to be a Gaussian, and nearly so also for a relaxing two level system cited by [1]. The components of the Gaussian modulate the $\tilde{\nu}_e$ energy in the same way as those of a complex spectrum of periodic lattice vibrations in a solid. As a result, the broad modulating Gaussian is reproduced in the *sidebands*, not the central line, The Gaussian sideband with the mean energy and width (many orders of magnitude wider than the natural width) is still well resolved from the sharp central line because of the long $T$ lifetime. The only effect is some loss of central line intensity. Using the same relaxation parameters as [1], the hypersharp $\tilde{\nu}_e$ intensity is $[J_o(\eta)]^4$ with $\eta = [(6.6 \times 10^{-11}/5 \times 10^{-11}) \sim 1.23] \sim 2 \times 10^{-1}$, not $\sim 10^{-11}$ as claimed by [1].

*Self-cancellation of final state energy differences.*—Inhomogeneous line broadening is a central consideration in classical ME. It presents a basically different perspective in hypersharp emission via the role of final state energy compensation specific to the $T \leftrightarrow$ He transition in addition to vibrational averaging.

The experiment is designed with a unique metal tritide host in which, the $T$ and He sit in well-defined isosymmetric tetragonal interstitial traps. The trap potential wells have different binding energies for $T$ and He (Table 2 in [3]). The atomic dynamics of $T$ and He are controlled by *discrete* vibrational excitations in the local potential wells. The recoilless fraction is determined only by these local excitations (known from neutron inelastic scattering [7]), not the Debye spectrum of the bulk lattice. The energies and dynamics of $T$ and He in the wells are largely insensitive to random impurities in the bulk for the small concentrations of $T$ and He. $\sim 2.5 \times 10^{-3}$. The nuclear quadrupole moments of $T$ and $^3$He are zero, thus electric interactions due to random lattice defects are absent. Inhomogeneous dipolar fields of neighboring atoms which depend on the atomic coordinate, are averaged out by lattice vibrations.

The $\tilde{\nu}_e$ energy is changed in emission and absorption by final state energies of atomic shells and trap binding and dynamics [such as zero point motion (ZPE)]. Each of these is different for $T$ and He. In each emission and *absorptionboth* $T$ and He participate. Thus the net final state energy change $\Delta$ in the decay $T \rightarrow$He changes sign in the absorption He$\rightarrow T$ and cancelled in the complete process $^3\mathrm{H} \leftrightarrow {}^3\mathrm{He}$. Further, all $r$-dependent energies such as the local well depths, binding energies and ZPE's are averaged to *unique hypersharp* central values. The self compensation is thus precise and the hypersharp resonance condition is maintained.

*Temperature dependence.*—The discrete excitation energies in the potential wells are known from neutron inelastic scattering [7]. The lowest vibrational level is at 72 meV, equivalent to $\sim$850 K, below which the atomic motions cannot excite a trap level. Thus at normal temperatures the hypersharp fraction is largely independent of temperature. There is thus little temperature dependent Josephson shift [8] either, only a shift in the constant ZPE. As discussed above, the ZPE shift changes sign in the complete process and cancelled. These aspects differ sharply from the temperature dependence of the classical

 

ME due to excitation of low frequencies in the bulk lattice Debye spectrum. The experience in maintaining ultra small temperature gradients in ton scale ultracold bar detectors of gravitational waves should be very helpful in eliminating residual temperature dependent shifts in the gram scale masses of our case.

*Lattice excitations due to static dilation?*—The formation of the trap is a local dilation extending at the most to the next two nearest neighbors. The dilation creates at the same time, local vibrational levels. The $\tilde{\nu}_e$ energy is changed by the trap energy (as discussed above) and possible excitation of states in the trap, also taken into account in the estimation of the resonance rates. For the dilation to also additionally excite states in the bulk lattice outside the trap (as suggested by [1]), the shock wave due to the dilation must sample the bulk. This can ocur only at the speed of sound, thus longafter $\tilde{\nu}_e$ emission. Note that this effect does *not affect the natural linewidth*—only the recoilless probability. Thus, even with the highly unlikely effect, the resonance cross section is still huge, $\sigma \sim 10^{-26}$ cm$^2$, adequate for most of the physics applications.

*Effect of external vibrations.*—Because of the extreme sharpness at issue [2] it is legitimate to raise the effect of external vibrations on the resonance via the Doppler effect. The latter can be estimated as $\delta E/E = v/c = 1/c(\delta l/l)fL = 3 \times 10^{-11} \times 10 \times 10^{-17} \sim 3 \times 10^{-27}$, where $L$ is the baseline ($\sim$10 cm) and $(\delta l/l)$ the fractional length change at the frequency $f$, taken to be: $(\delta l/l)f \sim 10^{-17}$ Hz from the data of the LIGO gravitational wave detector with a baseline of several km (thus much more demanding than our bench scale problem). Thus the vibrational effect is $\delta E/E \sim 3 \times 10^{-27}$ or $\sim$60 linewidths. This appears not as resonance detuning but a periodic *modulation* with $\eta = (\Omega_o/\Omega) = \delta E/(\hbar f 2\pi)$ which ranges from $10^{-5}$ to $10^{-11}$ for frequencies $f$ from $10^{-3}$ to $10^3$ Hz. The hypersharp fraction $[J_o(\eta)]^4$ is thus close to unity in the entire range even for $(\delta l/l)$ some 5 orders of magnitude larger. The sidebands with the small residual intensity occur at $\delta E/\Gamma \sim 60$ linewidths away from the central line. Indeed, the result suggests external modulation for averaging out problems similar to mechanical vibrations.

*Effect of time-energy uncertainty.*—If the dwell time of instability of a decaying state is uncontrolled, the energy spread of the $\tilde{\nu}_e$ is the natural width. If the dwell time of the state before decay is known, the ''age'' of the source forces the width to be broader than the natural width. Thus the signal at zero detuning in the tritium case is a spontaneous growth of the resonance signal with time as the source ages. The question is discussed recently in detail [9] and noticed also elsewhere [10].

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