Pragmatic Approach to the Little Hierarchy Problem: The Case for Dark Matter and Neutrino Physics

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We show that the addition of real scalars (gauge singlets) to the standard model can both ameliorate the little hierarchy problem and provide realistic dark matter candidates. To this end, the coupling of the new scalars to the standard Higgs boson must be relatively strong and their mass should be in the 1–3 TeV range, while the lowest cutoff of the (unspecified) UV completion must be $\gtrsim 5$ TeV, depending on the Higgs boson mass and the number of singlets present. The existence of the singlets also leads to realistic, and surprisingly reach, neutrino physics. The resulting light neutrino mass spectrum and mixing angles are consistent with the constraints from the neutrino oscillations.

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Introduction.—The goal of this project is to provide the most economic extension of the standard model (SM) for which the little hierarchy problem is ameliorated while retaining all the successes of the SM. We focus here on leading corrections to the SM, so we will consider only those extensions that interact with the SM through renormalizable interactions (below we will comment on the effects of higher-dimensional interactions). Since we concentrate on taming the quadratic divergence of the Higgs boson mass, it is natural to consider extensions of the scalar sector: when adding a new field φ, the gauge-invariant coupling $|\varphi|^2 H^\dagger H$ (where H denotes the SM scalar doublet) will generate additional radiative corrections to the Higgs boson mass that can serve to soften the little hierarchy problem. In this Letter we will consider a class of modest extensions by adding several real scalar fields which are neutral under the SM gauge group. The extension we consider, although renormalizable, will still be understood to constitute an effective low-energy theory valid up to energies $\sim$5–10 TeV; we shall not discuss the UV completion of this model.

The little hierarchy problem.—Within the SM the quadratically divergent 1-loop correction to the Higgs boson (h) mass is given by

$$\delta^{(\mathrm{SM})} m_h^2 = [3m_t^2/2 - (6m_W^2 + 3m_Z^2)/8 - 3m_h^2/8]\Lambda^2/(\pi^2 v^2) \quad (1)$$

where Λ is a UV cutoff (we use a cutoff regularization) and $v \simeq 246$ GeV denotes the vacuum expectation value of the scalar doublet (SM logarithmic corrections are small since we assume $v \ll \Lambda \lesssim 10$ TeV); the SM is treated here as an effective theory valid below the physical cutoff Λ, the scale at which new physics becomes manifest.

Since precision measurements (mainly from the oblique T_{obl} parameter [1]) require a light Higgs boson, $m_h \sim$ 120–170 GeV, the correction (1) exceeds the mass itself even for small values of Λ, e.g., for $m_h = 130$ GeV we obtain $\delta^{(\mathrm{SM})} m_h^2 \simeq m_h^2$ already for $\Lambda \simeq 580$ GeV. On the other hand constraints on the scale of new physics that emerge from analysis of operators of dim 6 require $\Lambda \gtrsim$ few TeV. This difficulty is known as the little hierarchy problem.

There are two ways to solve this problem: one adds new particles whose effects either (i) generate radiative corrections that partially cancel (1), as is done in supersymmetric theories (for which $\delta m_h^2 \ll m_h^2$ up to the GUT scale); or (ii) increase the allowed value of m_h by canceling the contributions to T_{obl} from a heavy Higgs boson (see e.g., [2]).

Here we follow the first strategy, but with a modest goal: we construct a simple modification of the SM within which δm_h^2 (the total correction to the SM Higgs boson mass squared) is suppressed only up to $\Lambda \lesssim$ 3–10 TeV. Since (1) is dominated by the fermionic (top) terms, the most economic way of achieving this is by introducing new scalars φ_i whose 1-loop contributions balance the ones derived from the SM. In order not to spoil the SM predictions we assume that φ_i are singlets under the SM gauge group. It is then easy to see that the oblique parameters will remain unchanged if $\langle \varphi_i \rangle = 0$ (which we assume hereafter), so that the SM prediction of a light Higgs is preserved. An extension of the SM by an extra scalar singlet was also discussed in [3], there however (classical) conformal symmetry was adopted to cope with the hierarchy problem.

The most general scalar potential consistent with $Z_2^{(i)}$ independent symmetries $\varphi_i \to -\varphi_i$ (imposed in order to prevent $\varphi_i \to hh$ decays) reads:

$$V(H,\varphi_i) = -\mu_H^2|H|^2 + \lambda_H|H|^4 + \sum_{i=1}^{N_\varphi}(\mu_\varphi^{(i)})^2\varphi_i^2 + \frac{1}{24}\sum_{i,j=1}^{N_\varphi}\lambda_\varphi^{(ij)}\varphi_i^2\varphi_j^2 + |H|^2\sum_{i=1}^{N_\varphi}\lambda_x^{(i)}\varphi_i^2. \quad (2)$$

In the following numerical computations we assume for simplicity that $\mu_\varphi^{(i)} = \mu_\varphi$, $\lambda_\varphi^{(ij)} = \lambda_\varphi$ and $\lambda_x^{(i)} = \lambda_x$, in which case (2) has an $O(N_\varphi)$ symmetry (small deviations from this assumption do not change our results qualitatively). The minimum of V is at $\langle H\rangle = v/\sqrt{2}$ and $\langle\varphi_i\rangle = 0$ when $\mu_\varphi^2 > 0$ and $\lambda_x, \lambda_H > 0$ which we now assume. The masses for the SM Higgs boson and the new scalar singlets are $m_h^2 = 2\mu_H^2$ and $m^2 = 2\mu_\varphi^2 + \lambda_x v^2$ ($\lambda_H v^2 = \mu_H^2$), respectively.

Stability (positivity) of the potential at large field strengths requires $\lambda_H\lambda_\varphi > 6\lambda_x^2$ at tree level. The high energy unitarity behavior (known [4] for $N_\varphi = 1$) implies $\lambda_H \le 4\pi/3$ (the SM requirement) and $\lambda_\varphi \le 8\pi$, $\lambda_x < 4\pi$. Note however that these conditions are derived from the behavior of the theory at energies $E \gg m$, where we do not pretend our model to be valid, so that neither the stability limit nor the unitarity constraints are applicable within our pragmatic strategy that aims at a modest increase of Λ to the 3–10 TeV range. These conclusions remain even if one includes higher-dimensional operators since such terms are subdominant unless the energies and/or field strengths are of order Λ—were the model is not valid; such operators can also generate spurious minima, but these have scale $\sim\Lambda$ and are not within the range of validity of the model. It is also fair to note that for $N_\varphi = 1$ the stability limit for $m_h > 115$ GeV implies $\lambda_\varphi > 12(\lambda_x v/m_h)^2 \gtrsim 55\lambda_x$. Then using $\lambda_\varphi \le 8\pi$ we find $\lambda_x \lesssim 0.68$; this does not allow for a significant cancellation of the SM contributions (1) and the little hierarchy problem remains. Increasing N_φ suppresses λ_x and relaxes the unitarity constrains.

The presence of φ_i generates additional radiative corrections to m_h^2. (The Λ^2 corrections to m^2 can also be tamed within the full model with additional fine tuning, but we will not consider them here, see [5]. However different ways of imposing the cutoff Λ (cutoff regularization, higher-derivative regulators, Pauli-Villars regulators, etc.) yield different expressions for the extra corrections; for large Λ, the coefficients of the Λ^2 and $m^2\ln(\Lambda^2/m^2)$ terms are universal, but the subleading, terms are not. Since the subleading contributions are small for $m \ll \Lambda$ (this is the range interesting for us) the differences between various regularization schemes are not relevant. Here we decided to adopt the simple UV cutoff regularization. Then the extra contribution to m_h^2 reads

$$\delta^{(\varphi)}m_h^2 = -[N_\varphi\lambda_x/(8\pi^2)][\Lambda^2 - m^2\ln(1+\Lambda^2/m^2)]. \quad (3)$$

Adopting the parameterization $|\delta m_h^2| = |\delta^{(\mathrm{SM})}m_h^2 + \delta^{(\varphi)}m_h^2| = D_t m_h^2$ [2], we can determine the value of λ_x needed to suppress δm_h^2 to a desired level (D_t) as a function of m, for any choice of m_h and Λ; examples are plotted in Fig. 1 for $N_\varphi = 6$.

It should be noted that (in contrast to SUSY) the logarithmic terms in (3) can be relevant in canceling large contributions to δm_h^2. (Note that in SUSY the corresponding logarithmic stop contributions survive and constitute a source of concern.) It is important to note that the required value of λ_x is smaller for larger m_h, and can also be reduced increasing the number of singlets N_φ. When $m \ll \Lambda$, the λ_x needed for the amelioration of the hierarchy problem is insensitive to m, D_t or Λ; as illustrated in Fig. 1; analytically we find

$$\lambda_x = N_\varphi^{-1}\{4.8 - 3(m_h/v)^2 + 2D_t[2\pi/(\Lambda/\mathrm{TeV})]^2\} \times [1 - m^2/\Lambda^2\ln(m^2/\Lambda^2)] + O(m^4/\Lambda^4). \quad (4)$$

Since we consider $\lambda_x \sim 1$, it is pertinent to estimate the effects of higher order corrections [6] to (1). In general, the fine tuning condition reads (m_h was chosen as a renormalization scale):

$$|\delta^{(\mathrm{SM})}m_h^2 + \delta^{(\varphi)}m_h^2 + \Lambda^2\sum_{n=1}f_n(\lambda_x,\ldots)[\ln(\Lambda/m_h)]^n| = D_t m_h^2, \quad (5)$$

where the coefficients $f_n(\lambda_x,\ldots)$ can be determined recursively [6], with the leading contributions being generated by loops containing powers of λ_x: $f_n(\lambda_x,\ldots) \sim [\lambda_x/(16\pi^2)]^{n+1}$. To estimate these effects consider the case where $\delta^{(\mathrm{SM})}m_h^2 + \delta^{(\varphi)}m_h^2 = 0$ at one loop then, keeping only terms $\propto \lambda_x^2$, we find, at 2 loops, $D_t \simeq [N_\varphi\lambda_x/(16\pi^2)]^2(\Lambda/m_h)^2$. Requiring $D_t \lesssim 1$ implies $\Lambda \lesssim 4\pi^2 m_h \simeq 5$–8 TeV for $m_h = 130$–210 GeV, respectively.

It should be emphasized that in the model proposed here the hierarchy problem is softened (by lifting the cutoff to $\sim$8 TeV) only if λ_x, Λ and m are appropriately fine-tuned; this fine-tuning, however, is significantly less dramatic than in the SM. One can investigate this issue quantitatively and determine the range of parameters that corresponds to a

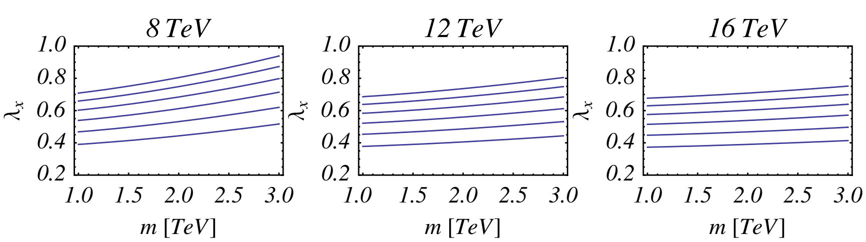

FIG. 1 (color online). Plot of λ_x corresponding to $D_t = 0$ and $N_\varphi = 6$ as a function of m for $\Lambda = 8$, 12, 16 TeV (as indicated above each panel). The various curves correspond to $m_h = 130$, 150, 170, 190, 210, 230 GeV (starting with the uppermost curve).

given level of fine-tuning as in [7]; we will return to this in a future publication [5].

Dark matter.—The singlets φ_i also provide natural dark matter (DM) candidates (see [8,9], for the one singlet case). Following [10] one can easily estimate the amount of the present DM abundance; we will assume for simplicity that all the φ_i are equally abundant (e.g., as in the $O(N_\varphi)$ limit). The thermal averaged cross section for singlet annihilations into SM final states $\varphi_i\varphi_i \to \mathrm{SM\,SM}$ in the nonrelativistic approximation, and for $m \gg m_h$, equals

$$\langle\sigma_i v\rangle \simeq \frac{\lambda_x^2}{8\pi m^2} + \frac{\lambda_x^2 v^2 \Gamma_h(2m)}{8m^5} \simeq \frac{1.73}{8\pi}\frac{\lambda_x^2}{m^2} \tag{6}$$

where the first contribution is from the hh final state (keeping only the s-channel Higgs exchange; the t and u channels can be neglected since $m \gg m_h$) while the second contribution is from all other final states; $\Gamma_h(2m) \simeq 0.48\ \mathrm{TeV}(2m/1\ \mathrm{TeV})^3$ is the Higgs width calculated when the Higgs boson mass equal $2m$.

From this the freeze-out temperature $x_f = m/T_f$ is given by

$$x_f = \ln[0.038 m_{Pl} m \langle\sigma_i v\rangle/(g_\star x_f)^{1/2}] \tag{7}$$

where $g_\star$ counts relativistic degrees of freedom at annihilation and m_{Pl} denotes the Planck mass. In the range of parameters we are interested in, $x_f \sim 12$–50 while $m \sim 1$–2 TeV, so that this is a case of cold dark matter. Then the present density of φ_i is given by

$$\Omega_\varphi^{(i)} h^2 = 1.06 \cdot 10^9 x_f/(g_\star^{1/2} m_{Pl}\langle\sigma_i v\rangle \mathrm{GeV}). \tag{8}$$

Finally, the requirement that the φ_i account for the inferred DM abundance, $\Omega_{\mathrm{DM}} h^2 = \sum_{i=1}^{N_\varphi} \Omega_\varphi^{(i)} h^2 = 0.106 \pm 0.008$ [1], can be used to fix $\langle\sigma_i v\rangle$, which translates into a relation $\lambda_x = \lambda_x(m)$ through the use of (6). Substituting this into $|\delta m_h^2| = D_t m_h^2$, we find a relation between m and Λ (for a given D_t), which we plot in Fig. 2 for $N_\varphi = 6$. It is important to stress that it is possible to find parameters Λ, λ_x and m such that *both* the hierarchy is ameliorated to the prescribed level *and* such that $\Omega_\varphi h^2$ is consistent with the DM requirement (we use a 3σ interval). It also is useful to note that the singlet mass (as required by the DM) scales with their multiplicity as $N_\varphi^{-3/2}$, therefore increasing N_φ implies smaller scalar mass, e.g., changing N_φ from 1 to 6 leads to the reduction of mass by a factor $\sim$15.

Neutrinos.—We now discuss consequences of the existence of φ for the leptonic sector, which we assume consists of the SM fields plus three right-handed neutrino fields ν_{iR} ($i = 1, 2, 3$) that are also gauge singlets; in this section we assume only one singlet for simplicity. (The arguments presented below remain essentially the same when a different number of right-handed neutrinos is present.) The relevant Lagrangian is then

$$\begin{aligned}\mathcal{L}_Y = &-\bar{L} Y_l H l_R - \bar{L} Y_\nu \tilde{H} \nu_R - \frac{1}{2}\overline{(\nu_R)^c} M \nu_R \\ &- \varphi \overline{(\nu_R)^c} Y_\varphi \nu_R + \mathrm{H.c.}\end{aligned} \tag{9}$$

where $L = (\nu_L, l_L)^T$ is a SM lepton isodoublet and l_R a charged lepton isosinglets (we omit family indices); we will assume that the see-saw mechanism explains the smallness of three light neutrino masses, and accordingly we require $M \gg M_D \equiv Y_\nu v/\sqrt{2}$. The symmetry of the potential under $\varphi \to -\varphi$ can be extended to (9) by requiring

$$L \to S_L L, \qquad l_R \to S_{l_R} l_R, \qquad \nu_R \to S_{\nu_R}\nu_R \tag{10}$$

where the unitary matrices S_{L, l_R, ν_R} obey

$$\begin{aligned} S_L^\dagger Y_l S_{l_R} &= Y_l, & S_L^\dagger Y_\nu S_{\nu_R} &= Y_\nu, \\ S_{\nu_R}^T M S_{\nu_R} &= +M, & S_{\nu_R}^T Y_\varphi S_{\nu_R} &= -Y_\varphi. \end{aligned} \tag{11}$$

In order to determine the consequences of this symmetry we find it convenient to adopt the basis in which M and Y_l are real and diagonal; for simplicity we will also assume that M has no degenerate eigenvalues. Then the last two conditions in (11) imply that S_{ν_R} is real and diagonal, so its elements are ± 1. For 3 neutrino species there are then two possibilities (up to permutations of the basis vectors): we either have $S_{\nu_R} = \pm\mathbb{1}$, $Y_\varphi = 0$, or, more interestingly,

$$S_{\nu_R} = \epsilon\,\mathrm{diag}(1, 1, -1); \qquad Y_\varphi = \begin{pmatrix} 0 & 0 & b_1 \\ 0 & 0 & b_2 \\ b_1 & b_2 & 0 \end{pmatrix}, \qquad \epsilon = \pm 1, \tag{12}$$

where $b_{1,2}$ are, in general, complex. The first conditions in (11) now require $S_{l_R} = S_L$ with

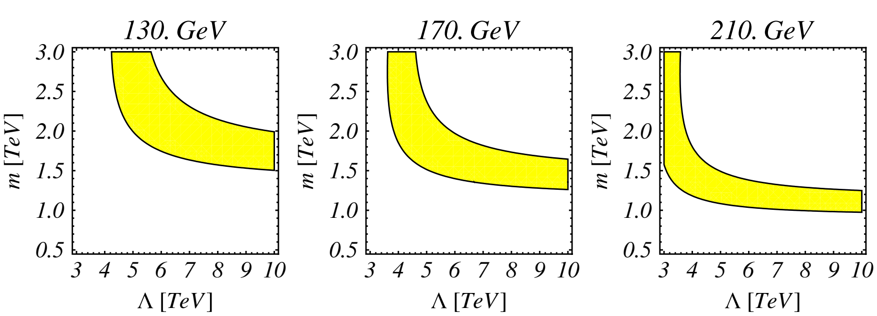

FIG. 2 (color online). The allowed region in the (m, Λ) plane for $D_t = 0$, $N_\varphi = 6$ and $\sum_{i=1}^{N_\varphi} \Omega_\varphi^{(i)} h^2 = 0.106 \pm 0.008$ at the 3σ level for $m_h = 130$, 170, 210 GeV (as indicated above each panel).

$$S_L = \mathrm{diag}(s_1, s_2, s_3), \qquad |s_i| = 1. \tag{13}$$

Before discussing the explicit solutions for Y_ν, we first diagonalize (to leading order in M^{-1}) the neutrino mass matrix in terms of the light (n) and heavy (N) eigenstates:

$$\mathcal{L}_m = -(\bar{n} M_n n + \bar{N} M N/2) \tag{14}$$

with $M_n = \mu^* P_R + \mu P_L$, $\mu = -4 M_D M^{-1} M_D^T$, where n and N are related to ν_R and ν_L through $\nu_L = n_L + (M_D M^{-1}) N_L$ and $\nu_R = N_R - (M^{-1} M_D^T) n_R$.

The remaining condition in (11) allows ten (up to permutations of the basis vectors) inequivalent solutions for Y_ν. (The conditions (11) where also investigated in [11].) Of those, assuming single massless neutrino and the absence of $\varphi \to n_i n_j$ decays, only one is acceptable; it corresponds to $s_{1,2,3} = \epsilon$ [cf. (12)]. To compare our results with the data, we use the so-called tri-bimaximal [12] lepton mixing matrix that corresponds to $\theta_{13} = 0$, $\theta_{23} = \pi/4$ and $\theta_{12} = \arcsin(1/\sqrt{3})$. One can undo the diagonalization of light neutrino mass matrix and check against the one implied by Y_ν as a consequence of (11). We find that there are only two possible forms of Y_ν that are consistent with (11) and independent of M, and that agree with tri-bimaximal mixing:

$$Y_\nu = \begin{pmatrix} a & b & 0 \\ -a/2 & b & 0 \\ -a/2 & b & 0 \end{pmatrix}, \qquad \begin{aligned} m_1 &= -3v^2 a^2/M_1 \\ m_2 &= -6v^2 b^2/M_2 \\ m_3 &= 0 \end{aligned}$$

$$\text{and} \quad Y_\nu = \begin{pmatrix} a & b & 0 \\ a & -b/2 & 0 \\ a & -b/2 & 0 \end{pmatrix}, \qquad \begin{aligned} m_1 &= -3v^2 b^2/M_2 \\ m_2 &= -6v^2 a^2/M_1 \\ m_3 &= 0 \end{aligned} \tag{15}$$

where a and b are real (for simplicity) parameters. The resulting mass spectrum is consistent with the observed pattern of neutrino mass differences, see e.g., [13]. For this solution only N_3 and φ are odd under the Z_2 symmetry hence the φ will be absolutely stable if $m < M_3$.

It is noteworthy that the presence of Y_φ also leads to an additional contribution $-(\Lambda/\pi)^2 \mathrm{tr} Y_\varphi^2$ to δm^2 (we assumed Y_φ real for simplicity) so the neutrinos can be used to ameliorate the little hierarchy problem associated with m (for this however Y_φ cannot be too small) thereby "closing" the solution to the little hierarchy problem in a spirit similar to supersymmetry. This interesting scenario will be discussed elsewhere [5].

Conclusions.—We have shown that the addition of real scalar singlets φ_i to the SM may ameliorate the little hierarchy problem (by lifting the cutoff Λ to multi-TeV range) and also provide realistic candidates for DM. In the presence of right-handed neutrinos this scenario allows a light neutrino mass matrix texture that is consistent with experimental data while preserving all the successes of leptogenesis as an explanation for the baryon asymmetry.

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