Dynamics of Charged Events

Constantin Bachas,[1] Claudio Bunster,[3] and Marc Henneaux[2,3]

[1]*Laboratoire de Physique Théorique de l'Ecole Normale Supérieure, 24 rue Lhomond, 75231 Paris cedex, France**
[2]*Physique Théorique et Mathématique, Université Libre de Bruxelles and International Solvay Institutes,*
ULB Campus Plaine C.P. 231, 1050 Bruxelles, Belgium
[3]*Centro de Estudios Científicos (CECS), Avenida Arturo Prat 514, Valdivia, Chile*

In three spacetime dimensions the world volume of a magnetic source is a single point, an event. We make the event dynamical by regarding it as the imprint of a flux-carrying particle impinging from an extra dimension. This can be generalized to higher spacetime dimensions and to extended events. We exhibit universal observable consequences of the existence of events and argue that events are as important as particles or branes. We explain how events arise on the world volume of membranes in *M* theory, and in a Josephson junction in superconductivity.

The principle of electric-magnetic duality states that electric and magnetic fields must be treated on equal footing. It implies that one must consider both electric and magnetic sources in the Maxwell equations.

When the dimension of the electric source is increased, the dimension of the dual magnetic source decreases. In the extreme case of an electric object whose world volume has dimension $D - 2$, in D spacetime dimensions, the world volume of the corresponding magnetic object has dimension zero. The world volume is just a point. We shall say then that the magnetic source is an *event*. The simplest example of an event is the dual of an ordinary point electric charge in $D = 3$ spacetime dimensions. Endowing such an object with dynamical properties seems to be a contradiction in terms. Thus, so far, events have been considered as external sources [1,2]. This is unsatisfactory because it violates the principle of action and reaction: the source acts on the electromagnetic field, but the field does not react on the source. In order to have a closed physical system one must, therefore, assign dynamics to the source. A way to achieve this is the following: One considers the event as the imprint of a flux-carrying particle which comes in from an extra dimension, hits spacetime, and spreads its flux over it. It will be shown below that this idea can indeed be carried through successfully.

The idea uses two key ingredients: (i) The electromagnetic field must be localized on a $D = 3$ slice of a higher-dimensional spacetime, and in addition (ii) magnetic flux can escape in the extra dimensions where it is carried by pointlike particles. Two examples in which these ingredients are present include the membrane of *M* theory and S-I-S Josephson junctions. We will identify dynamical events in these familiar systems.

Solving in this way the problem of the dynamics of a point source in spacetime opens a treasure chest. A point event is the simplest member of a family which includes also *extended events*. An extended event is the imprint left on spacetime when a *p* brane with $p \geq 1$ hits it. Just as point events, extended events do not obey an equation of motion in the D-dimensional spacetime, because their dynamics takes place in the extra dimensions. This is the key difference between events and particles, or ordinary extended objects. For example, a closed one-dimensional event is a loop in the D-dimensional spacetime with arbitrary shape, it can be timelike, spacelike, null [3], or a combination of all three. It can also go forward and backwards in time with no violation of causality. The worldline is not a history. However, it still produces fields in the D-dimensional spacetime and interacts with them within a closed physical system.

It is the main point of this article to put forward the idea that events should play as central a role in physics as that played by particles or ordinary branes. It should be stressed here that events occur in Lorentzian spacetime and should be distinguished from instantons, i.e., solutions of the Euclidean equations that describe tunneling. This will become more clear at the end.

With no essential loss of generality we will restrict ourselves to the magnetic point events of $D = 3$ electrodynamics, which motivated the introduction of the concept. The generalization to extended events will be straightforward. A simple action is the sum of three terms,

$$I = -\frac{1}{2} \int d^3x \, {}^*F_\mu {}^*F^\mu - m \int_{-\infty}^0 d\tau \sqrt{-\dot{z}_M \dot{z}^M}$$
$$+ g \int d^3x \, \delta^{(3)}(x - \bar{z}) \varphi(x). \tag{1}$$

Here ${}^*F_\mu = \partial_\mu \varphi$ with φ the magnetic potential, and $z^M(\tau)$ is the worldline of the flux-carrying particle coming in from the past and absorbed at $\tau = 0$. The coordinates of the event are, in self-explanatory notation, $z^M(0) = (\bar{z}^\mu, 0)$. The third axis runs along the extra dimension and the signature is $(-, +, +, +)$. For a particle being emitted (an inverse event) one should reverse the limits of the τ integration as well as the sign of g.

The equation of motion for φ reads

$$\Box\varphi = g\,\delta^{(3)}(x - \bar{z}). \tag{2}$$

The general solution of (2) is the sum $\varphi = \varphi_0 + gG$, where φ_0 is the general solution of the homogeneous equation ($g = 0$) and G is a Green function of the wave operator. At the classical level a natural choice is to take for G the retarded Green function which vanishes outside the future light cone of the event,

$$G_R(x) = \frac{1}{2\pi}\frac{1}{\sqrt{-x_\mu x^\mu}}\theta(-x_\mu x^\mu)\theta(x^0), \tag{3}$$

where we have here set $\bar{z} = 0$. If one takes $\varphi_0 = 0$ the situation as seen from within the $D = 3$ spacetime is the following: For $x^0 < 0$ there is no field. At $x^0 = 0$, suddenly a flash of light emerges and propagates to the future [4]. This situation will be the most probable classically because it corresponds to a flux-carrying particle impinging from the extra dimension, without any precise control ("fine-tuning") of its initial conditions [5]. The time-reversed process, where one would replace the retarded Green function G_R by the advanced one G_A, corresponds to a precisely prepared ("fine tuned") pulse of radiation converging on the spacetime point $\bar{z} = 0$, and disappearing as a flux-carrying particle into the extra dimension. This situation would be classically improbable.

The action (1) is not fully satisfactory because it does not allow a proper treatment of energy-momentum balance. First, the energy of the event field (2) diverges. In order to make it finite one must smear the magnetic source over a finite region of the $D = 3$ spacetime. This smearing should be physically related to the approach and absorption of the flux-carrying particle at $z^3 = 0$. There appears to be no natural description of this process, consistent with Poincaré invariance, within the present simple effective model. We therefore prefer to leave the issue of energy-momentum conservation to an eventual underlying microscopic theory. An analogous difficulty actually arises when one tries to compute the mass of a magnetic pole within the Dirac theory.

A more satisfactory situation is encountered in connection with the conservation of magnetic flux and of angular momentum. The action (1) is invariant, up to a surface term, under the global gauge transformation $\varphi \to \varphi + c$, where c is a constant. The first two terms of (1) are obviously invariant, so one needs to analyze only the change in the third, coupling term. This is just $-g$ times the shift c. In order to apply Noether's theorem one rewrites this change as c times the integral over the extended spacetime of the divergence of a current, i.e., $\Delta I = c \int d^4x\, \partial_M J^M_{\text{particle}}$ where

$$J^M_{\text{particle}} = g\int_{-\infty}^{0} d\tau\,\delta^{(4)}(x - z(\tau))\frac{dz^M}{d\tau}. \tag{4}$$

It is easy to verify that $\partial_M J^M_{\text{particle}} = -g\,\delta^{(4)}(x - \bar{z})$, and $\int d^3x\, J^0_{\text{particle}} = g\theta(\bar{z}^0 - x^0)$ with θ the step function.

Furthermore, the contribution to the current from the Maxwell term is given by

$$J^M_{\text{field}} = (^*F^\mu, 0)\delta(x^3). \tag{5}$$

On account of the field equation (2) this obeys $\partial_M J^M_{\text{field}} = g\delta^{(4)}(x - \bar{z})$ and $\int d^3x\, J^0_{\text{field}} = g\theta(x^0 - \bar{z}^0)$. The divergences of the two currents cancel, and one obtains the flux conservation law $\partial_M(J^M_{\text{particle}} + J^M_{\text{field}}) = 0$.

In conclusion, the conserved charge due to the (global) gauge invariance of the action is the magnetic flux g, which is being transferred to the electromagnetic field by the impinging particle. Unlike the situation with energy and momentum, this deposited flux is finite. It constitutes the main signature of the event, independent of any microscopic details.

The most striking consequence of the existence of magnetic events is the quantization rule for the product of the electric charge e and of the event charge g,

$$eg = 2\pi\hbar n, \qquad n \text{ integer}. \tag{6}$$

For the magnetic pole in $3 + 1$ spacetime dimensions, this result was first obtained by Dirac in 1931, and rederived by him in 1948 from the quantum-mechanical implications of a classical point-particle action principle. Later on Dirac's 1948 derivation was proven to remain valid for extended objects [1]. The quantization rule (6) has survived the embedding of the effective point-particle theory in detailed microscopic models. Since its derivation is independent of whether the electric and magnetic sources obey equations of motion or not, it remains valid for events of any dimension.

The fact that magnetic events imply the quantization of electric charge is, conceptually, even more striking than the analogous implication of magnetic poles. Indeed, for example, in four-dimensional space time, one could have in the whole history of the Universe a single magnetic event, which could be a very small loop carrying a quantum g of magnetic flux. That small loop could lie entirely in the very far past (or, for that matter, in the very far future) and yet all electric charges, at all times, should have to be quantized according to (6). One could say that magnetic events are like the Cheshire cat, they vanish leaving only their smile behind.

There exists one more property of magnetic events which, like the conservation of magnetic flux and the quantization rule of electric charge, is simple and universal; i.e., it is expected to survive in any underlying microscopic theory. This is angular momentum. Consider an electric particle with worldline $y^\mu(\tau)$. The particle starts feeling the influence of the event after it enters its future light cone. The conserved angular momentum is the sum of the orbital angular momentum of the electric particle $L^{\mu\nu}$ and of the angular momentum stored in the electromagnetic field $J^{\mu\nu}_{\text{field}} = \int_\Sigma(T_\mu{}^\nu x^\rho - T_\mu{}^\rho x^\nu)d\Sigma^\mu$. The latter vanishes before the interaction, and it is given afterwards by

$$J_{\text{field}}^{\mu\nu} = eg/2\pi(y_\rho/\sqrt{-y^2})\varepsilon^{\mu\nu\rho}. \tag{7}$$

The electromagnetic field whose energy-momentum tensor enters in the expression for $J_{\text{field}}^{\mu\nu}$ is the sum of the retarded field (3) of the magnetic event, and of the field of the electric particle. The only contribution comes from the cross term. To evaluate the integral it is necessary to regularize the event field on the light cone. The result uses only the Gauss law and does not depend on the details of the regularization.

Since total angular momentum is conserved, the change in the orbital angular momentum of the particle is given by $\Delta L^{\mu\nu} = -J_{\text{field}}^{\mu\nu}$ [6]. The dual pseudovector $^*\Delta L^\mu = \frac{1}{2}\epsilon^{\mu\nu\rho}\Delta L_{\nu\rho}$ points from the event to the electric charge and, in view of the quantization rule (6), its magnitude is an *integer* multiple of $\hbar$,

$$|^*\Delta L| = eg/2\pi = n\hbar. \tag{8}$$

This result mirrors the well-known fact that, in $3 + 1$ dimensions, the angular momentum stored in the field of an electric and a magnetic charge is a multiple of $\hbar/2$.

As the last development in the context of the effective classical theory (1) of events, one may consider the addition of a Chern-Simons term to the Maxwell action. The modified field equations take the form

$$\partial_\mu F^{\mu\nu} + \kappa^* F_\nu = j_{\text{el}}^\nu \quad \text{and} \quad \partial_\mu{}^* F^\mu = j_{\text{mag}}, \tag{9}$$

where κ is the coefficient of the Chern-Simons term and we have here allowed for generic sources. Taking the divergence of the left-hand equation gives $\partial_\nu j_{\text{el}}^\nu = \kappa j_{\text{mag}}$. Hence, a magnetic event deposits automatically κg units of electric charge. Furthermore, since this latter is quantized, one concludes that κ must be an integer multiple of $2\pi\hbar/g^2$. It is worthwhile noting here that the presence of magnetic events in a Chern-Simons theory not only quantizes electric charge, but requires its existence.

Although in $2 + 1$ dimensions the quantization of the Chern-Simons term is automatic if the photon field comes from a spontaneously broken non-Abelian gauge theory [7], this property was first established, in terms of what we now call an event, in [2]. For higher dimensions the quantization of the coupling was derived in [8] (see also [9]) by using dimensional reduction and the Witten effect, namely, the acquisition of electric charge in the presence of a topological θ term. One arrives at the same conclusion without having to dimensionally reduce the theory, along lines that generalize the analysis in $2 + 1$ dimensions. The argument in $D = 2n + 1$ dimensions uses n extended events whose codimensions span the entire space.

So far we have dealt with universal properties of events as described by a simplified effective theory. Next we will exhibit two examples of their occurrence in instances where the underlying microscopic theory already exists.

The first example is the impinging of D particles on D2 branes in string theory. The Mawxell field lives on the world volume of the D2 brane, which is in turn embedded in a ten-dimensional spacetime [10]. Bulk D particles feel the gravitational attraction of the D2 brane, but this is negligibly weak at distances $\gg \ell_s$, the characteristic string length. However, whenever a D particle gets sufficiently close to the D2 brane, a tachyonic instability develops [11] and it gets rapidly absorbed. To an observer on the D2 brane the process looks precisely like a magnetic event: the particle deposits a flux $\int F = T_0/T_2$, which spreads at the speed of light. Here T_0 is the mass of the D particle and T_2 is the tension of the D2 brane. If one normalizes canonically the photon field the magnetic charge of the event is $g = T_0/\sqrt{T_2}$. The minimal electric charge, which corresponds to the end point of an open string, is given by $e = T_F/\sqrt{T_2}$, where $T_F \equiv \hbar/2\pi\ell_s^2$ is the fundamental-string tension. Using Polchinski's values for the tensions [10] one then finds $eg = 2\pi\hbar$. The capture of a D particle corresponds, indeed, to a magnetic event with the minimal allowed charge.

One may view the impinging D particle as a small spherical D2 brane carrying one unit of magnetic flux. Events and inverse events acquire then a geometric meaning, as the joining and splitting of membranes. It is furthermore possible to induce a Chern-Simons coupling by inserting one (or more) D8 branes between the D2 brane and the Minkowski vacuum [12]. When the D particle traverses a D8 brane an open string stretching between them is automatically created [13]. After the D particle has been dissolved, these open strings remain attached to the D2 brane. They are the electric charges deposited by the event. All universal features of events are, thus, neatly illustrated in this string theory example.

Our second example is taken from the theory of superconductivity. The setup is that of a S-I-S Josephson junction, consisting of a thin insulating layer sandwiched between two superconducting bulk pieces. The (history of the) insulating layer will be our $D = 3$ spacetime, while the adjacent (history of the) superconductor will be the extended spacetime. This setup was proposed in [14] as an example of "localization" of the Maxwell field. To see why, recall that the superconductor may be described to a first approximation as a macroscopic quantum fluid of Cooper pairs of charge $q = 2e$, where e is the charge of the electron. The Cooper pairs condense in the ground state, as expressed by the nonvanishing expectation value of a complex scalar field, $\langle\Psi\rangle = |\Psi_0|e^{i\omega}$. This condensate screens all electric charges and expels the magnetic field from the bulk. The longitudinal electric field and the normal magnetic field continue to vanish inside the insulating layer, while the remaining nonzero fields can be identified as those of the effective $D = 3$ electrodynamics,

$$^*F^\mu \equiv F^{3\mu} = \tfrac{1}{2}\epsilon^{\mu\nu\rho}\,^*F_{\nu\rho}. \tag{10}$$

Here math-style and text-style symbols denote the three- and four-dimensional fields. The latter are evaluated at the position ($x^3 = 0$) of the junction, which is sufficiently thin so as to neglect, in its interior, all field variations along the

x^3 direction. Notice that the identification (10) has exchanged the roles of "electric" and "magnetic."

The magnetic events of the $D = 3$ theory correspond to electric charges crossing the insulating layer. This follows from (10) and the Maxwell equation $\partial_\mu F^{3\mu} = -J^3$. Integrating over the junction gives, for an isolated system, the conserved global charge $\int d^2 x^* F^0 + Q_{\text{right}}$, where Q_{right} is the charge in the right-hand side of the junction. The magnetic flux g deposited by the event is therefore equal to the total electric charge transferred. The effective $D = 3$ theory has also electric pointlike charges if the superconductors are of type II. These charges are the end points of Abrikosov vortex tubes which spread their magnetic flux in the insulating layer. The quantum of flux, $\Phi_0 = h/q$, is the effective electric charge in three dimensions. The quantization condition (6) is thus valid, provided the charge carriers crossing the junction are Cooper pairs [15].

The most striking feature of a Josephson junction is the existence of a (Josephson) current at zero voltage drop [16]. This arises because of the quantum tunneling of Cooper pairs across the insulating layer. The effect is described mathematically by *instantons*, which are the solutions of Eq. (2) in imaginary time. The action of an instanton controls the strength of the Josephson current, $J^3 = J_{\text{max}} \sin(q\varphi/\hbar)$ with $J_{\text{max}} \sim e^{-I_{\text{inst}}}$, as well as the nonperturbatively small mass of φ in the effective three-dimensional theory.

Although closely related, instantons should be distinguished from real-time events. Which of the two is more relevant depends on the detailed physical context. The example of the D2 brane in string theory helps to illustrate clearly this point: an event describes the capture of a D particle, whereas instantons describe the tunneling of flux from a neighboring D brane. Thus, an observer living on a D2 brane in a D-particle gas will witness numerous real-time events, but there will be no tunnelings if there is no other D brane nearby.

We conclude by remarking that, even though superconductivity makes events tangible, the more striking implications of their existence might reside in astrophysics and cosmology. The ideas of this Letter cannot, however, be simply transposed to gravity. Nevertheless, the distinction between instantons and events acquires special meaning in cosmology: Was our Universe created out of nothing, by quantum tunneling, or was the big-bang an extended (and most dramatic) event [17]?

*Unité mixte de recherche (UMR 8549) du CNRS et de l'ENS, associée à l'Université Pierre et Marie Curie et aux fédérations de recherche FR684 et FR2687.

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