Time-Resolved Measurement of Landau-Zener Tunneling in Periodic Potentials

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We report time-resolved measurements of Landau-Zener tunneling of Bose-Einstein condensates in accelerated optical lattices, clearly resolving the steplike time dependence of the band populations. Using different experimental protocols we were able to measure the tunneling probability both in the adiabatic and in the diabatic bases of the system. We also experimentally determine the contribution of the momentum width of the Bose condensates to the temporal width of the tunneling steps and discuss the implications for measuring the jump time in the Landau-Zener problem.

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Tunneling is one of the most striking manifestations of quantum behavior and has been the subject of intense research in both fundamental and applied physics [1]. While tunneling *probabilities* can be calculated accurately and have an intuitive interpretation as statistical mean values of experimental outcomes, the concept of tunneling *time* and its computation are still the subject of debate even for simple systems [2,3]. The time it takes a quantum system to complete a tunneling event (which in the case of cross-barrier tunneling can be viewed as the time spent in a classically forbidden area) has been widely investigated and measured recently for electrons ionized by attosecond radiation [4]. It is related to the time required for a state to evolve to an orthogonal state, and an observation, i.e., a quantum mechanical projection on a particular basis, is required to distinguish one state from another [3]. The measured time depends both on the type of observation (e.g., a temporal modulation of the potential in the classically forbidden region [5]) and on the quantum mechanical basis used, as derived in [6] for Landau-Zener (LZ) tunneling [7,8], in which a quantum system tunnels across an energy gap at an avoided crossing of the system's energy levels. Similarly to the tunneling time in real space, the LZ tunneling time measures the duration of the quantum mechanical evolution (which plays an important role, e.g., in quantum control [9]). In a given quantum basis for the LZ Hamiltonian, Vitanov [6] defined the "jump time" required to evolve a state to an orthogonal one, following previous works [10,11]. The role of the different bases was also emphasized by Berry [12], who introduced a superadiabatic basis with a universal time evolution.

In this Letter we directly measure the dynamics of LZ tunneling. The tunneling process is frozen at different times by performing a projective quantum measurement on the states of a given basis. The jump time is then derived from the survival probability in the initial state as function of time [6]. In our experiments, backed up by numerical simulations, we use ultracold atoms forming a Bose-Einstein condensate (BEC) inside an optical lattice [13,14].

For cold atoms, LZ tunneling in optical lattices was used [15,16] for detecting deviations from an exponential decay law at short times. In contrast to these experiments, our BEC has an initial width in momentum space that is much smaller than $p_B = 2p_{\text{rec}} = 2\pi\hbar/d_L$, the width of the first Brillouin zone of a periodic potential with lattice constant d_L. This enables us to observe the full dynamics for single or multiple LZ crossings [17], the only limitation being the initial momentum width of the condensates and nonlinear effects. Our experiments are similar to recent studies of LZ transitions in a solid-state artificial atom [18], but the high level of control over the light-induced periodic potential also allowed us to measure the tunneling dynamics in different eigenbases (adiabatic and diabatic).

In our experiments we created BECs of 5×10^4 ^{87}Rb atoms inside an optical dipole trap (mean trap frequency around 80 Hz). A one-dimensional optical lattice created by two counterpropagating, linearly polarized Gaussian beams was then superposed on the BEC by ramping up the power in the lattice beams in 100 ms. The wavelength of the lattice beams was $\lambda = 842$ nm, leading to a sinusoidal potential with lattice constant $d_L = \lambda/2$. A small variable frequency offset between the two beams introduced through the acousto-optic modulators in the setup allowed us to accelerate the lattice in a controlled fashion.

The time-resolved measurement of LZ tunneling was done [see Fig. 1(a)] by first loading the BEC into the ground state energy band of an optical lattice of depth V_0. The lattice was then accelerated with acceleration a_{LZ} for a time t_{LZ} to a final velocity $v = a_{\text{LZ}} t_{\text{LZ}}$, resulting in a force F_{LZ} on the atoms in the lattice rest frame [19]. During t_{LZ} the quasimomentum of the BEC swept the Brillouin zone, and at multiples of half the Bloch time $T_B = 2\pi\hbar(M a_{\text{LZ}} d_L)^{-1}$ (where M is the atomic mass), i.e., at times $t = (n + 1/2)T_B$ $(n = 0, 1, 2, \ldots)$ when the sys-

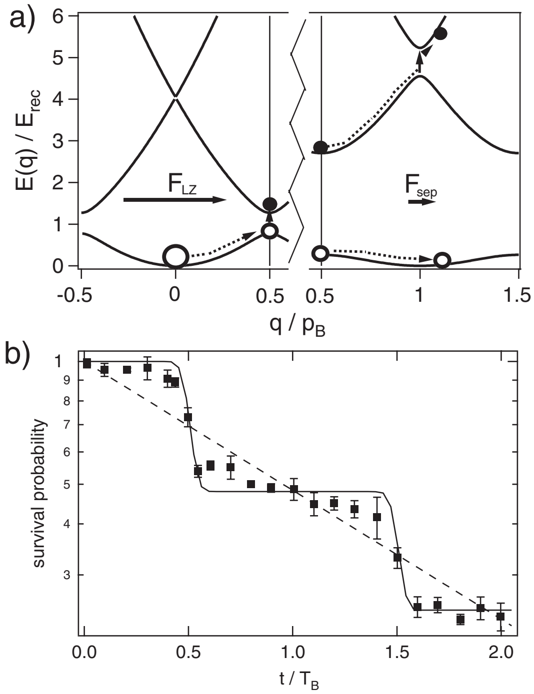

FIG. 1. Time-resolved measurement of LZ tunneling. (a) Experimental protocol [shown in the band-structure representation of energy $E(q)$ versus quasimomentum q]. Left: The lattice is accelerated, (partial) tunneling occurs. Right: The acceleration is then suddenly reduced and the lattice depth increased so as to freeze the instantaneous populations in the lowest two bands; finally, further acceleration is used to separate, and measure, these populations in momentum space. (b) Experimental results for $V_0 = 1E_{\rm rec}$ and $F_0 = 0.383$ ($a_{\rm LZ} = 13.52$ ms^{-2}), giving $T_B = 0.826$ ms. The solid and dashed lines are a numerical simulation of our experimental protocol and an exponential decay curve for our system's parameters, respectively.

tem was close to the Brillouin zone edge, tunneling to the upper band became increasingly likely. At $t = t_{\rm LZ}$ the acceleration was abruptly reduced to $a_{\rm sep} \ll a_{\rm LZ}$ and the lattice depth was increased to $V_{\rm sep}$ in a time $t_{\rm ramp} \ll T_B$. These values were chosen in such a way that at $t = t_{\rm LZ}$ the probability for LZ tunneling from the lowest to the first excited energy band dropped from between ≈ 0.1–0.9 (depending on the initial parameters chosen) to less than ≈ 0.01, while the tunneling probability from the first excited to the second excited band remained high at about 0.95. This meant that at $t = t_{\rm LZ}$ the tunneling process was effectively interrupted and for $t > t_{\rm LZ}$ the measured survival probability $P(t) = N_0/N_{\rm tot}$ (calculated from the number of atoms N_0 in the lowest band and the total number of atoms $N_{\rm tot}$) reflected the instantaneous value $P(t = t_{\rm LZ})$.

The lattice was then further accelerated for a time $t_{\rm sep}$ such that $a_{\rm sep}t_{\rm sep} \approx 2np_{\rm rec}/M$ (typically $n = 2$ or 3). In this way, atoms in the lowest band were accelerated to a final velocity $v \approx 2np_{\rm rec}/M$, while atoms that had tunneled to the first excited band before $t = t_{\rm LZ}$ tunneled to higher bands with a probability >0.95 and were, therefore, no longer accelerated. At $t_{\rm sep}$ the lattice and dipole trap beams were suddenly switched off and the expanded atomic cloud was imaged after 23 ms. In these time-of-flight images the two velocity classes 0 and $2np_{\rm rec}/M$ were well separated, from which N_0 and $N_{\rm tot}$ could be measured directly. Since the populations were "frozen" inside the energy bands of the lattice, which represent the adiabatic eigenstates of the system's Hamiltonian, this experiment effectively measured the time dependence of P_a in the *adiabatic* basis. A typical result is shown in Fig. 1(b). One clearly sees two "steps" at times $t = 0.5T_B$ and $t = 1.5T_B$, which correspond to the instants at which the atoms cross the Brillouin zone edges, where the lowest and first excited energy bands exhibit avoided crossings. For comparison, the result of a numerical simulation (integrating the linear Schrödinger equation for the experimental protocol) as well as an exponential decay as predicted by LZ theory are also shown.

The LZ tunneling probability can be calculated by considering a two-level system with the adiabatic Hamiltonian

$$H_a = H_d + V = \alpha t \sigma_z + \frac{\Delta E}{2}\sigma_x, \quad (1)$$

where σ_i are the Pauli matrices. The eigenstates of the diabatic Hamiltonian H_d, whose eigenenergies vary linearly in time, are mixed by the potential V characterized by the energy gap ΔE. Applying the Zener model [8] to our case of a BEC crossing the Brillouin zone edge leads to a band gap $\Delta E = V_0/2$ and to $\alpha = 2v_{\rm rec}Ma_{\rm LZ} = 2F_0E_{\rm rec}^2/(\pi\hbar)$, with $E_{\rm rec} = \hbar^2\pi^2/(2Md_L^2)$ the recoil energy and $F_0 = Ma_{\rm LZ}d_L/E_{\rm rec}$ the dimensionless force. The limiting value of the adiabatic and diabatic LZ survival probabilities (for t going from $-\infty$ to $+\infty$) in the eigenstates of H_a and H_d, respectively, is

$$P_a(t \to +\infty) = 1 - P_d(t \to +\infty) = 1 - P_{\rm LZ}, \quad (2)$$

where the standard LZ tunneling probability is

$$P_{\rm LZ} = e^{-\pi/\gamma} \quad (3)$$

with the adiabaticity parameter $\gamma = 4\hbar\alpha(\Delta E)^{-2}$ [20].

Figure 2(a) shows the first LZ tunneling step for different lattice depths V_0, measured in units of $E_{\rm rec}$ at a given acceleration. The steps can be well fitted with a sigmoid function

$$P_a(t) = 1 - \frac{h}{1 + \exp[(t_0 - t)/\Delta t_{\rm LZ}]}, \quad (4)$$

where t_0 is the position of the step (which can deviate slightly from the expected value of $0.5T_B$, e.g., due to a nonzero initial momentum of the condensate), h is the step height, and $\Delta t_{\rm LZ}$ represents the width of the step. Equations (2) and (3) correctly predict the height h of the step, as tested in the experiment for a variety of values of V_0 and F_0 [see Fig. 2(b)].

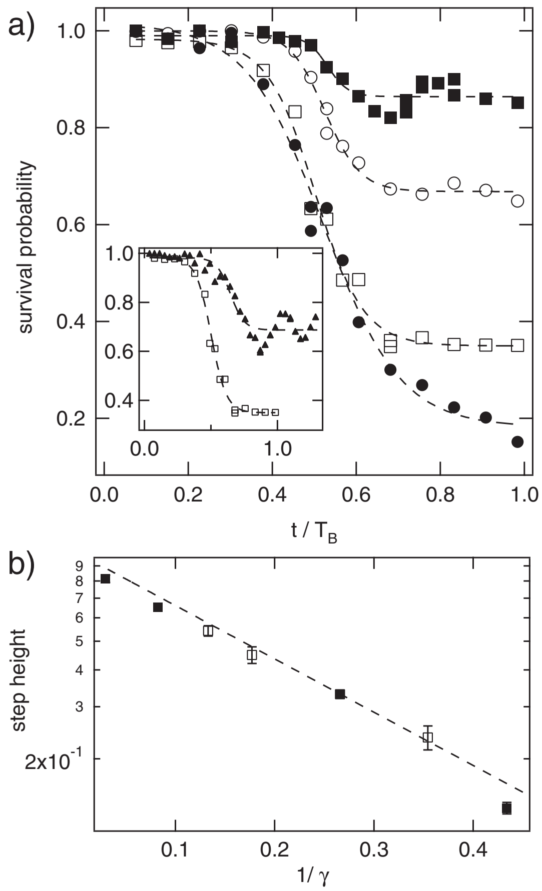

FIG. 2. (a) LZ survival probability in the adiabatic basis for a fixed force $F_0 = 1.197$ ($a_{\mathrm{LZ}} = 42.25\ \mathrm{ms}^{-2}$) and different lattice depths (filled squares: $V_0 = 2.3E_{\mathrm{rec}}$; open circles: $V_0 = 1.8E_{\mathrm{rec}}$; open squares: $V_0 = 1E_{\mathrm{rec}}$; filled circles: $V_0 = 0.6E_{\mathrm{rec}}$). The dashed lines are sigmoid fits to the experimental data. Inset: Survival probability in both the adiabatic (open squares) and diabatic (filled triangles) bases for $V_0 = 1E_{\mathrm{rec}}$ and $F_0 = 1.197$. (b) Step height h as a function of the inverse adiabaticity parameter $1/\gamma$ for varying lattice depth and $F_0 = 1.197$ (open symbols), and for varying force with fixed $V_0 = 1.8E_{\mathrm{rec}}$ (filled symbols). The dashed line is the prediction of Eq. (3) for the LZ tunneling probability.

While the experimental protocol described above measures the LZ tunneling probability in the *adiabatic basis*, it is possible to make analogous measurements in the *diabatic basis* of the unperturbed free-particle wave functions (plane waves with a quadratic energy-momentum dispersion relation) by abruptly switching off the lattice and the dipole trap after the first acceleration step (with the BEC initially prepared in the adiabatic basis, which, far away from the band gap, is almost equal to the diabatic basis). In this case, after a time-of-flight the number of atoms in the $v = 0$ and $v = 2p_{\mathrm{rec}}/M$ velocity classes are measured and from these the survival probability in the $v = 0$ velocity class is calculated. The inset of Fig. 2(a) (filled triangles) shows such a measurement. Again, a step around $t = 0.5T_B$ is clearly seen, as well as strong oscillations for $t > 0.5T_B$. While weaker oscillations are also seen in the adiabatic basis [see Fig. 2(a) with $V_0 = 2.3E_{\mathrm{rec}}$], they are much stronger and visible for a wider range of parameters in the diabatic basis [6]. These oscillations, also known as the Stokes phenomenon, are due to the discrepancy between the diabatic basis in which we measure the tunneling event and the ideal superadiabatic basis in which they are absent and the tunneling time is minimized [12]. They were also predicted for LZ tunneling in atomic Rydberg states [21] and experimentally observed in a wave-optical two-level system [22].

The widths Δt_{LZ} of the steps shown in Fig. 2(a) reflect the "jump time" for LZ tunneling $\Delta t_{\mathrm{LZ}} = \Delta v_{\mathrm{LZ}}/a_{\mathrm{LZ}}$ during which the probability of finding the atoms in the lowest energy band goes from $P_a(t = 0) = 1$ to its asymptotic LZ value $1 - P_{\mathrm{LZ}}$. Vitanov [6] defines the jump time in the adiabatic basis as

$$\tau_a^{\mathrm{jump}} = \frac{P_a(t = +\infty)}{P_a'(t = t_0)}, \tag{5}$$

where $P_a'(t = t_0)$ denotes the time derivative of the tunneling probability $P_a(t)$ evaluated at the crossing point of H_a. A sigmoidal function for $P_a(t)$ leads to $\tau_a^{\mathrm{jump}} = 4\Delta t_{\mathrm{LZ}}$. For large values of γ, which is the regime of our experiments, the theoretical jump time is given by

$$\tau_a^{\mathrm{jump}} \approx T_B \frac{\Delta E}{4E_{\mathrm{rec}}}. \tag{6}$$

This time, which coincides with the LZ traversal time of [10], is taken by the force to transfer the barrier energy to the system. It increases with ΔE and decreases with F_0.

From our sigmoidal fits we retrieve $\tau_a^{\mathrm{jump}}/T_B \approx 0.15$–$0.35$ (corresponding to absolute jump times between 50 and 200 μs), whereas the theoretical values for our experimental parameters are in the region of 0.1–0.15. This discrepancy is due to the fact that in our experiment the condensate does not occupy one single quasimomentum but is represented by a momentum distribution of width $\Delta p/p_B \gtrsim 0.1$ due to the finite number of lattice sites (around 50) it occupies and the effects of atom-atom interactions.

In order to test the dependence of Δt_{LZ} on Δp we created initial distributions of different widths using a dynamical instability [23]. The condensate was loaded into a lattice moving at a finite velocity corresponding to quasimomentum $q = -0.3p_B$ and held there for up to 3 ms. During this time the dynamical instability associated with the negative effective mass at that q led to an increase in Δp. After this preparatory stage, the LZ dynamics was measured as described above and Δt_{LZ} was extracted [see Fig. 3(a)]. As expected, Δt_{LZ} increases with Δp [Fig. 3(b)]. This was confirmed by a numerical integration of the Schrödinger equation in which Δp was varied by changing the initial trap frequency. The simulation also showed that

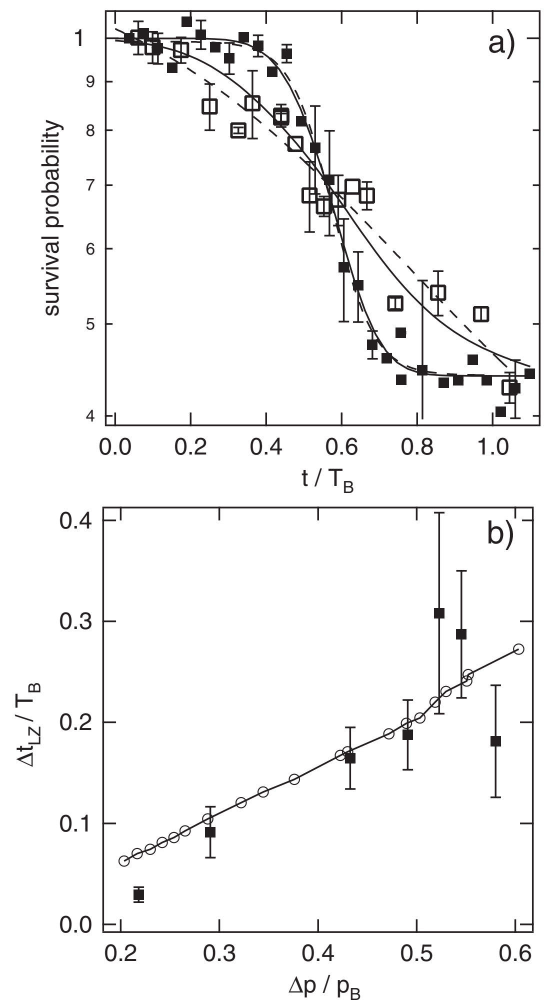

FIG. 3. LZ transition for different momentum widths of the condensate. (a) Survival probability for $\Delta p/p_B = 0.2$ (filled squares) and $\Delta p/p_B = 0.6$ (open squares). The solid and dashed lines are the results of a numerical simulation and of a sigmoid fit, respectively. (b) Step width $\Delta t_{\rm LZ}$ as a function of Δp. The open symbols (connected by a solid line for clarity) are the results of a numerical simulation.

for $\Delta p \rightarrow 0$, $\Delta t_{\rm LZ}$ remains finite and in that limit directly reflects the jump time given by Eq. (6).

In summary, we have measured the LZ dynamics of matter waves in an accelerated optical lattice in the adiabatic and diabatic bases. In both bases the steplike behavior as well as oscillations of the survival probability were clearly seen and agree with theoretical predictions. In future investigations one could reduce the initial momentum width, which currently limits the resolution of our experiment, by using, e.g., appropriate trap geometries or by controlling the nonlinearity through Feshbach resonances. This would enable a comparison with theoretical results related to the minimum time for a single LZ crossing limited by fundamental quantum (or wave, see [22]) mechanical properties [24]. Also, clearer observations of the short-time oscillations as seen in Fig. 2(a) should be possible in this way. Our method can also be used to study multiple LZ crossings, e.g., in order to observe Stückelberg oscillations.

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