Resonant Pair Tunneling in Double Quantum Dots

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We present exact results on the nonequilibrium current fluctuations for 2 quantum dots in series throughout a crossover from non-Fermi liquid to Fermi liquid behavior described by the 2 impurity Kondo model. The result corresponds to resonant tunneling of carriers of charge $2e$ for a critical interimpurity coupling. At low energy scales, the result can be understood from a Fermi liquid approach that we develop and use to also study nonequilibrium transport in an alternative double dot realization of the 2 impurity Kondo model under current experimental study.

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Introduction.—Measurements of nonequilibrium shot noise in current fluctuations in electronic devices [1] has became a practical tool to probe strongly correlated systems with elementary excitations whose charge, e^*, possibly differs from the electron charge e, the prominent examples being the observation of the Cooper-pair charge $e^* = 2e$ in normal metal-superconductor junctions [2], or fractional charges in quantum Hall samples [3]. Remarkably, many-body physics with unusual emergent excitations arises even when the interactions occur only at a single point, e.g., in an impurity in a metal, or in a nanoscale quantum dot (QD) connecting few leads. Theoretical studies of various quantum impurity problems encountered the notion of non-Fermi liquid (NFL) behavior, with inelastic scattering of an incoming electron into multiple particle and hole states even at zero temperature T [4]. It is of general interest to study shot noise in QD systems showing such elusive NFL behavior.

Nontrivial effective charges emerge even for quantum impurity problems showing regular Fermi liquid (FL) behavior. For example in the basic single impurity Kondo model, realized by a single QD coupled to leads, studies of shot noise [5] lead to a prediction of a universal fractional charge [6] $e^* = 5e/3$ in the low temperature regime which was detected experimentally [7], reflecting a combination of single electron and two-electron backscattering. A generalization for a variety of Kondo models was achieved recently [8]. The crossover which is typically addressed in experiment [9] is very rarely understood theoretically. In this Letter we find that a simple and yet unusual "noninteractinglike" picture for transport of particles with effective charge $e^* = 2e$ emerges along an entire crossover from NFL to FL behavior occurring in double QDs in series [10–12] exhibiting the physics of the 2-impurity Kondo model (2IKM).

The simplest 2IKM consists of two impurity spins (S_L, S_R), coupled to two channels of conduction electrons and interacting with each other through an exchange interaction K; see Fig. 1. After the standard "unfolding transformation" [13], reducing the two spin-1/2 channels to four chiral Dirac fermions, $\psi_{i\alpha}(x)$, $i = 1, 2 = L, R$, $\alpha = \uparrow, \downarrow$, $x \in \{-\infty, \infty\}$, the Hamiltonian becomes $H = H_0 + H_K$ where $H_0 = \sum_{j,\alpha} \int dx \psi^\dagger_{j\alpha} i\partial_x \psi_{j\alpha}$ and

$$H_K = J_L(\psi^\dagger_L \vec{\sigma} \psi_L) \cdot \vec{S}_L + J_R(\psi^\dagger_R \vec{\sigma} \psi_R) \cdot \vec{S}_R + K\vec{S}_L \cdot \vec{S}_R. \tag{1}$$

$\vec{\sigma}(\vec{\tau})$ is a vector of Pauli matrices acting in spin (channel) space. For this model a NFL quantum critical point (QCP) was found at $K = K_c \sim T_K$ [14] separating a local singlet FL phase at $K > K_c$ from a Kondo screened FL phase at $K < K_c$ (see Fig. 1). However, more realistic models containing interchannel tunneling,

$$H_{PS} = V_{LR}\psi^\dagger_L \psi_R + \text{H.c.}, \tag{2}$$

where H.c. stands for Hermitian conjugate, [or, $H_{PS} = \text{Re}V_{LR}(\psi^\dagger \tau^1 \psi) - \text{Im}V_{LR}(\psi^\dagger \tau^2 \psi)$], with implicit sum over spin and channel indices, do not show a critical point [15,16]. The reason for this is that Eq. (2) results in a relevant perturbation with dimension 1/2 at the QCP [17], leading to an energy scale $T^* \propto T_K|\nu V_{LR}|^2 + (K - K_c)^2/T_K$ which is finite even at $K = K_c$, below which an effective FL theory takes over. Here ν is the density of

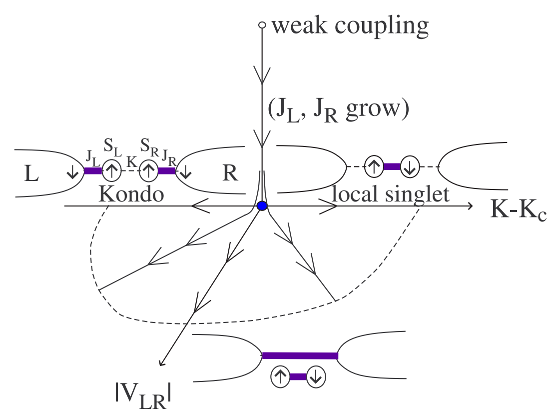

FIG. 1 (color online). Phase diagram: as temperature is reduced the system flows from the weak coupling limit ($T \gg T_K$, empty dot) to the vicinity of the QCP ($T^* \ll T \ll T_K$, filled blue dot) and finally to the FL line of fixed points ($T \ll T^*$, dashed line).

states of the conduction electrons. This crossover from NFL to FL behavior is reflected in the conductance of double QDs [10–12,18], and in particular geometries, e.g., the series geometry, we were able to calculate it exactly [12]. However this information is not sufficient to uncover the nature of the transport.

In this Letter we study the full counting statistics [19,20] (FCS) for charge transfer through a series double QD along the full NFL to FL crossover. In general, charge is transferred in units of e or $2e$. We note that this feature occurs also along the Kondo crossover in a single QD in the unrealistic Toulouse limit [21], as was first realized in the case of finite magnetic field where it can become energetically favorable to tunnel electron pairs through the impurity rather than single electrons [22]. A peculiar situation occurs in the double QD at $K \to K_c$, where $2e$ becomes the basic unit of charge along the full crossover. This striking behavior is not captured in a slave-boson mean field calculation [23].

We also derive a local Fermi liquid Hamiltonian governing the physics below T^*. Using an enhanced understanding of this crossover we go beyond previous works [24–26] in determining all coupling constants in this effective Hamiltonian and obtain a universal theory depending only on an energy scale, T^*, similar to Nozières FL theory (FLT) for the single impurity problem [27], and on the new FL boundary condition associated with the ratio $|K - K_c|/(\nu|V_{LR}|T_K)$. This approach helps us to understand the charge $2e$ carriers.

In the geometry proposed by Zaránd *et al.* [18], where transport proceeds between two leads connected via one QD side coupled to a second QD coupled to another lead, exact results on the crossover are not available. Nonetheless, we use our FLT to calculate universal nonequilibrium transport and noise properties at low energies when the NFL critical behavior is destabilized by a nonzero $K - K_c$. Our predictions can be probed experimentally [28].

Full counting statistics.—We will obtain the full charge transfer distribution in a series double QD tuned to the 2IKM regime, along the crossover from NFL to FL behavior, using the formulation of Ref. [12].

In terms of Abelian bosonization one can write the original free fermion theory with $H_K \to 0$ and $H_{PS} \to 0$ in terms of 8 chiral Majorana fermions χ_j^A, $\chi_1^A = \frac{\psi_A^\dagger + \psi_A}{\sqrt{2}}$, $\chi_2^A = \frac{\psi_A^\dagger - \psi_A}{\sqrt{2}i}$, associated with the real ($j = 1$) and imaginary ($j = 2$) parts of the charge, spin, flavor and spin-flavor fermions ($A = c, s, f, X$); for a definition of these fermions, see Ref. [12]. Then the free Hamiltonian is $H_0[\{\chi'\}] = \frac{i}{2}\sum_{j=1}^{8}\int dx \chi'_j \partial_x \chi'_j$, where $\{\chi'\} = \{\chi_2^X, \chi_1^f, \chi_2^f, \chi_1^X, \chi_1^c, \chi_2^c, \chi_1^s, \chi_2^s\}$. The Fermi operator $\psi_f^\dagger(\psi_f)$ increases (decreases) $Y = (N_L - N_R)/2$ by 1, N_i being the total fermion number in lead $i = L, R$.

Turning on H_K, the QCP is obtained at $K = K_c$ from the free case by a change in boundary condition occurring only for the first Majorana fermion, $\chi_1(0^-) = -\chi_1(0^+)$. For energies $\ll T_K$, the leading terms in the Hamiltonian describing deviations $K - K_c$ as well as finite V_{LR} can be written [12] in a new basis $\{\chi\}$, where $\chi_1(x) = \chi_1'(x)\mathrm{sgn}(x)$ and $\chi_i = \chi_i'$, ($i = 2, \ldots, 8$), as $H_{\mathrm{QCP}} = H_0[\{\chi\}] + \delta H_{\mathrm{QCP}}$ where [29]

$$\delta H_{\mathrm{QCP}} = i\sum_{j=1}^{2}\lambda_j\chi_j(0)a. \qquad (3)$$

Here a is a local Majorana fermion, $a^2 = 1/2$, and

$$\lambda_1 = c_1\frac{K - K_c}{\sqrt{T_K}}, \qquad \lambda_2 = c_2\sqrt{T_K}|\nu V_{LR}|, \qquad (4)$$

where c_1 and c_2 are constant factors of order 1. Those couplings determine two energy scales λ_1^2, λ_2^2, and the total crossover scale is $\lambda^2 = \lambda_1^2 + \lambda_2^2 = T^*$. The operators in δH_{QCP} have scaling dimension 1/2; hence, they destabilize the QCP; below the crossover scale T^* the system flows to FL fixed points whose nature depend on the ratio λ_1/λ_2.

By definition the FCS is obtained from the cumulant generating function $\chi(\mu)$ for the probability distribution function $P(Q)$ to transfer Q units of charge during the waiting time $\mathcal{T}$ (which is sent to infinity), $\chi(\mu) = \sum_Q e^{iQ\mu}P(Q)$. The cumulants $\langle\delta^n Q\rangle$ can be found from $\langle\delta^n Q\rangle = (-i)^n \frac{\partial^n}{\partial\mu^n}\ln\chi(\mu)|_{\mu=0}$. In fact, due to a formal equivalence of our nonequilibrium formulation and that of Schiller and Hershfield [22] for a single QD tuned to the Toulouse limit, we can borrow directly the results of Komnik and Gogolin for the FCS for that model [21]; translating between the parameters of the two models in the limit $T^* \ll T_K$, we obtain

$$\frac{\ln\chi(\mu)}{\mathcal{T}} = \int_{-\infty}^{\infty}\frac{d\epsilon}{4\pi}\ln\Bigg[1 + \sum_{n=-2}^{2}A_n(\epsilon)(e^{i\mu n} - 1)\Bigg]. \qquad (5)$$

Here $A_1(\epsilon) = \frac{2\lambda_1^2\lambda_2^2}{4\epsilon^2+\lambda^4}[n_F(1 - n_L) + n_R(1 - n_F)]$, $A_2(\epsilon) = \frac{\lambda_2^4}{4\epsilon^2+\lambda^4}n_L(1 - n_R)$, $A_{-n} = A_n|_{L\leftrightarrow R}$, $n_F = (1 + e^{\epsilon/T})^{-1}$, $n_{L,R} = n_F(\epsilon \mp eV)$, and V is the source-drain voltage. The presence of one particle as well as two-particle transport processes in our model is apparent from the μ dependence of the two terms $\propto(e^{\pm i\mu} - 1)$ and $\propto(e^{\pm 2i\mu} - 1)$ in Eq. (5), respectively. At $K = K_c$, giving $A_1 = A_{-1} = 0$, Eq. (5) is equivalent to the formula for the FCS of spinless noninteracting fermions of charge $2e$ transmitted though a resonant level of width $\sim T^*$, namely, the noninteracting formula [19,21] is obtained from Eq. (5) by the replacement $2\mu \to \mu$, $n_{L,R} \to n_F(\epsilon \mp eV/2)$ and adding an overall factor of 2.

Intuitive picture.—Unfortunately, in NFLs a simple picture in terms of the original electrons is absent. This makes the interpretation of the charge $2e$ in our problem in terms of pairing of the bare electrons incorrect in general, except in the FL regime, as discussed below.

We may think of the impurity spins as having 2 degenerate states when $K = K_c$: singlet and triplet. The impurity entropy of $S = (1/2)\ln 2$ [30] implies that this degeneracy is only partially lifted by the Kondo interactions with electrons in the leads. We may roughly represent these 2 states by an effective $S = 1/2$ spin (to avoid the formal notion of a Majorana fermion).

We consider two different transport processes: (i) single *electron* tunneling with amplitude V_{LR}, Eq. (2), which is not sensitive to the effective spin, and leads to a constant and small current $\sim(e^2/h)(\nu V_{LR})^2 V$; (ii) charge tunneling accompanied with *flipping the effective spin*. This is indeed Eq. (3) with $j = 2$, $\delta H_{\rm QCP}|_{K=K_c} = \frac{i}{\sqrt{2}}\lambda_2(\psi_f^\dagger + \psi_f)a$, which is formally generated from the basic tunneling Eq. (2) to first order in V_{LR}. The operator a flips the effective spin. Most importantly, this spin flip tunneling (SFT) intimately influences the QCP, since it eventually removes the entropy associated with it, as represented by the $|V_{LR}|$ axis in Fig. 1. Therefore, the essential features of the current close to the QCP arise from the SFT. The charge tunneling operator in the SFT is the collective flavor excitation $\chi_1^f = (\psi_f^\dagger + \psi_f)/\sqrt{2}$, where $\psi_f^\dagger(\psi_f)$ moves charge $e(-e)$ from right to left lead.

The doubling of charge may be understood by noting the analogy between the SFT operator $\delta H_{\rm QCP}|_{K=K_c}$ and a noninteracting model where a resonant level d is coupled to the leads $\delta H = \lambda'(\psi_L^\dagger + \psi_R^\dagger)d + {\rm H.c.}$ [22]. The two models are essentially identical. The difference is that the fermions appearing in δH change $Y[= (N_L - N_R)/2]$ by $\pm 1/2$ (1 electron tunnels onto or off of the resonant level), while the fermions appearing in $\delta H_{\rm QCP}|_{K=K_c}$ change Y by ± 1 (charge e tunneling). Since the noninteracting resonant level model results in current fluctuations and FCS with basic charge e, this analogy explains the origin of resonant tunneling of charge $2e$ in terms of resonant scattering of flavor collective "particle" excitations by the effective spin at the QCP.

Shot noise and effective charge.—The two-particle processes can be probed by looking simultaneously at the current $I = e\langle\delta^1 Q\rangle/\mathcal{T}$ and noise $S = 2e^2\langle\delta^2 Q\rangle/\mathcal{T}$. Equation (5) gives the $T = V = 0$ conductance $G = \frac{dI}{dV} = g_0 t$ where $t = \lambda_2^2/\lambda^2 = \frac{|\nu V_{LR}|^2}{|\nu V_{LR}|^2 + (c_1/c_2)^2(\frac{K-K_c}{T_K})^2}$ and $g_0 = 2e^2/h$ (or setting $\hbar = 1$ as in the rest of the Letter, $g_0 = e^2/\pi$). Using Eq. (5), in Fig. 2 we plot shot noise $S(V)$ along the crossover from FL ($eV \ll T^*$) to NFL ($eV \gg T^*$) regimes for various values of $K - K_c$ (determined by t). Experiments [7] extract effective charges by fitting shot noise measurements with the formula

$$S_{\rm fit} = 2e^* g_0 \int_0^V t(V')[1 - t(V')]dV', \tag{6}$$

where $t(V)$ is extracted from nonlinear conductance measurements $t(V) = \frac{1}{g_0}\frac{dI}{dV}$, $t(0) = t$, and e^* is the effective charge. For $K \neq K_c$ ($t < 1$), $S_{\rm fit}$ with $e^* = e$ gives a good fit for sufficiently small V; see inset of Fig. 2. When $|K - K_c| \gg T_K|\nu V_{LR}|$ ($t \ll 1$), this fit with $e^* = e$ becomes reasonably good along the full crossover; see curve with $t = 0.15$ in Fig. 2. For $K \to K_c$ ($t \to 1$) the fit with $e^* = e$

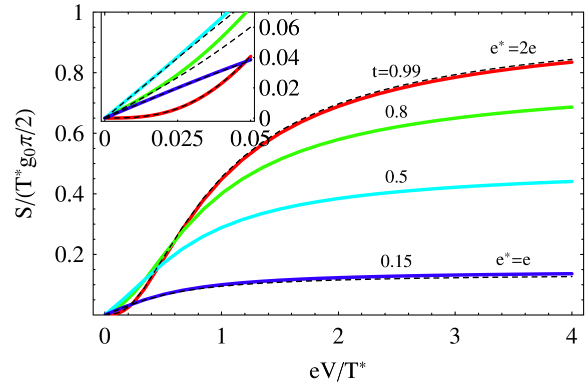

FIG. 2 (color online). Shot noise versus voltage for several values of $K - K_c$ (determined by t) at $T = 0$. Data are fitted in dashed lines using Eq. (6). The inset blows up the FL region, with fits of $e^* = e$ for $t = 0.15$, 0.5, 0.8 and $e^* = 2e$ for $t = 0.99$ ($K \to K_c$, red curve).

works only for an extremely small range $eV \ll \sqrt{T^*/T_K}|K - K_c| \to 0$, and, remarkably, the *full* curve fits with $S_{\rm fit}$ with $e^* = 2e$; see curve with $t = 0.99$.

Fermi liquid theory.—The line of FL fixed points (dashed line in Fig. 1) should have a simpler effective interacting theory written essentially in terms of the original fermions $\psi_{j\alpha}$. The derivation will be published elsewhere; the result is $H_0^{\rm FL} = \int dx \Psi^\dagger i\partial_x \Psi$ (indices summed), and

$$\delta H_{\rm FL} = \frac{\cos^2(2\delta)\mathcal{O}_{11} + \sin^2(2\delta)\mathcal{O}_{22} + \sin(4\delta)\mathcal{O}_{12}}{2T^*}\Big|_{x=0},$$
$$\mathcal{O}_{11} = \frac{16}{3}(\vec{J}_L^2 + \vec{J}_R^2) - 4(\vec{J}_L + \vec{J}_R)^2,$$
$$\mathcal{O}_{22} = (J_L - J_R)^2 - :\Psi_{L\alpha}^\dagger \epsilon_{\alpha\beta}\Psi_{L\beta}^\dagger \Psi_{R\gamma}\epsilon_{\gamma\delta}\Psi_{R\delta} + {\rm H.c.}:,$$
$$\mathcal{O}_{12} = i\sum_{j=L,R} :\Psi_{L\alpha}^\dagger \epsilon_{\alpha\beta}\Psi_{R\beta}^\dagger \Psi_{j\gamma}\epsilon_{\gamma\delta}\Psi_{j\delta}: + {\rm H.c.}, \tag{7}$$

where $J_j = :\Psi_j^\dagger \Psi_j:$, $\vec{J}_j = \Psi_j^\dagger \frac{\vec{\sigma}}{2}\Psi_j$ ($j = L, R$) and $\epsilon_{\alpha\beta}$ is the antisymmetric tensor. Here $\Psi_{j\alpha}$ are single particle scattering states incoming from lead (channel) j with spin $\alpha = \pm 1$ ($x > 0$ corresponds to the incoming part in our left moving convention),

$$\Psi_{j\alpha}(x) = \theta(x)\psi_{j\alpha}(x) + \sum_{j'}\theta(-x)s_{jj'}[\alpha]\psi_{j'\alpha}(x), \tag{8}$$

with $s[\alpha]_{jj'} = \cos(2\delta)\delta_{jj'} - i\alpha\sin(2\delta)(\tau^1)_{jj'}$. We also give a formula for the FL phase shift,

$$2\delta = \arg(\lambda_1 + i\lambda_2). \tag{9}$$

This universal FL Hamiltonian follows from a large symmetry emerging close to the QCP [17], and is valid for energies $\ll T^* \ll T_K$, $|\nu V_{LR}| \ll 1$, and $|K - K_c| \ll T_K$.

The emergence of the basic transport charge $2e$ for $K = K_c$ ($\delta = \pi/4$) in the series geometry follows at low energies $\ll T^*$ because in this case $\delta H_{\rm FL}$ in Eq. (7) is dominated by O_{22}; this operator produces scattering of two Ψ_L

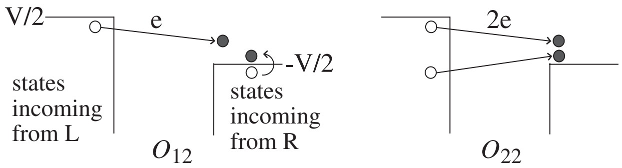

FIG. 3. Transport processes along the line of FL fixed points in Fig. 1. At $K = K_c$ the $2e$ backscattering dominates.

fermions to two Ψ_R fermions and vice versa. Since at $\delta = \pi/4$ the transmission is $t = 1$, we refer to this as an inelastic $2e$ *backscattering*; see Fig. 3. Away from $K = K_c$ the operator O_{12} in $\delta H_{\rm FL}$ adds also single electron processes.

To calculate transport properties for the Zaránd *et al.* double QD geometry [18] which is under current experimental study [28], it is necessary to calculate the single particle Green's function in the 2IKM. Because the electron field cannot be expressed in terms of the Majorana fermions, we have not been able to calculate this throughout the crossover, results being necessarily restricted to the vicinity of the NFL or FL critical points corresponding to an interimpurity singlet or Kondo screened behavior. Here we consider this system at K slightly different than K_c in the FL regimes, T, $eV \ll T^*$, where our FLT can be applied, ignoring particle-hole breaking ($V_{LR} = 0$). The conductance of this system can be expanded as $G_{\rm singlet} = g_0[\sin^2\delta_1 + \beta_1\{(T/T^*)^2 + \alpha(eV/T^*)^2\}]$, and $G_{\rm screened} = g_0[\cos^2\delta_1 - \gamma_1\{(T/T^*)^2 + \alpha'(eV/T^*)^2\}]$, where δ_1 is a small phase shift associated with marginal potential scattering operators. We assume parity symmetry of the device. Using Eq. (7), after a lengthy but straightforward calculation, we determine universal relations: $\beta_1 = \gamma_1 = O(1)$, $\alpha = \alpha' = 9/10\pi^2$. We also calculate the shot noise in the FL regime, and define effective charges $(e^*/e) = S/2I$ for the local singlet FL regime ($K > K_c$), and $(e^*/e)' = S/2(g_0V - I)$ in the Kondo screened phase ($K < K_c$), defined in the limit $\delta_1 \to 0$. Using Eq. (7) we obtain $(e^*/e) = (e^*/e)' = 11/9$. Our predictions should be contrasted with the measurements on single QDs with $\alpha = 3/2\pi^2$ [31] and $e^*/e = 5/3$ [7].

Recently it was shown [32] that the rate of entanglement production at a point contact is determined by the FCS. Our results suggest the exciting possibility of experimentally measuring entanglement near a NFL critical point.

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